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PHASE TRANSITIONS IN GLOBALLY COUPLED MAPS

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ABSTRACT

Transition to intermittency is investigated in globally coupled maps. The distribution of laminar phases for the various transition regions obey –3/2 power law decay. The difference of the emerging two clusters at the various transition regions show features of on-off intermittency. By applying noise to the system at the transition regions, the small cluster attractor reduces to a large number of clusters. The differences of the clusters also show features of on-off intermittency.

INTRODUCTION

The transition routes to chaos in low-dimensional non-linear dynamical systems have been well understood. One of them, the intermittency route was classified into three types by Pomeau and Manneville [1]. The essential feature of intermittency is that a simple periodic orbit is replaced by a chaotic attractor, where the chaotic behaviour resembles that before the transition in an intermittent fashion. Recently, a different type of intermittency called " on-off" intermittency has been reported in some low-dimensional non linear dynamical systems [2-7]. This intermittency is characterized by a two-state nature. One is the "off" state which is nearly constant, and remains so for very long periods of time and is suddenly changed by a burst, the so called "on" state which departs quickly from and returns quickly to the "off" state. Moreover, a power law characterizing the on-off intermittency has been obtained and discussed by Platt et al [6].

A self-organised on-off spatio temporal intermittency has also been reported in a system of coupled maps via nearest-neighbors interaction [8]. Fagen Xie and Cerdeira [9] have investigated and found a new intermittency transition from a coherent chaotic state to a two-cluster chaotic attractor in globally coupled systems. In this paper we study most of the transitions that take place in globally coupled systems.

THEORETICAL ANALYSIS

Globally coupled systems are ubiquitous in nature. They arise naturally in studies of Josephson junction arrays, multimode laser, charge-density wave,

oscillatory neuronal system and so on [10-12]. As one of the simplest globally coupled map (GCM) has been the subject of intensive research in recent years. Some rather surprising and novel results, such as clustering, splay state, collective chaotic behaviour, and violation of the law of large numbers in the turbulent regime are revealed in the GCM model [13-20]. However, to the best of our Knowledge, the mechanism of transitions among the dynamical phases in the GCM has never been discussed. In this paper, we will study the transition to intermittency in the GCM model, which takes place between the coherent and the ordered phases, coherent and glassy phases and the ordered and turbulent phases.
Specifically, we use the following form of GCM.

$$X_{n+1}(i) = (1-\varepsilon)f\big(X_n(i)\big)+\frac{\varepsilon}{N}\sum_{j=1}^{N} f\big(X_n(j)\big) \qquad (1)$$

Where n i and ε are discrete time step, the index of elements and the coupling coefficient respectively. The mapping function $f(x)=1-ax^2$, and a is the nonlinear parameter. N is the total number of elements or system size.
An important concept in GCM model is "clustering". This means that even when the interactions between all elements are identical, the dynamics can break into different clusters, each of which consists of fully synchronized elements. After the system falls in an attractor, we say that the elements i and j belong to the same cluster $X_n^i \equiv X_n^j$
Therefore, the behaviour of the whole system can be characterized by the number of clusters n_{cl} and the number of elements of each cluster ($M_1, M_2, \ldots Mn_{cl}$) [21]
As the nonlinearity or coupling is varied, the system exhibits successive phase transitions among coherent, ordered and turbulent phases [21].
In the coherent region the system is homogeneous in space i.e. $X^i \equiv X^j$,v i, j. It is characterized by only one cluster i.e., $n_1=1, M_1=N$. The motion of each element is equivalent to that of the single logistic map. The stability condition for coherent state is the modulus of all eigenvalues of NxN stability metrix $J=\prod^{m}_{n=1} f^1(X_n)J_o^m$ has magnitude less than one. Here $f^1(X_n)$ is the derivative of the n^{th} iteration of the logistic map. M is taken as the periodic number
J_o is an NxN constant matrix

$$J_0 = Mcirc\left(1-\varepsilon+\frac{\varepsilon}{N},\frac{\varepsilon}{N},\frac{\varepsilon}{N},\frac{\varepsilon}{N},\ldots,\frac{\varepsilon}{N},\frac{\varepsilon}{N}\right) \qquad (2)$$

The eigenvalues of the stability matrix are

$$\mu_1 = \prod_{N=1}^{M} f^1(x_n), \qquad \mu_r \equiv (1-\varepsilon)^m \prod_{n=1}^{M} f(x_n), \quad r = 2,3\ldots,N \qquad (3)$$

The eigenvector corresponding to eigenvalue is given by $\left(\frac{1}{\sqrt{N}}\right)(1,1,...,1)^T$ Thus, the amplification of a disturbance along this eigenvector does not destroy the coherence Eigenvectors for the other N -1 identical eigenvalues are not uniform, the amplification along these eigenvectors destroys the coherent phase. Therefore, the stability condition of the coherent attractor is decided by the N-1 identical eigenvalues.

Their corresponding lyapunov exponents are

$$\lambda = \lambda_r \equiv \ln(1-\varepsilon) + \lambda_0, \quad r = 2,3...,N \tag{4}$$

where λ_0 is the lyapunov exponent of the single logistic map. Therefore the critical stability condition is given by $\lambda = 0$, i.e. $\varepsilon c = 1\text{-e} -\lambda_0$. When ε is larger than ε_c all elements quickly evolve to the same motion (the homogeneous state) after a short transient process, since $\lambda < 0$. Starting at the coherent region =0.4, the coherent attractor occupies all the basin volumes for a<1.84. The basin for 2 cluster attractor increase with a, for a>1.84. By interating equation [1] with initial condition of each element randomly chosen in the uniform interval [0,1], and gradually increasing the non linearity 'a' at the transition region, it is observed that the system oscillates between the coherent state and the 2 cluster attractor state as nonlinearity is gradually increased. Furthermore the difference of the two clusters $(X^1\text{-}X^2)$ shows some very interesting and complex features.

RESULTS AND DISCUSSION

Fig.1 shows a time evolution of $X^1\text{-}X^2$ for a=1.899 and ε = 0.4 for the GCM with M_1=102, M_2=98, N=200, + ε_c =0.42

In the region ε =0.3, where the coherent atractor is known to occupy a ≤ 1.54 and the system is in the glassy phase from 1.56≤ a ≤1.80, the same oscillation between the coherent state and the two cluster attractor state is observed at the transition region. The complex features of the difference of the two clusters $(X^1\text{-}X^2)$ are also observed. Fig, 2 shows a time evolution of $X^1\text{-}X^2$ for a=1.6705 and ε = 0.3 for the GCM with M_1=78, M_2=122, N=200.ε_C=0.32.

In the region ε=0.2, a=1.893 where there are four clusters M_1=20, M_2=21, M_3=74, M_4=85. The difference of the two smallest clusters M_1 and M_2 only give the complex feature.

Fig.3 shows a time evolution of $X^1\text{-}X^2$ for ε =0.2 and a=1.893

In the region for ε =0.1, for a< 1.62 where the ordered phase appears, the complex feature is observed at a=1.62 for the GCM with M_1=21, M_2=41, M_3=62, M_4=76, N=200.

Fig 4 shows a time evolution of X^1-X^2 for ε=0.1 and a=1.62 and X^1-X^2 is the difference of the two smallest clusters M_1 and M_2.
It is observed that X^1-X^2 remains along time near zero, and suddenly departs from it and quickly returns after some random bursts. In order to characterize the intermittent behaviour, we have calculated numerically the statistical distribution of the duration of laminar phase X^1-X^2 shown in figs 5,6,7 and 8.
For threshold $\tau = 10^{-3}$, a total of 1×10^{7} iterations of equation (1) were computed to obtain the curves. P_n represents the probability of the laminar phase of length n, namely $Pn=M_n/M$ where M is the total number of segments of laminar phase, M_n the number of those of length n.
The distribution obeys an asymptotic power law with exponent –3/2.
By adding small amplitude to the system, the model is changed to

$$x_{n+1}(I) = (1-\varepsilon)f(X_n(i)) + \frac{\varepsilon}{N}\sum_{j=1}^{N} f(X_n(j)) + \sigma\xi \qquad (5)$$

Where ξ is a random variable in the interval [0,1] and σ is the intensity of the noise.
The intermittent behaviour is observed much earlier at (ε =0.4, a=1.87899), (ε =0.3, a=1.67), (ε =0.2, a=1.47 and a= 1.91) $\lambda\varepsilon$ =0.1, a= 1.6227) and σ =0.008 see figs 9, 10, 11, 12 and 13.
The 2-cluster attractor reduces to a large number of clusters with all Mi's small (1 or 2). The difference of the clusters also showed the features of intermittency at the transition regions. The coherent state at ε =0.2, a=1.47 also reduces to large number of clusters with the addition of noise. Intermittency is also observed.

CONCLUSION

In conclusion, we have investigated the intermittent attractions from a coherent state to a two cluster attractor, coherent to glassy state and the ordered to turbulent states.
The on-off intermittency of low dimensional systems, the two state on-off characteristic of motion and the -3/2 power law scaling are observed. The spatiotemporal on-off intermittency is essentially a global behaviour at transition regions of extended system.

ACKNOWLEDGEMENT

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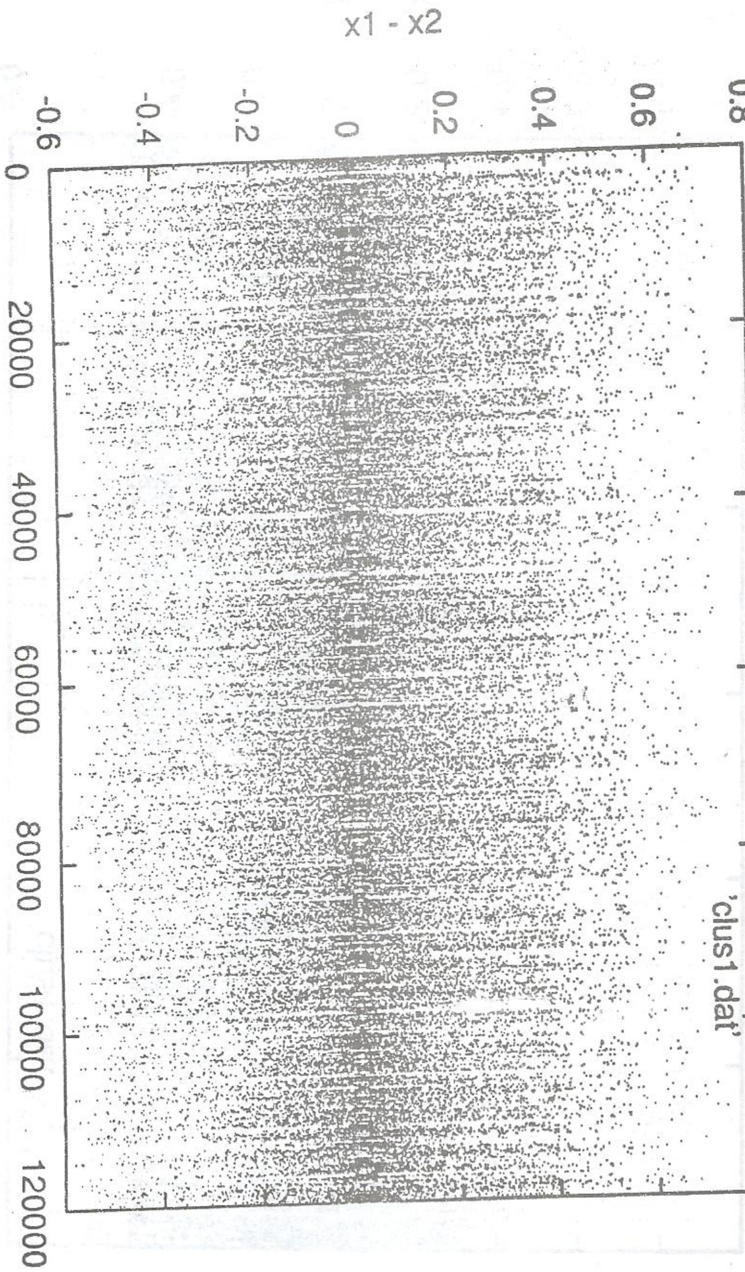


FIG. 1. The evolution of the difference of the two clusters at e=0.4,a=1.899.

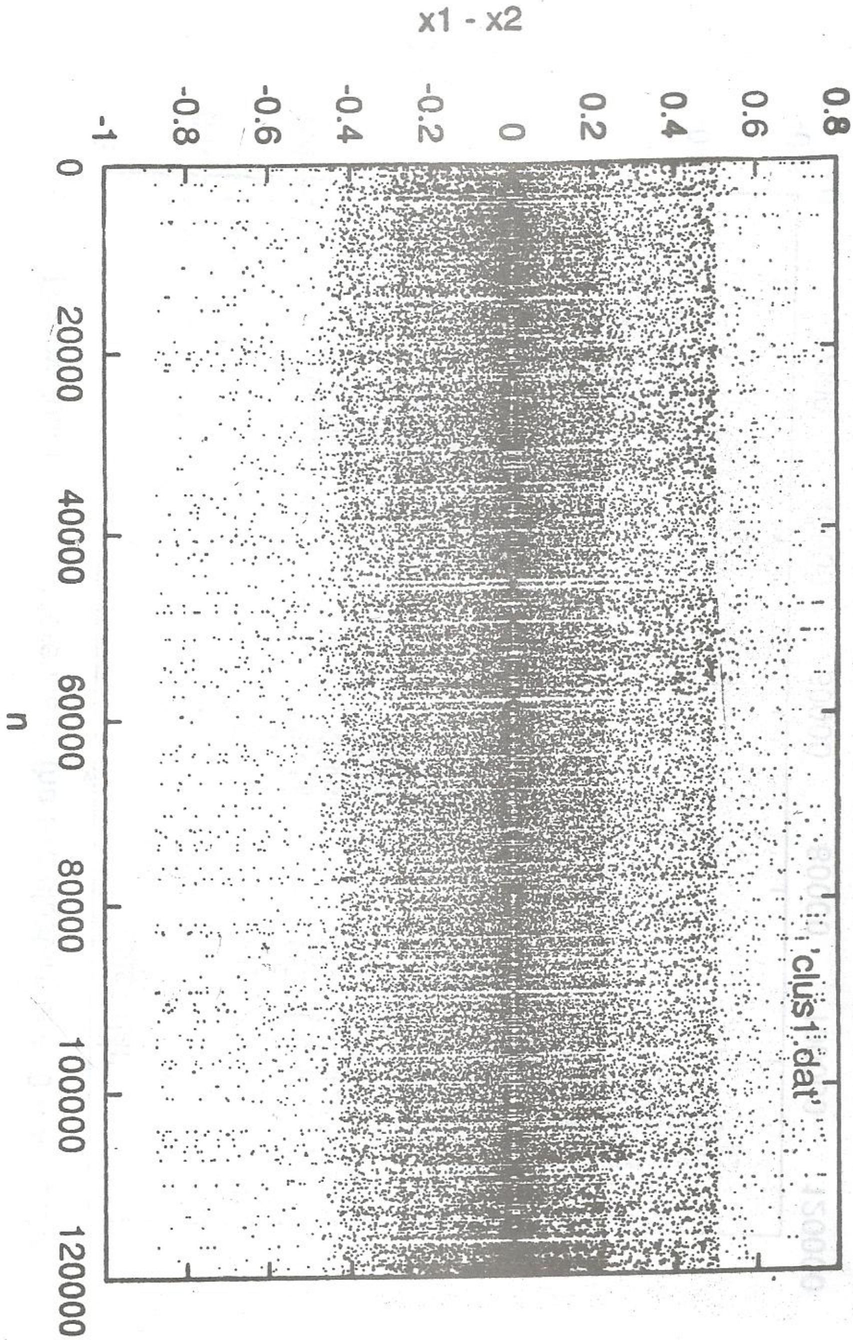


FIG. 2. The evolution of the difference of the two clusters at e=0.3,a=1.6705.

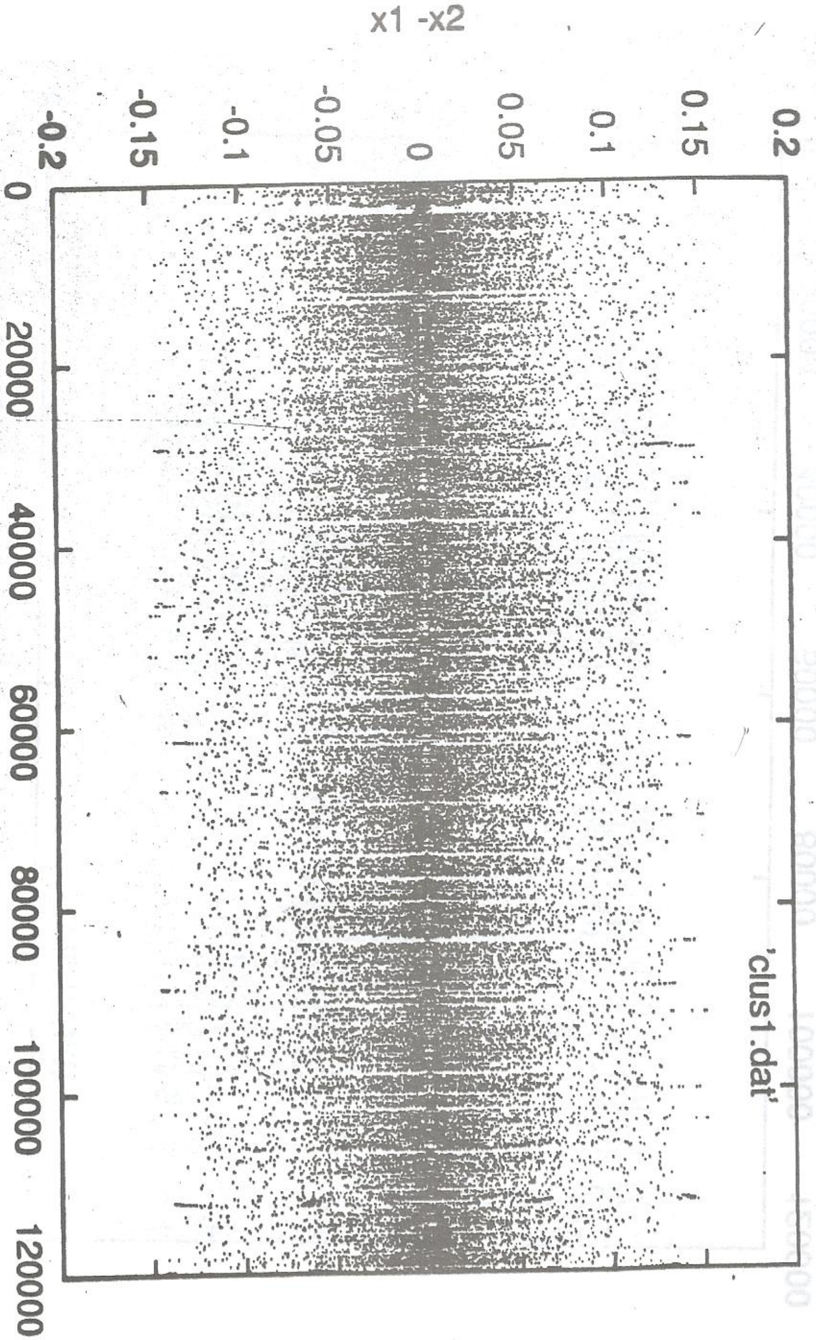


FIG. 3. The evolution of the difference of the two clusters at e=0.2,a=1.893

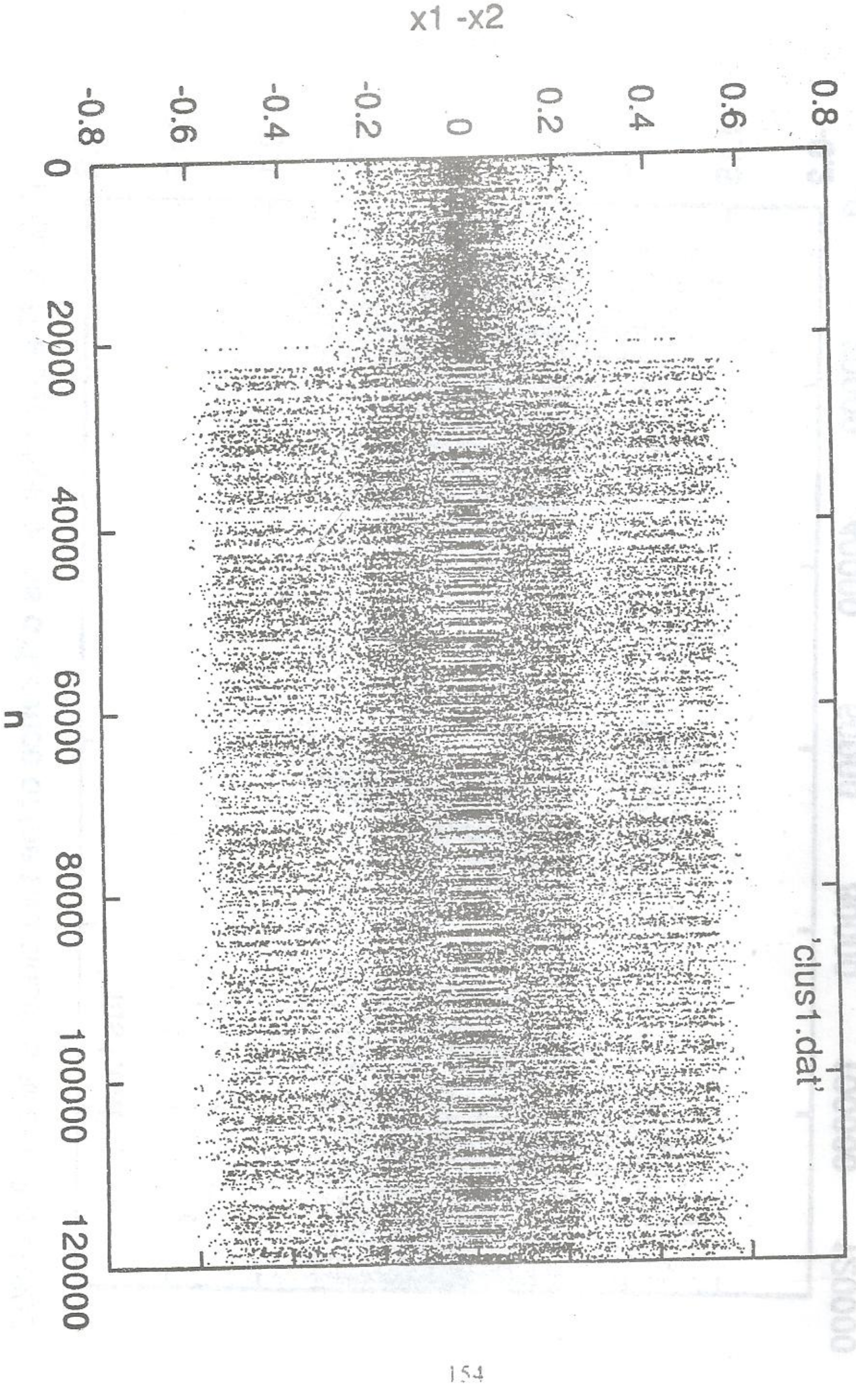


FIG. 4.The evolution of the difference of the two clusters at e=0.1,a=1.62.

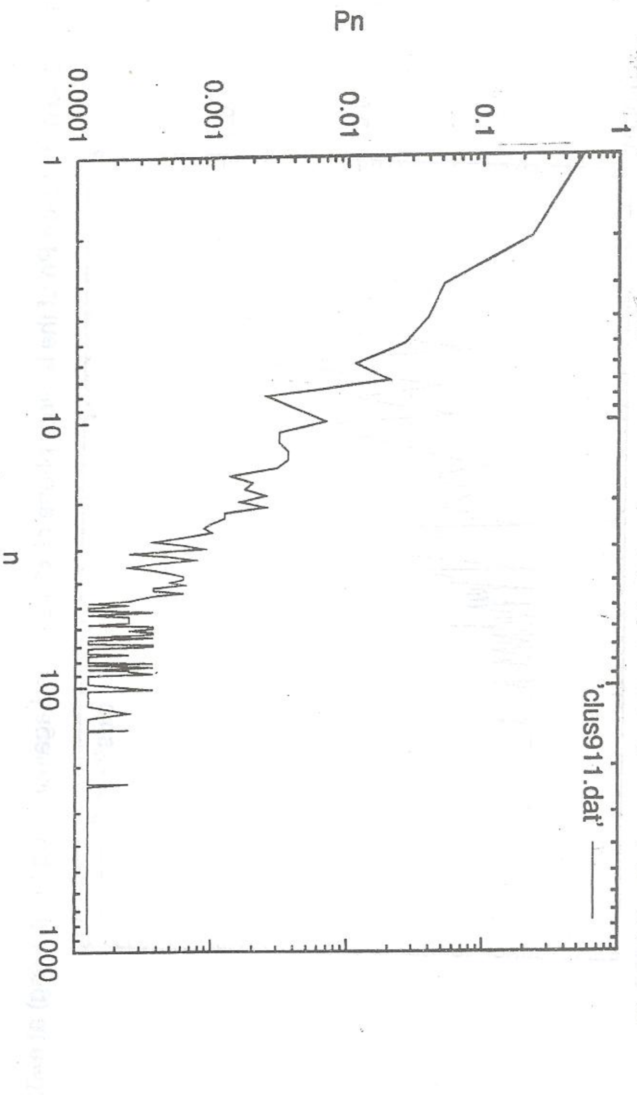


FIG. 5. The relative distribution Pn of the laminar phase of x1-x2 plotted against n (log-log plotting) at e=0.4,a=1.899.

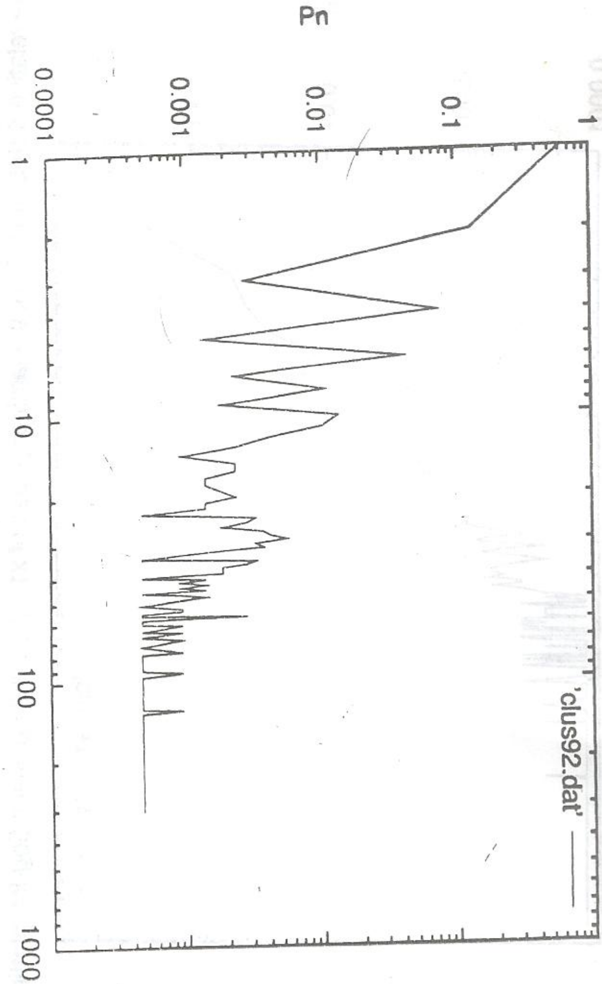
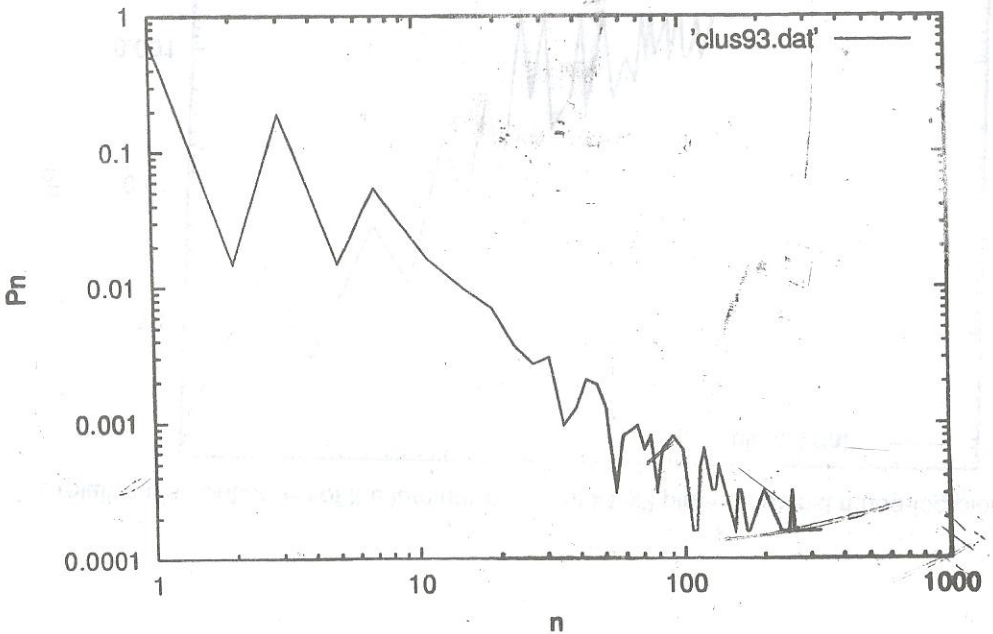


FIG. 6. The relative distribution Pn of the laminar phase of x1-x2 plotted against n (log-log plotting) at e=0.3,a=1.6705.

FIG.7. The relative distribution Pn of the laminar phase of x1-x2 plotted against n (log-log plotting) at e=0.2,a=1.893.



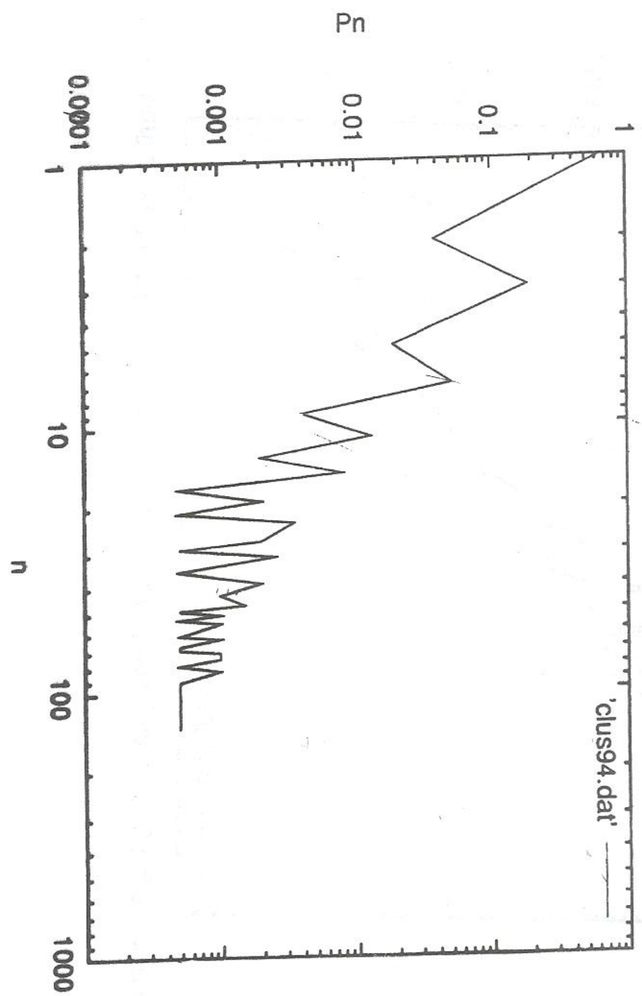


FIG.8. The relative distribution Pn of the laminar phase of x1-x2 plotted against n (log-log ploting) at e=0.1,a=1.62.

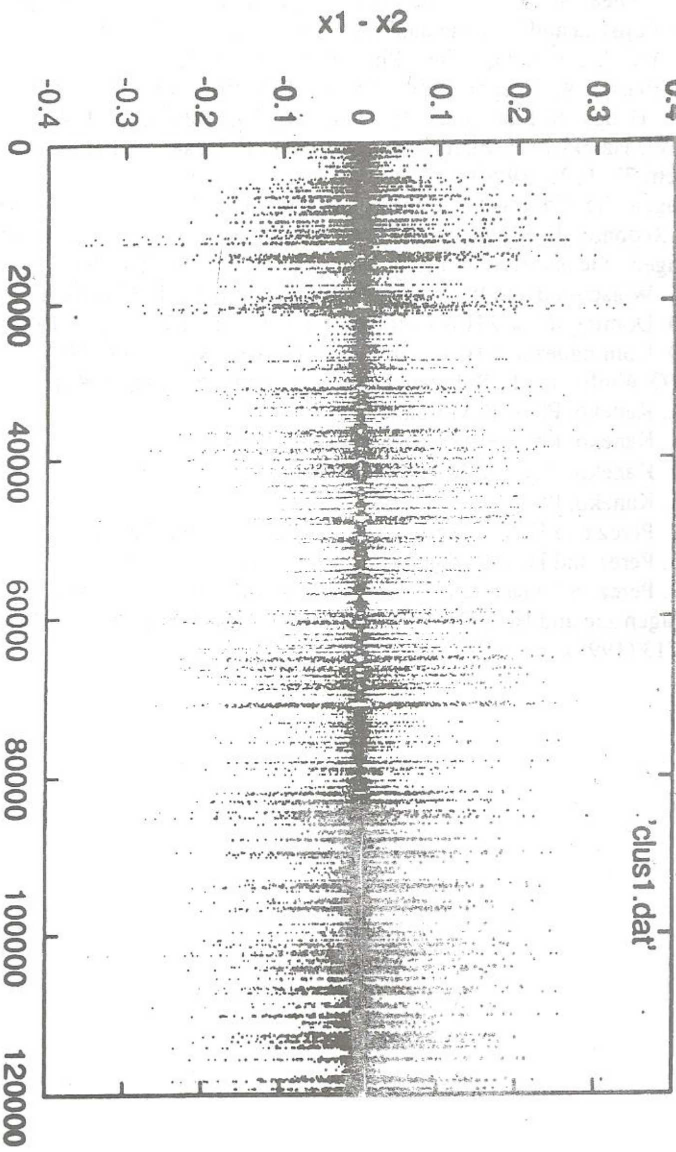


FIG. 9.The evolution of the difference of the two clusters with addition of noise at e=0.2,a=1.47.

REFERENCES

[1]. Y. Pomeau and Manneville, Commun Math. Phys. 74, 189 (1980).
[2]. H. Fujisaka and T. Yamada, Prog. Theor. Phy 69, 32 (1983)
[3]. L. Yu, E. Ott, and Q. Chen, Phys, Rev. Lett 65, 2935.
[4]. N. Platt, E.A. Spiegel, and C. Tresser, Phy. Rev. Lett. 70, 279 (1993).
[5]. J.F. Heagy, N. Platt, and S.M. Hammel, Phys, Rev. E 49, 1140 (1994)
[6]. P.W. Hammer, N. Platt, S.M. Hammel, J.F. Heagy and B.D Lee Phys. Rev. Lett. 73, 1095 (1994).
[7]. Fagen Xie, Gang Hu and Zhilin Qu, Phys. Rev. E. 52, R1265 (1995).
[8]. J. Redondo, E. Roldan and G.J. de Valcarcel, Phys. Lett A 210-301 (1996).
[9]. Fagen Xie and Cerdeira H.A. Phys Rev. E. 54, 33235 (1996).
[10]. K. Wiesenfeld and P. Hadley, Phys Rev. Lett. 62, 1335 (1989).
[11]. D. Dominguez and H.A Cerdeira, Phys. Rev.Lett 71, 3359 (1993).
[12]. D. Dominguez and H.A. Cerdeira Phys. Rev. B52, 513 (1995)
[13]. H.G Winful and L. Rahman, Phys. Rev. Lett. 65, 1575 (1990).
[14]. K. Kaneko, Phys. Rev. Lett. 63, 219 (1989).
[15]. K. Kaneko, Phys. Rev. Lett. 65, 1391 (1990).
[16]. K. Kaneko, Physica D 41,137 (1990).
[17]. K. Kaneko, Physica D 55, 368 (1992).
[18]. G. Perez and H.A Cerdeira Phys. Rev. A 46, 7492 (1992)
[19]. G. Perez and H.A Cerdeira Phys. Rev. E 49, R 15 (1994)
[20]. G. Perez, S Sinha and H.A Cerdeira, Physica D 63, 341 (1993).
[21]. Fagen Zie and H.A. Cerdeira, Int. Journal of Bifurcation and Chaos Vol. 8, 1713 (1998).