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COMMENTS ON SUPERSPACE AND SUPERFIELDS

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ABSTRACT
Two commutator relations useful in the discussion of superspace and superfields are proved.

1. INTRODUCTION

In the supersymmetric extension of the Poincare algebra in the two-component Weyl formalism, we have the following anticommutators as elements of the super-Poincare algebra [1]:

$$\{Q_A, Q^B\} = 0 \qquad A, B = 1,2 \tag{1}$$

$$\{\overline{Q}^{\dot{A}}, \overline{Q}_{\dot{B}}\} = 0 \qquad \dot{A}, \dot{B} = \dot{1},\dot{2} \tag{2}$$

$$\{Q_A, \overline{Q}_{\dot{B}}\} = 2\sigma^{\mu}_{A\dot{B}} P_\mu \tag{3}$$

$$\{\overline{Q}^{\dot{A}}, Q^B\} = 2\overline{\sigma}^{\mu\dot{A}B} P_\mu \tag{4}$$

Here $\sigma^\mu = (\sigma^0, \sigma)$ and $\overline{\sigma}^\mu = (\sigma^0, -\sigma)$

Where σ^0 is the unit 2x2 matrix, while $\sigma = (\sigma^1, \sigma^2, \sigma^3)$ is the 3-tuple of Pauli matrices. Moreover, $P_\mu = -i\partial_\mu$. In obtaining these anticommutators the close connection between four-component Majorana spinors and two-component Weyl spinors has been invoked.

The super-Poincare algebra, a graded Lie algebra has 14 generators and comprises both commutators and anticommutators. The inclusion of an internal symmetry group in a non-trivial way in this superalgebra has necessitated the addition of four spinor charges Q_A and $\overline{Q}_{\dot{A}}$ to give N = 1 supersymmetry. In order to obtain a formalism in which supersymmetry is inherently manifest, it has been necessary to introduce the Grassmann variables θ_A and $\overline{\theta}_{\dot{B}}$ [2]. By superspace $(x^\mu, \theta, \overline{\theta})$ we mean an eight-dimensional space with the four space time coordinates x^μ and the Grassman variables θ_A and $\overline{\theta}_{\dot{B}}$ which together constitute the supercoordinates. The general superfield $\Phi(x, \theta, \overline{\theta})$ which is a Lorentz scalar or pseudoscalar, is an operator-valued function defined on superspace. The supercoordinates obey the following anticommutation relations:

$$\{\theta_A, \theta_B\} = 0 \qquad (5)$$
$$\{\bar{\theta}_{\dot{A}}, \bar{\theta}_{\dot{B}}\} = 0 \qquad (6)$$
$$\{\theta_A, \theta_{\dot{B}}\} = 0 \qquad (7)$$

In order to obtain a regular Lie superalgebra, it is necessary to convert the anticommutators in the super-Poincare algebra into commutators. Two of these commutators will be derived in this paper.

2 THE COMMUTATOR RELATIONS

We now show that

$$[P_\mu, (\theta Q)] = 0 \qquad (8)$$
$$[P_\mu, (\bar{\theta}\bar{Q})] = 0 \qquad (9)$$

In order to prove Eq. (8), we start with the element of the super-Poincare algebra

$$[P_\mu, Q_A] = 0$$

i.e.

$$P_\mu Q_A - Q_A P_\mu = 0$$

On multiplying this equation on the left by θ^A we obtain

$$0 = \theta^A P_\mu Q_A - \theta^A Q_A P_\mu = -i\theta^A \partial_\mu Q_A + i\theta^A Q_A \partial_\mu \qquad (10)$$

Let us now note that a superfield is written as follows as a power series expansion in the Grassmann variables θ and $\bar{\theta}$

$$\Phi(x,\theta,\bar{\theta}) = f(x) + \theta\phi(x) + \bar{\theta}\bar{\chi}(x) + (\theta\theta)m(x) + (\bar{\theta}\bar{\theta})n(x) + (\theta\sigma^\mu\bar{\theta})V_\mu(x) + (\theta\theta)\bar{\theta}\bar{\lambda}(x) + (\bar{\theta}\bar{\theta})\theta\psi(x) + (\theta\theta)(\bar{\theta}\bar{\theta})d(x) \qquad (11)$$

Where $f(x), \phi(x), \bar{\chi}(x), m(x), n(x), V_\mu(x), \bar{\lambda}(x), \psi(x)$ and $d(x)$ are called component fields. Eq. (11) can be expressed in general as

$$\Phi(x,\theta,\bar{\theta}) = \sum_i g_i(\theta,\bar{\theta})h_i(x) \qquad (12)$$

where g_i and h_i are arbitrary functions of their variables. It is clear that for the commutator acting on any function of the form of Eq. (12):

$$\theta^A \partial_\mu = \partial_\mu \theta^A \tag{13}$$

Hence, Eq. (10) becomes

$$0 = -i\partial_\mu \theta^A Q_A + i\theta^A Q_A \partial_\mu = P_\mu(\theta Q) - (\theta Q)P_\mu \tag{14}$$

i.e.

$$[P_\mu(\theta Q)] = 0$$

This completes the proof of Eq. (8).

In order to prove Eq. (9) let us note in passing that in summing over undotted spinors such as in Eq. (14) the 'summation toward the south-east convention' is observed while in summing over dotted indices as in Eq. (15) below the 'summation towards the north-east convention' is observed. We also note that

$$[P_\mu, \overline{Q}_{\dot{A}}] = 0 \text{ implies}$$
$$[P_\mu, \overline{Q}^{A}] = \varepsilon^{\dot{A}\dot{B}}[P_\mu, \overline{Q}_{\dot{B}}] = 0$$

We therefore start with the equation

$$[P_\mu, \overline{Q}^{\dot{A}}] = 0 = P_\mu \overline{Q}^{\dot{A}} - \overline{Q}^{\dot{A}} P_\mu = -i\partial_\mu \overline{Q}^{\dot{A}} + i\overline{Q}^{\dot{A}}\partial_\mu$$

On multiplying this equation on the left by $\overline{\theta}_{\dot{A}}$, we obtain

$$0 = -i\overline{\theta}_{\dot{A}}\partial_\mu \overline{Q}^{\dot{A}} + i\overline{\theta}_{\dot{A}}\overline{Q}^{\dot{A}}\partial_\mu \tag{15}$$

We also note that for the commutator acting on any function of the form of Eq. (12)

$$\partial_\mu \overline{\theta}_{\dot{A}} = \overline{\theta}_{\dot{A}}\partial_\mu$$

Eq. (15) thus becomes

$$0 = -i\partial_\mu \overline{\theta}_{\dot{A}}\overline{Q}^{\dot{A}} + i\overline{\theta}_{\dot{A}}\overline{Q}^{\dot{A}}\partial_\mu = P_\mu(\overline{\theta}\,\overline{Q}) - (\overline{\theta}\,\overline{Q})P_\mu$$

i.e. $[P_\mu(\overline{\theta}Q)] = 0$

This completes the proof of Eq. (9).

REFERENCES

[1]. Muller-Kirsten, H.J.W., Wiedermann, A., (1987), Supersymmetry, World Scientific, Singapore.

[2] Salam, A., Strathdee, J. (1974), Nucl. Phys., B76, 477.