J. Nig. Ass. Math. Phys.
Vol. 4 (2000)

FAILURE-TIME MODEL FOR A COMPONENT WITH LIFE CONTROL DEVICE.

BASHIR M. YAKASAI.
DEPARTMENT OF MATHEMATICAL SCIENCES.
BAYERO UNIVERSITY, KANO.

ABSTRACT.
In this paper we propose a failure-time model for a component whose working-time length can be exploited optimally, to a certain level by the use of life control device. For realization of the problem we provide a prototype example, where we use the data obtained from the experiment conducted on the life span of battery sets installed in a particular brand of radio.
Keywords: failure-time; failure rate; control- level,;Weibull distribution.

1.0 INTRODUCTION.

In literature various contributors have modeled life length of components under different work conditions. The objective is to maximize the use of such components for an optimum time length before retirement and to maintain certain level of continuity in the system operation. Barlow and Proschan[1] have discussed extensively on block replacement policy; Bhat[3] introduced for the first time in literature the application of used items in replacement decision making, in order to minimize replacement cost. Bashir[2] suggested a cast-off replacement policy for a component demanded by two independent, not necessarily identical units. The purpose of that policy is to exhaust the life of the component to an insignificant level before it is cast off. Tosch and Holmes[6] suggested a bivariate failure model in which the residual lifetime of one component is dependent on the working status of the other.

In this paper we propose a failure-time model for a component with life regulator mechanism. We assume throughout this paper that

1. The life of the component is controllable to a certain extent.
2. The life of the component depends on the length of time it is working and the level of the control device.
3. The failure-time t is a function of the control level v.
4. The failure rate $r(t)$ is proportional to failure-time t; thus if $t\uparrow$ then $r(t)\uparrow$.
5. The failure-time is inversely proportional to the control level v; thus if $v\uparrow$ then $t\downarrow$.
6. The component has increasing failure rate (IFR).

For realization of the problem we provide a prototype example using the data obtained by Hassana[5] from the experiment conducted on the working life length of a particular brand of battery used in a particular brand of radio. We have considered the radio volume as the control device in this case (see table 1 below).

2.0 FAILURE-TIME MODEL.

From assumption 5 above, we can express the rate of change of the failure-time $t(v)$ with respect to the small change in the control level v as

$$\frac{dt}{dv} = -\frac{\alpha}{v} \tag{1}$$

where $\alpha > 0$ is a constant and $v > 0$. By solving this simple differential equation we obtain that

$$t(v) = -\alpha \log v + A \tag{2}$$

where A is a constant.

Suppose v_h denotes the highest level of the control mechanism. And let t_h be the failure-time at v_h. Then, from equation (2), at boundary point $v = v_h$ we have

$$A = t_h + \alpha \log v_h$$

Substitute in equation 2, we get

$$t(v) = t_h + \alpha \log\left(\frac{v_h}{v}\right) \tag{3}$$

provided that t_h, $v_h > 0$ and known.

Equation (3), give us a mathematical model that is deterministic. To make the problem more realistic, suppose that we have n number of components each observe at a particular control level, v. Then, the time to failure at that level of control is a random variable, T, say. Let $t_1, t_2, \ldots, t_n$ be n copies of T and $F(t)$ probability distribution function of T. Hence, we can state the following theorem.

Theorem.

If the failure rate $r(t)$ is proportional to failure-time $t(v)$, then the probability distribution function $F(t)$ is a Weibull distribution with shape parameter 2 and scale parameter $\frac{\beta}{2}$.

Proof.

$r(t) = \beta t$ where β is a constant of proportionality. According to Cox[4], the age specific failure rate is given by

$$r(t) = \frac{f(t)}{\bar{F}(t)} \tag{4}$$

where $\bar{F}(t) = 1 - F(t)$ is the survivor function and $f(t)$ is the probability mass function of the random variable T. Thus

$$\frac{f(t)}{\bar{F}(t)} = \beta t \tag{5}$$

Barlow and Proschan[1] provide the following relations.

From equation (5), equations (6) become

$$\bar{F}(t) = \exp\left\{-\frac{\beta t^2}{2}\right\} \quad \text{and} \quad f(t) = \beta t \exp\left\{-\frac{\beta t^2}{2}\right\} \; ; \; t > 0 \qquad (7)$$

This give us a Weibull distribution with parameters 2 and $\frac{\beta}{2}$. Hence, the mean residual life (MRL) is

$$\int_t^{\infty} \frac{\bar{F}(x)dx}{\bar{F}(t)} = \frac{1}{\beta t} \quad \text{(see Barlow and Proschan[1])} \qquad (8)$$

Since t is a function v, we can express equations (7) in terms of v. Thus

$$\bar{F}(v) = \exp\left\{-\frac{\pi}{4}\left(\frac{t_h + \alpha z}{\tau_h + \alpha z}\right)^2\right\} \text{ and } f(v) = \beta(t_h + \alpha z)\exp\left\{-\frac{\beta}{2}(t_h + \alpha z)^2\right\} \qquad (9)$$

where $v_l \leq v \leq v_h$ and $\beta(v) \geq 0$

From the theorem, we can determine the expected value of T at v_h. That is $\tau_h = E(t_h)$, which we substitute in equation (3) to get the mean time to failure for a given level of control v as

$$\tau(v) = \tau_h + \alpha \log\left(\frac{v_h}{v}\right) \qquad (10)$$

To use this model we require a suitable value of α and β. The optimum α can be chosen if the mean time to failure at the lower boundary point is known. Thus, taking $v = v_l$, the lower boundary point, we have from equation (10)

$$\tau_l = \tau_h + \alpha \log\left(\frac{v_h}{v_l}\right)$$

Hence

$$\alpha = \frac{\tau_l - \tau_h}{\log\left(\frac{v_h}{v_l}\right)} \qquad (11)$$

Equation (10) can be expressed as

$$E(t) = \tau_h + \alpha z = \sqrt{\frac{\pi}{2\beta}} \qquad (12)$$

where $z = \log\left(\frac{v_h}{v}\right)$. Therefore, solving for β, we obtain for a given control level

$$\beta(v) = \frac{\pi}{2(\tau_h + \alpha z)^2} \quad ; \; v \geq 0 \tag{13}$$

In particular, if $v = v_l$

$$\beta = \frac{\pi}{2\tau_l^2} \tag{14}$$

And if $v = v_h$

$$\beta = \frac{\pi}{2\tau_h^2} \tag{15}$$

Thus at each control level v we obtain a suitable scaling parameter β.

3.0 PROTOTYPE EXAMPLE.

We consider the data obtained by Hassana[5] in this paper for prototype example. New battery sets of the same brand were used in the particular brand of radio and the volume of the radio was fixed at two different levels; the highest level v_h and the lowest level v_l. The radio is allowed to work until the battery set fails to work satisfactorily, according to the experimenter's subjective judgment. Hence, the battery set is considered failed. The experiment was repeated three times at the two different levels. The data recorded is summarized in table 1.

From table 1, we obtain α=289.337. Hence, we have

$$\beta(t) = \frac{\pi}{2\left[643.33 + 289.337\log\left(\frac{5}{v}\right)\right]^2} \quad ; \; v > 0$$

$$F(v) = 1 - \exp\left\{-\frac{\beta}{2}\left[690 + 289.337\log\left(\frac{5}{v}\right)\right]^2\right\} \quad ; \; v > 0$$

and $$t(v) = 690 + 289.337\log\left(\frac{5}{v}\right) \quad ; \; v > 0$$

These equations lead to table 2.

4.0 DISCUSSION.

Table 2 indicates that the life-length $t(v)$ of the battery sets decrease significantly as the control level increases. The scaling parameter $\beta(v)$ varies at different control levels v; and the variation is directly proportional to the control level v. A straight line would be obtained when β is plotted against v (see Fig. 2 in the appendix). The mean residual life of the battery set is much higher at the lowest control level than it is at the highest control level. The cumulative distribution function $F(v)$ of the failure-time $t(v)$ is not changing significantly at different levels of the life control device. For instance, it is approximate that $F(v) = 0.58$ for $v = 1,2,3$ and $F(v) = 0.60$ for $v = 4, 5$. In other word, at control levels 1,2 and 3 the battery sets would work for 20 hours, 16 hours and 13 hours respectively, with probability of survival 0.42 each. And at v equal to 4 and 5, the battery sets would work satisfactorily for the time length of 13 hours and 12 hours respectively, with probability of survival 0.40 each. Surface graphics for the failure-time distribution $F(v,\beta)$ is plotted using computer software, the Mathematica for Students 3.0 (see appendix). Figures 1(a) and 1(b) are the same, we only exchange the axes of β and v in order to obtain clear picture of the graph.

5.0 CONCLUSION.

This paper proposes a failure-time model for a component whose working-life time can be controlled by a certain device. The model can be used to make prediction on the time to failure of a component when the control device is fixed at certain point. We have shown that as the failure rate of the component is proportional to the failure-time, then the cumulative distribution function of the failure-time is a special case of a well known classical distribution, the Weibull distribution with shape parameter 2 and scale parameter β/2. As time to failure is dependent on the control level, the distribution function of $t(v)$ is therefore obtained as a function of the control level v. In the prototype example, it is observed that the scaling parameter β varies with v. Thus, β is a function of the control level v. And the relation between β and v is linear with positive covariance (see fig.2 in the appendix).

Table1: **Life-lengths of the battery sets (in min.) at fixed control levels v_l and v_h.**

Experiment	1	2	3	Total	Mean
v_l	1060	1134	1133	3327	1109.00
v_h	690	610	630	1930	643.33

Table 2: **Failure-time distribution function $F(v)$ and working age $t(v)$ of the battery sets at different control levels**

v	1	2	3	4	5
$F(v)$	0.58	0.58	0.58	0.60	0.60
β	1.3×10^{-6}	1.9×10^{-6}	2.5×10^{-6}	3.1×10^{-6}	3.8×10^{-6}
t(v)	1155.67	955.12	837.80	754.56	690.00
MRL	665.61	551.05	477.44	427.51	381.39

ACKNOWLEDGEMENT.

The author would like to extend his thanks to Dr. S. S. Duwa of the Department of Physics, Bayero University, Kano for the surface graphics provided in this paper.

Appendix: Surface Graphics for F(V, B)

Fig.1a

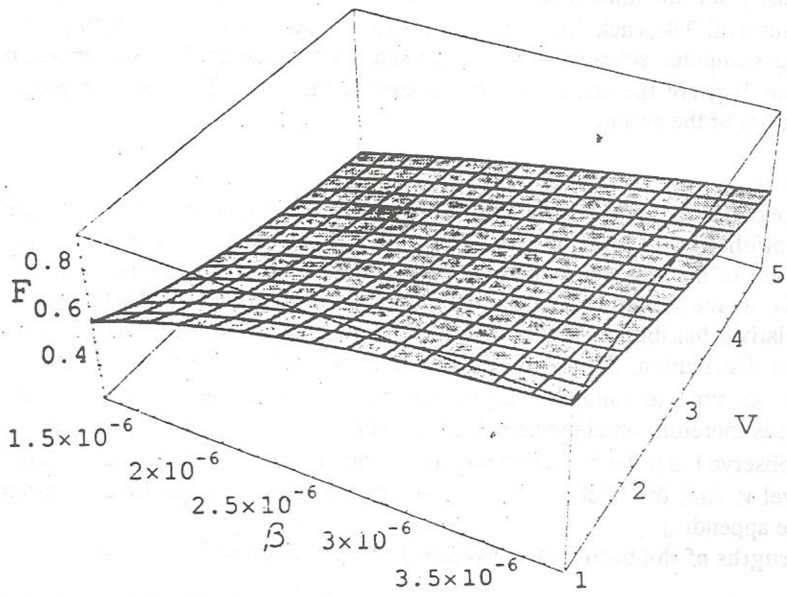

Fig 1b