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INTERPRETATION OF VERTICAL ELECTRICAL SOUNDINGS AT AFUZE AND EME-ORA USING RESISTIVITY TRANSFORM FUNCTIONS.

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ABSTRACT

Geophysical survey work was recently carried out at Afuze, Owan East and Eme-Ora, Owan West Local Government areas of Edo State using vertical electrical sounding technique. It is usually difficult interpreting result in terms of lithological variation with depth. One of the latest methods of interpretation is by matching practical field curves with theoretically generated curves from resistivity kernel functions. Researchers have formulated several kernel functions and research work is still going on with the aim of identifying an optimum kernel function. This work proposes resistivity transform function as an acceptable kernel function. Detailed theory of the function is given and typical result using kernel function for interpretation for the two towns are presented. Spontaneous potential loggings were carried out in nearby boreholes in the two towns for correlation purposes. The three sets of results – the available driller's logs, the geophysical logs and the interpretation by method of kernel function all correlated very well within limits of experimental errors.

INTRODUCTION

Abortive boreholes are common in Edo north. To forestall waste of resources on abortive boreholes it is necessary to carry out detailed geophysical survey for the identification of suitable water borehole sites. Even if suitable sites are identified and promising boreholes are drilled, wrong positioning of the screen inside the boreholes may lead to an early failure of the boreholes. Geophysical logging forestalls wrong positioning of screen in the borehole.

The Schlumberger electrode array system was used for this survey, Asokhia et. al. (1999). The method of investigation was by vertical electrical sounding (VES). The interpretation was by use of kernel function. For correlation purposes, the spontaneous potential (SP) logging was used. The theories of resistivity transform derivatives as a form of kernel function and the theories of SP logging are presented below.

THEORY

An earth model of n layers is characterised by 2n-1 parameters (P_j:j=1, 2,…,n-1, n, n+1,…, 2n-1). The first n-1 parameters are thicknesses (h_i:i=1, 2,…, n-1) and the next n parameters are resistivities ($\ell_i : i = 1,2,\ldots,n$). The resistivity transform function is defined by:

$$T_{i+1} = \frac{T_i + T_{n-i}^1}{1 + T_i T_{n-i}^1 / \ell_{n-i}^2} \qquad (i = 2,3,\ldots,n-1) \quad \ldots\ldots(1)$$

where $$T_{n-i}^1 = \ell_{n-i}\left(\frac{1-\exp(-2\lambda h_{n-i})}{1+\exp(-2\lambda h_{n-i})}\right) = \ell_{n-i}\tanh(\lambda h_{n-i}) \quad \ldots\ldots(2)$$

and $$T_2 = \ell_1\left(\frac{1+k_{12}\exp(-2\lambda h_1)}{1-k_{12}\exp(-2\lambda h_1)}\right) \quad \ldots\ldots(3)$$

The resistivity transform derivative with respect to any one-model parameter is defined by:

$$\left(\frac{\partial T_{i+1}}{\partial P_j}\right) = \frac{\left(1+T_iT_{n-i}^1/\ell_{n-i}^2\right)\frac{\partial}{\partial P_j}\left(T_i + T_{n-i}^1\right) - \left(T_i + T_{n-i}^1\right)\frac{\partial}{\partial P_j}\left(T_iT_{n-i}^1/\ell_{n-i}^2\right)}{\left(1+T_iT_{n-i}^1/\ell_{n-i}^2\right)^2} \quad \ldots(4)$$

where the partial derivatives on the right simplify as follows:-

$$\frac{\partial}{\partial P_j}\left(T_i + T_{n-i}^1\right) = \left(\frac{\partial T_i}{\partial P_j}\right) + \left(\frac{\partial T_{n-i}^1}{\partial P_j}\right) \quad \ldots(5)$$

$$\frac{\partial}{\partial P_j}\left(T_iT_{n-i}^1/\ell_{n-i}^2\right) = \left(T_{n-i}^1/\ell_{n-i}^2\right)\left(\frac{\partial T_i}{\partial P_j}\right) + \left(T_i/\ell_{n-i}^2\right)\left(\frac{\partial T_{n-i}^1}{\partial P_j}\right) - \left(2T_iT_{n-i}^1/\ell_{n-i}^3\right)\left(\frac{\partial \ell_{n-i}}{\partial P_j}\right) \quad ..(6)$$

$$\left(\frac{\partial T_{n-i}^1}{\partial P_j}\right) = \tanh(\lambda h_{n-i})\left(\frac{\partial \ell_{n-i}}{\partial P_j}\right) + \ell_{n-i}\frac{\partial}{\partial P_j}\left(\tanh(\lambda h_{n-i})\right)$$

$$= \tanh(\lambda h_{n-i})\left(\frac{\partial \ell_{n-i}}{\partial P_j}\right) + \ell_{n-i}\operatorname{sec}h^2(\lambda h_{n-i})\frac{\partial}{\partial P_j}(\lambda h_{n-i})$$

$$= \frac{1-\exp(-2\lambda h_{n-i})}{1+\exp(-2\lambda h_{n-i})}\left(\frac{\partial \ell_{n-i}}{\partial P_j}\right) + \frac{4\lambda \ell_{n-i}\exp(-2\lambda h_{n-i})}{[1+\exp(-2\lambda h_{n-i})]^2}\left(\frac{\partial h_{n-i}}{\partial P_j}\right) \quad \ldots(7)$$

By using equations (5), (6) and (7) equation (4) becomes:

$$\left(\frac{\partial T_{i+1}}{\partial P_j}\right)=\left[\begin{array}{l}\left(\frac{\partial T_i}{\partial P_j}\right)\left(1+\left(T_iT^1_{n-i}/\ell^2_{n-i}\right)-\left(T_i+T^1_{n-i}\right)T^1_{n-i}/\ell^2_{n-i}\right)+\left(\frac{\partial h_{n-i}}{\partial P_j}\right)\left(\lambda\ell_{n-i}\right)\sec h^2\left(\lambda h_{n-i}\right)\\ \left(1+\left(T_iT^1_{n-i}/\ell^2_{n-i}\right)-\left(T_i+T^1_{n-i}\right)T_i/\ell^2_{n-i}\right)+\left(\frac{\partial \ell_{n-i}}{\partial P_j}\right)\left(\begin{array}{l}\tanh\left(\lambda h_{n-i}\right)\left(\begin{array}{l}1+\left(T_iT^1_{n-i}/\ell^2_{n-i}\right)\\-\left(T_i+T^1_{n-i}\right)T_i/\ell^2_{n-i}\end{array}\right)+\\ 2\left(T_i+T^1_{n-i}\right)T_iT^1_{n-i}/\ell^3_{n-i}\end{array}\right)\end{array}\right]$$

$$+\left(1+\left(T_iT^1_{n-i}/\ell^2_{n-i}\right)\right)^2 \qquad .....(8$$

Finally, the transform derivative is given recursively by:

$$\left(\frac{\partial T_{i+1}}{\partial P_j}\right)=\left[\begin{array}{l}\left(\frac{\partial T_i}{\partial P_j}\right)\left(1-\left(\frac{T^1_{n-i}}{\ell_{n-i}}\right)^2\right)+\left(\frac{\partial h_{n-i}}{\partial P_j}\right)\left(\frac{4\lambda\ell_{n-i}\exp\left(-2\lambda h_{n-i}\right)}{\left(1+\exp\left(-2\lambda h_{n-i}\right)\right)^2}\left(1-\left(\frac{T_i}{\ell_{n-i}}\right)^2\right)\right)+\left(\frac{\partial \ell_{n-i}}{\partial P_j}\right)\\ \left(\left(\frac{1-\exp\left(-2\lambda h_{n-i}\right)}{1+\exp\left(-2\lambda h_{n-i}\right)}\right)\left\{1-\left(\frac{T_i}{\ell_{n-i}}\right)^2\right\}+\frac{2T_iT^1_{n-i}\left(T_i+T^1_{n-i}\right)}{\ell^3_{n-i}}\right)\end{array}\right]$$

$$+\left(1+\frac{T_iT^1_{n-i}}{\ell^2_{n-i}}\right)^2 \qquad ......(9)$$

Note, in particular, that:

$$\left(\frac{\partial h_{n-i}}{\partial P_j}\right)=\begin{vmatrix}0 & P_j\neq h_{n-i}\\ 1 & P_j=h_{n-i}\end{vmatrix} \qquad (10)$$

$$\left(\frac{\partial \ell_{n-i}}{\partial P_j}\right)=\begin{vmatrix}0 & P_j\neq \ell_{n-i}\\ 1 & P_j=\ell_{n-i}\end{vmatrix} \qquad (11)$$

In the particular case where $P_j \neq \ell_{n-i},\ h_{n-i}$ the transform derivative reduces to:

$$\left(\frac{\partial T_{i+1}}{\partial P_j}\right)=\frac{\ell_{n-i}^{2}\left(\ell_{n-i}^{2}-T_{n-i}^{1\ 2}\right)}{\left(\ell_{n-i}^{2}+T_iT_{n-i}^{1}\right)^{2}}\left(\frac{\partial T_i}{\partial P_j}\right) \quad ..(12)$$

Equation (9) is not a complete definition of the transform derivative because it does not include the terminal value (i=1). This is found by differentiating equation (3):

$$\left(\frac{\partial T_2}{\partial P_j}\right)=\left(\frac{1+k_{12}\exp(-2\lambda h_1)}{1-k_{12}\exp(-2\lambda h_1)}\right)\left(\frac{\partial \ell_1}{\partial P_j}\right)+\ell_1\frac{\partial}{\partial P_j}\left(\frac{1+k_{12}\exp(-2\lambda h_1)}{1-k_{12}\exp(-2\lambda h_1)}\right)$$

$$=\left(\frac{1+k_{12}\exp(-2\lambda h_1)}{1-k_{12}\exp(-2\lambda h_1)}\right)\left(\frac{\partial \ell_1}{\partial P_j}\right)+\frac{2\ell_1}{(1-k_{12}\exp(-2\lambda h_1))^2}\frac{\partial}{\partial P_j}[k_{12}\exp(-2\lambda h_1)] \quad ..(13)$$

The partial derivative on the far right of equation (13) reduces as follows:

$$\frac{\partial}{\partial P_j}[k_{12}\exp(-2\lambda h_1)]=\exp(-2\lambda h_1)\left(\frac{\partial k_{12}}{\partial P_j}\right)+k_{12}\frac{\partial}{\partial P_j}[k_{12}\exp(-2\lambda h_1)]$$

$$=\exp(-2\lambda h_1)\frac{\partial}{\partial P_j}\left(\frac{\ell_2-\ell_1}{\ell_2+\ell_1}\right)-2\lambda k_{12}\exp(-2\lambda h_1)\left(\frac{\partial h_1}{\partial P_j}\right) \quad ..(14)$$

$$where: \quad \frac{\partial}{\partial P_j}\left(\frac{\ell_2-\ell_1}{\ell_2+\ell_1}\right)=(\ell_2+\ell_1)^{-2}\left((\ell_2+\ell_1)\frac{\partial}{\partial P_j}(\ell_2-\ell_1)-(\ell_2-\ell_1)\frac{\partial}{\partial P_j}(\ell_2+\ell_1)\right)$$

$$=2(\ell_2+\ell_1)^{-2}\left(\ell_1\left(\frac{\partial \ell_2}{\partial P_j}\right)-\ell_2\left(\frac{\partial \ell_1}{\partial P_j}\right)\right) \quad ..(15)$$

Applying equations (14) and (15) to equation (13), the terminal derivative becomes:

$$\left(\frac{\partial T_2}{\partial P_j}\right)=\left(\frac{\partial h_1}{\partial P_j}\right)\left(\frac{-4\ell_1\lambda k_{12}\exp(-2\lambda h_1)}{\{1-k_{12}\exp(-2\lambda h_1)\}^2}\right)+\left(\frac{\partial \ell_2}{\partial P_j}\right)\left(\frac{2\ell_1\exp(-\lambda h_1)}{(\ell_1+\ell_2)\{1-k_{12}\exp(-2\lambda h_1)\}}\right)^2$$
$$+\left(\frac{\partial \ell_1}{\partial P_j}\right)\left(\frac{(\ell_1+\ell_2)^2\{1-k^2{}_{12}\exp(-4\lambda h_1)\}-4\ell_1\ell_2\exp(-2\lambda h_1)}{(\ell_1+\ell_2)^2\{1-k_{12}\exp(-2\lambda h_1)\}^2}\right) \quad (16)$$

The three partial derivatives on the right of equation (16) are either zero or unity: generalising h_1, ℓ_1 *and* ℓ_2 *by* p_k,

$$\left(\frac{\partial P_k}{\partial P_j}\right)=\delta_{jk}=\begin{vmatrix}0 & j\neq k\\ 1 & j=k\end{vmatrix} \quad (17)$$

Koefoed (1970), Dash and Ghosh (1974), Johansen (1975), Meinardus (1970). Equations (1) to (17) above are used in the resistivity transform function for the purpose of interpreting field results in terms of lithological variation with depth.

A basic equation in electrical and spontaneous potential logging is the empirical formula due to Archie (1942) given by

$$\ell_e = a\phi^{-m}s^{-n}\ell_w \text{..} (18)$$

where ℓ_e is resistivity of porous rock, ϕ is the fraction of pore volume (porosity), S is the fraction of pores containing water, ℓ_w is the resistivity of water, n is equal to 2 for practical purposes and a, m are constants

Archie's empirical formula can be modified into three significant equations. The first expresses the bulk water – wet resistivity of a rock sample ℓ_w, and the resistivity of the water contained in its pores, ℓ_o, in terms of a formation resistivity factor, F:

$$F = \ell_o/\ell_w \text{..} (19)$$

Secondly, it was also shown that the formation factor is a function of the porosity and, to a lesser degree, of the permeability of the sample. The relation is

$$F = 1/\phi^m, \text{..} (20)$$

where ϕ is the porosity of the material and M is a cementation factor whose value lies between 1.3 and 2.6. An alternative form of this expression, called the Humble formula, applicable to many granular rocks is

$$F = 0.62\,\phi^{-2.15} \quad \text{.....(21)}$$

The third empirical equation by Archie accounts for partial water saturation of the rock; if S_w is the fraction of the pore volume filled with water,

$$S_W = \left({\rho_0}/{\rho_t} \right)^{1/n} \quad \text{.....(22)}$$

where ρ_a is the true resistivity of the sample, derived by applying corrections for tool dimensions and configuration, borehole diameter, mud resistivity, etc. to the measured (or apparent) resistivity, ρ_t, and n the saturation exponent, which lies between 1.5 and 3.0; n is usually assumed to be 2 where there is no evidence to the contrary.

Shaly formations

Clays and shales, however, are not rare, and they do contribute to formation conductivity. Shale exhibits conductivity because of the electrolyte that it contains and because of an ion-exchange process whereby ions move under the influence of the impressed electric field between exchange sites on the surface of the clay particles Poupon et al, (1963).

Evaluation of shaly formations, usually shale sands, is somewhat complex. All logging measurements are influenced by the shale, and corrections for shale content are required Worthington (1975). Most shaly sand interpretation models employ a weighted-average technique to evaluate the relative contributions the sand and shale phases to the overall shaly sand response. For example, in the case of bulk density as measured by a density log, the relationship is

$$\sigma_b = \phi\left(S_{X0}\sigma_{mf} + S_{hr}\sigma_h\right) + V_{sh}\sigma_{sh} + \left(1 - \phi - V_{sh}\right)\sigma_{ma} \quad \text{.....(23)}$$

where V_{sh} is the bulk - volume fraction of shale

σ_{sh} is the density of shale

σ_h is the apparent density of the material

σ_{ma} is matrix density

σ_{mf} is mud filtrate density.

There are many formulae that relate resistivity to water saturation in shaly sands. Many are generally of the form

where V_x is a term related to the volume, or some specific volumetric characteristic, of the shale or clay; is a term related to the resistivity of the shale or clay; and C, if it occurs in the formula, is a term that relates to the water saturation, Sw.

When the shale volume is zero (i.e. a clean sand), eqn, (24) reduces to the Archie water saturation equation given by

$$S_W{}^2 = \frac{F\ell_W}{\ell_t} \qquad \text{...(25)}$$

This is true for all shally sand water saturation interpretation techniques.

EXPERIMENTAL WORK

Afuze, one of the towns where this research was carried out is situated around 6.9°N and 6.1°E while Eme-Ora the second town is situated around 6.8°N and 5.95°E. .1. The Schlumberger electrode array system was used for the fieldwork, Osemeikhian and Asokhia (1994). The ABEM TERRAMETER SAS 300B manufactured in Sweden was used for taking surface resistivity readings. This equipment contains a borehole logging component that is attached for the purpose of logging the nearby boreholes. The mode of logging was the spontaneous potential method.

RESULTS AND DISCUSSION

The apparent resistivity curves for both Afuze and Eme-Ora are Q-type curves ($\ell_1 > \ell_2 > \ell_3$) and are shown in VES 1 and VES 2 respectively. The interpretation for each curve is given below the curve. The resistivity curve for Afuze, VES 1, shows that the earth surface is rather identical in structure (consisting of laterite and sand) for about 10m from the surface of the earth. The driller's log and the SP log, fig. 1(a) and fig. 1(b) agree well with this interpretation. Thereafter, the resistivity curves continued to fall sharply, indicative of the presence of clay. This result too was confirmed by the driller's log and the SP log.

On the other hand, the gradual fall of the resistivity curve for Eme-Ora was right from the surface of the earth. The driller's log and the SP log for Eme-Ora fig.2(a) and fig. 2(b) are indicative of the presence of clay minerals (usually resulting in low resistivity values) right from the surface of the earth.

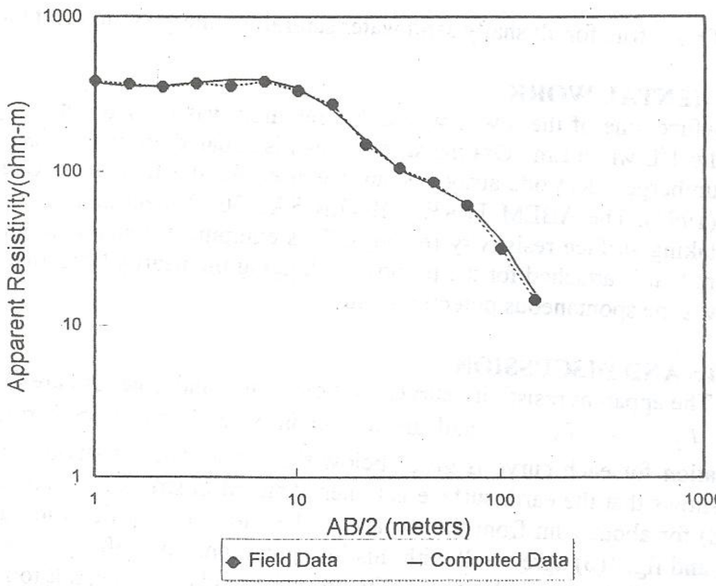
The SP log is positive for Afuze, indicative of low salinity but negative for Eme- Ora, indicative of high salinity.

CONCLUSION

The resistivity transform function used in the interpretation of the vertical electrical sounding data in this work is very reliable. The correlation of results obtained in the two towns in Edo north- Afuze and Eme-Ora with results obtained from drillers' logs and SP logs are so close within experimental errors, that no risk is involved in using this method of interpretation for vertical electrical sounding.

PROJECT: Field and theoretical curve for VES 1

SITE: Afuze



Project:

Resistivity Sounding Interpretation

Field and Theoretical Data

AB/2 values (metres)	Observed values (ohm-metre)	Computed values (ohm-metre)	Log difference
1.00	378.00	367.65	0.01
1.47	364.00	353.35	0.01
2.15	349.00	353.48	-0.01
3.16	365.00	371.01	-0.01
4.64	354.00	391.73	-0.04
6.81	377.00	390.82	-0.02
10.00	330.00	343.57	-0.02
14.70	271.00	251.64	0.03
21.50	148.00	159.32	-0.03
31.60	105.00	103.92	0.00
46.40	85.10	80.50	0.02
68.10	59.00	60.41	-0.01
100.00	31.30	35.43	-0.05
147.00	14.70	16.32	-0.05

Model Parameters

Geoelectric Layer	Specific Resistivity (ohm-metre)	Thickness (metres)	Cumulative Thickness (metres)
1	398.00	0.62	0.62
2	264.00	0.70	1.32
3	500.00	5.32	6.64
4	65.50	11.48	18.12
5	132.00	19.40	37.52
6	6.90	infinity	infinity

Field Measurments by: Ujuanbi O.

Computer Interpretations by: Ujuanbi O.

RMS error (%) 2 75

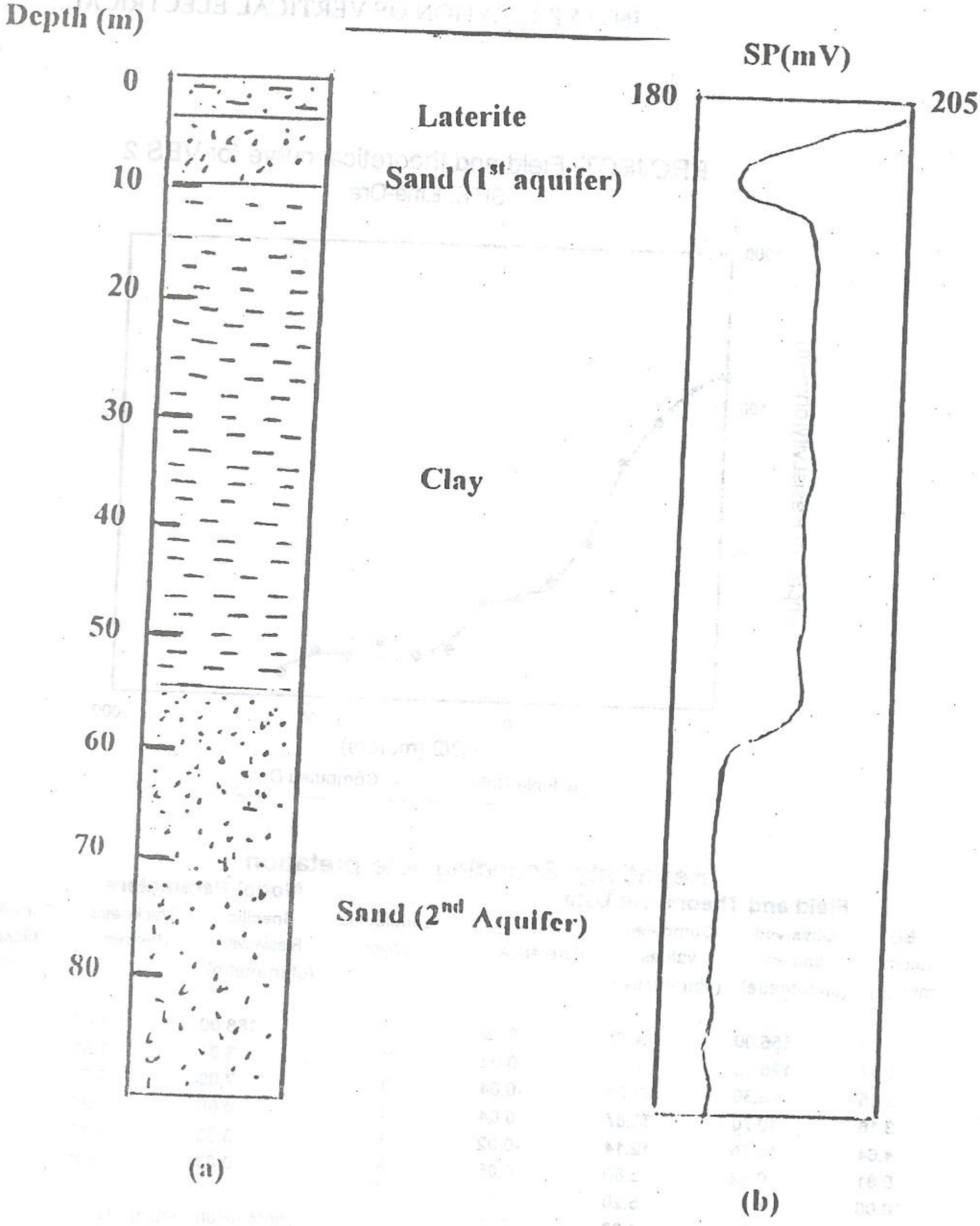
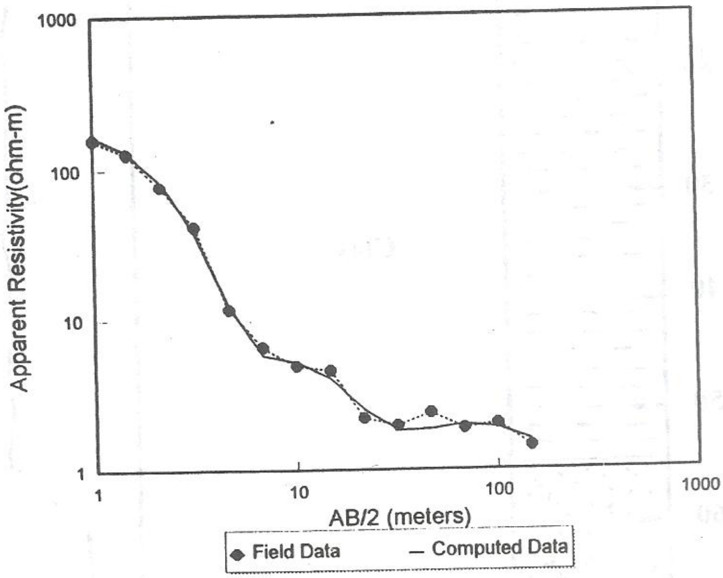


Fig. 1. Driller's/SP log in Afuze

PROJECT: Field and theoretical curve for VES 2
SITE: Eme-Ora



Resistivity Sounding Interpretation

Field and Theoretical Data				Model Parameters			
AB/2 values metres)	Observed values (ohm-metre)	Computed values (ohm-metre)	Log difference	Geoelectric Layer	Specific Resistivity (ohm-metre)	Thickness (metres)	Cumulative Thickness (metres)
1.00	156.00	164.17	-0.02	1	188.00	1.07	1.07
1.47	126.00	131.77	-0.02	2	3.81	1.51	2.58
2.15	76.30	83.08	-0.04	3	7.09	5.35	7.93
3.16	40.70	36.87	0.04	4	0.66	8.98	16.91
4.64	11.50	12.14	-0.02	5	3.33	28.48	45.39
6.81	6 52	5.80	0.05	6	0.83	infinity	infinity
10.00	4.86	5.20	-0.03				
14.70	4.54	4.02	0.05				
21.50	2.18	2.52	-0.06				
31.60	1.95	1.81	0.03				
46.40	2.36	1.84	0.11				
68.10	1.86	1.98	-0.03				
100.00	2.01	1.88	0.03				
147.00	1.42	1.57	-0.04				

Field Measurments by: Ujuanbi O.

Computer Interpretations by: Ujuanbi O.

RMS error (%) 4.69

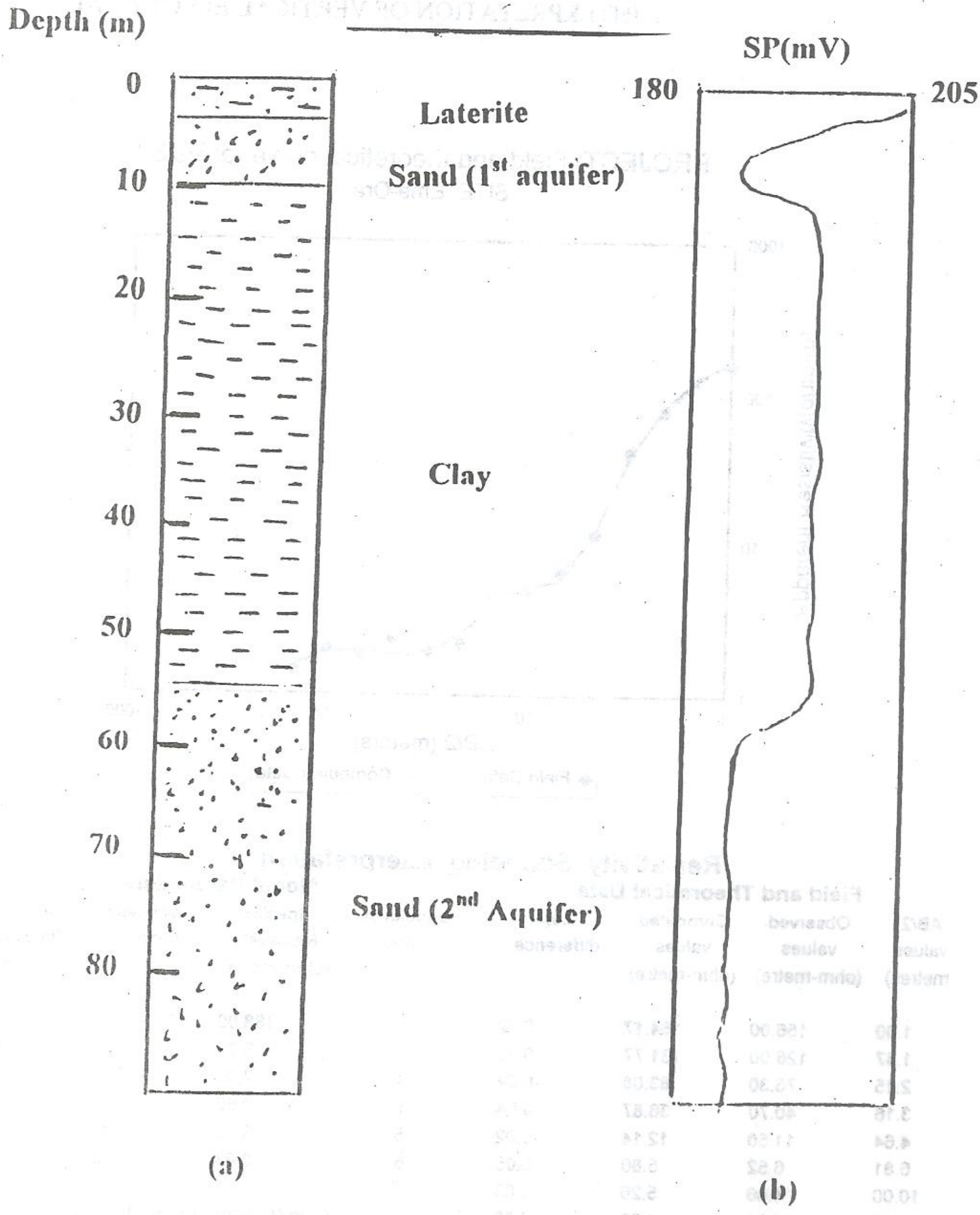


Fig. 1. Driller's/SP log in Afuze

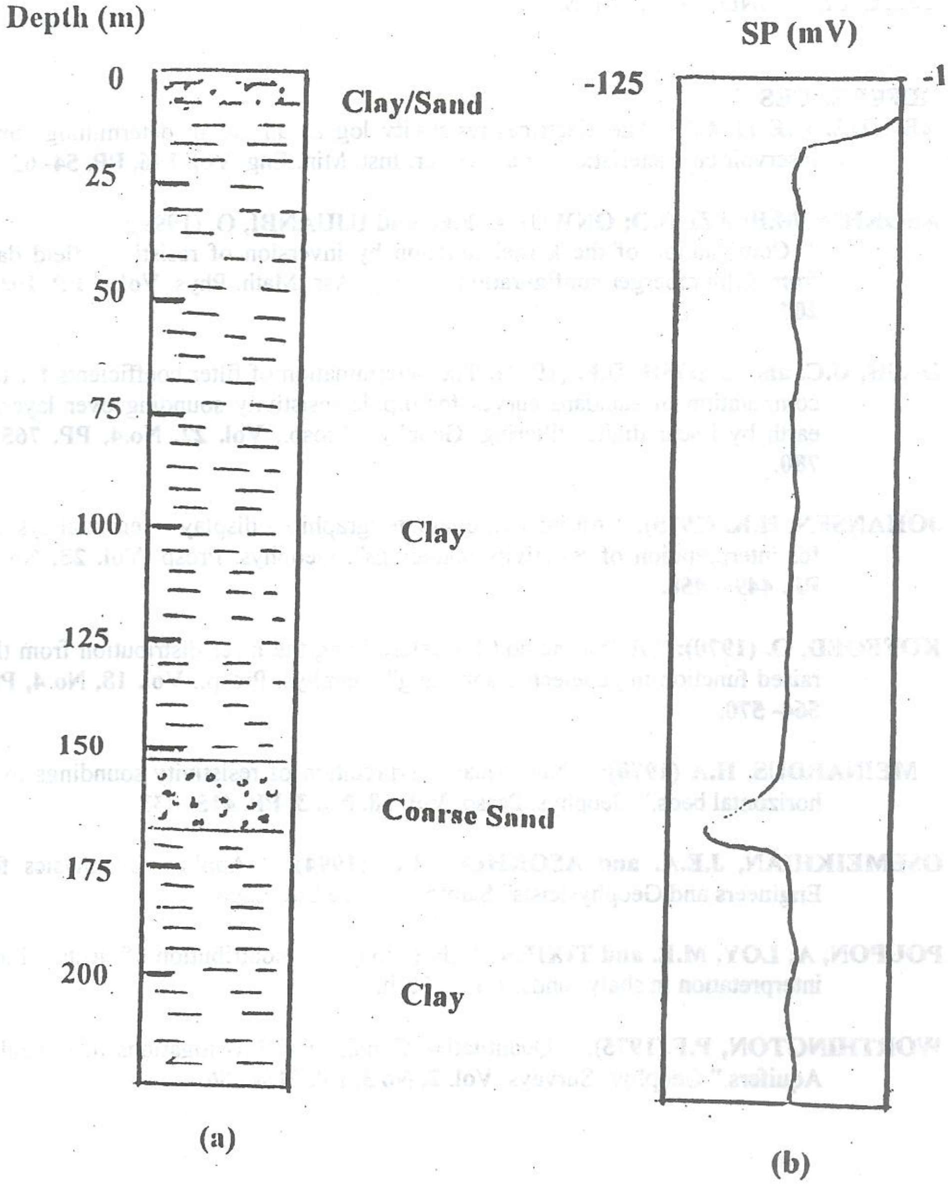


Fig. 2. Driller's/ SP log in Eme- Ora

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