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ESTIMATION OF MAGNETO- TELLURIC NOISE BY METHOD OF POLARISATION ANALYSIS

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ABSTRACT
Magneto – telluric (MT) survey is one of the latest methods of geophysical investigation. The first paper on this method was published by Cagniard in 1953. The method has the advantage of being able to probe up to 100km of the interior of the earth. A major problem is interference of data by noise. Noise is a function of coherence of signals. A common method of computing the noise present so far is by method of normalized transformed functions of magnetic and telluric signals. This work presents an alternative method of achieving the same purpose by method of polarisation analysis of magnetic and telluric signals. The already existing theories were used in interpreting the coherence of MT data from Sweden. The coherence values of the same data were also computed by the method of the proposed polarisation analysis. The results were so close that one could be substituted for the other within limits of experimental error. It is suggested that more research should be carried out to assertain the better method of analysing noise in MT. work.

INTRODUCTION
Airy (1868) produced the first available record of what appeared to be magneto – telluric (MT) data. The first published paper in MT survey was produced by Cagniard (1953). Berdichevsky (1963) as well as other workers expressed the relationship between magnetic and telluric fields in form of an impedance tensor

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix} \begin{bmatrix} H_x \\ H_y \end{bmatrix} \qquad (1)$$

where E is electric field intensity , H is magnetic field intensity and Z is impedance.

MT data are prone to interference by noise. Only high quality MT data can give meaningful interpretation. Coherence is a function of noise and interpretable data should have a coherence value of 0.5 or above. Usual methods of computing coherence values presently are by methods of normalised transformed coherence functions. This work presents an alternative method of coherence analysis by method of polarisation of both magnetic and telluric signals after Fowler et. al. (1967).

The usual theories for coherence analysis by method of normalised transformed coherence functions are first reviewed. The proposed alternative theories of achieving the same purpose by method of polarisation analysis of both magnetic and telluric signals are then presented.

THEORY.

A common technique of noise estimation in MT investigations is by method of normalised transformed coherence functions. If x(t), the signal input and y(t), the output are time series with power spectra $P_{xx}(f)$ *and* $P_{yy}(f)$ respectively and cross – spectrum $P_{xy}(f)$, then the coherence between the two time series is defined by

$$\gamma_{xy}^{2}(f)\,\gamma_{xy}^{2}(f)=\frac{\left|p_{xy}\right|^{2}}{\left(p_{xx}\quad p_{yy}\right)}\quad\ldots\ldots(2)$$

where $0\leq\gamma_{xy}\ \leq1$ *for all frequencies*

Foster and Guinzy (1967). Jones (1977) defined signal to noise ratio as

$$\frac{S(f)}{N(f)}=\frac{\hat{\gamma}_{xy}(f)}{1-\gamma_{xy}^{2}(f)}\quad\ldots\ldots(3)$$

where $\hat{\gamma}_{xy}$ is complex valued coherency and $\gamma_{xy}^{2}(f)$ coherency

The coherence for two input/ one output linear system is given by Jenkins and Watts (1968) as

$$\gamma_{y12}^{2}=\frac{P_{11}\left|P_{2y}\right|^{2}+P_{22}\left|P_{1y}\right|^{2}-2\,\mathrm{Re}\left(P_{12}P_{2y}P_{y1}\right)}{P_{22}\left(P_{11}P_{22}-\left|P_{12}\right|^{2}\right)}$$

In order to derive the time coherence between the output and one of the inputs, the effect of the other input must be removed. This is done in the least – square sense by computing the spartial coherence functions. These functions are given by Bendat and Piersol (1972) as

$$\gamma_{y1\cdot2}=\frac{\gamma_{y12}^{2}-\gamma_{y2}^{2}}{1-\gamma_{y2}^{2}}\quad\ldots\ldots(5)$$

and

$$\gamma_{y2\cdot1}^{2}=\frac{\gamma_{y12}^{2}-\gamma_{y1}^{2}}{1-\gamma_{y1}^{2}}\quad\ldots\ldots(6)$$

where $\gamma_{y1\cdot2}^{2}$ is the spartial coherence input x_1 (t) and the output y (t) with the effect of x_2(t) removed

It is necessary to normalise coherence function to unity so as to indicate directly the coherence signal to noise ratio. Normalised coherence functions are defined thus:

Normalised ordinary coherence function $= \hat{\gamma}^2_{yx}\left(\frac{n}{2}\right)$ ....(7)

Normalised multiple coherence function $= \hat{\gamma}^2_{y12}\left(\frac{n-2}{4}\right)$ ....(8)

Normalised partial coherence function $= \hat{\gamma}^2_{y1.2}\frac{(n-2)^2}{(2n+4)}$ ....(9)

where n is the number of degrees of freedom associated with the estimate, Jenkins and Watts (1968), Kendal and Stuart (1958). However, the original coherence function and hence the normalised coherence function defined by equations (7) to (9) do not have a normal distribution. As such, it is difficult to calculate confidence limits and other statistical limits. Applying Fisher's Z – transform we have the normalised transformed coherence functions given by

Normalised transformed ordinary coherency, NTOC

$$N_{yx} = \frac{\operatorname{arctan} h|\hat{\gamma}_{yx}|}{\operatorname{arctan} h\sqrt{2/n}} \qquad (10)$$

Normalised transformed multiple coherency, NTMC

$$\hat{N}_{y12} = \frac{\operatorname{arctan} h|\hat{\gamma}_{y12}|}{\operatorname{arctan} h\sqrt{4/(n-2)}} \qquad (11)$$

Normalised transformed partial coherency, NTPC

$$\hat{N}_{y1.2} = \frac{\operatorname{arctan} h|\hat{\gamma}_{y1.2}|}{\operatorname{arctan} h\frac{(\sqrt{2n+4})}{(n-2)}} \qquad (12)$$

Hald (1952), Enochson and Goodman (1965), Benignus (1969).

Equations (2) to (12) are the usual equations for estimating the noise present in MT data. Fowler (`1967) proposed some theories for analysing noise in the following types of data:

(i) Complex wave forms generated utilising combination of sinusoids.
(ii) Computer generated random number time series, and
(iii) Geomagnetic micropulsations resulting from nuclear events.

Fowler's methods are modified in this work to make them applicable in the analysis of noise in a fourth type of data sets – the magneto – telluric data. The method was used for analysing data from station K_3 S_{10} in Sweden.

The polarisation of any wave field can be described in terms of the coherent matrix elements of the wave field. For the case of the magnetic field this matrix is expressed as;

$$J = \begin{bmatrix} \dfrac{<H_x H_x^*>}{<H_y H_x^*>} & \dfrac{<H_x H_y^*>}{<H_y H_y^*>} \end{bmatrix} = \begin{bmatrix} J_{xx} & J_{xy} \\ J_{yx} & J_{yy} \end{bmatrix} \quad \text{.......(13)}$$

where H is the Fourier coefficient of the magnetic field components, the sharp brackets indicate time averages and the asterisks denote the complex conjugate. The matrix elements represent the smoothed auto – and cross – power estimates.

As the determinant of the coherence matrix is zero for a totally polarised wave, this property can be used to separate a partially polarised wave into its polarized and unpolarised parts, Fowler et. al. (1967):

$$J = \begin{bmatrix} J_{xx} & J_{xy} \\ J_{yx} & J_{YY} \end{bmatrix} = \begin{bmatrix} P_{xx} & P_{xy} \\ P_{yx} & P_{yy} \end{bmatrix} + \begin{bmatrix} U_{xx} & U_{xy} \\ U_{yx} & U_{yy} \end{bmatrix} \quad \text{.......(14)}$$

where P is the polarised signal and U is the unpolarised portion.

For the polarised portion

$$U_{xy} = U_{yx} = 0 \quad \text{.......(15)}$$

and

$$U_{xx} = U_{yy} = U \quad \text{.......(16)}$$

otherwise a degree of polarisation would exist. As such, we can express the matrix J as

$$J = \begin{bmatrix} J_{xx} & J_{xy} \\ J_{yx} & J_{yy} \end{bmatrix} = \begin{bmatrix} P_{xx} + U & P_{xy} \\ P_{yx} & P_{yy} + U \end{bmatrix} \quad \text{.......(17)}$$

From where we see that

$$\begin{aligned} P_{xx} + U &= J_{xx} \\ P_{xy} &= J_{xy} \\ P_{yx} &= J_{yx} \\ P_{yy} + U &= J_{yy} \end{aligned} \quad \text{.......(18)}$$

Solving for U we have

$$U = \frac{1}{2}\left[J_{xx} + J_{yy}\right] - \frac{1}{2}\left[P_{xx} + P_{yy}\right] \quad \text{.....................................(19)}$$

The intensity of the total signal T_i is

$$T_i[P] = P_{xx} + P_{yy} = \left[(J_{xx} + J_{yy})^2 - 4|J|\right]^{0.5} \quad \text{.....................................(20)}$$

where $|J|$ is the determinant of J

The degree of polarisation, R_1 is defined as the ratio of the polarised intensity to the total intensity, i.e.

$$R_1 = \frac{P_{xx} + P_{yy}}{J_{xx} + J_{yy}} = \left[1 - \frac{4|J|}{(J_{xx} + J_{yy})^2}\right]^{0.5} \quad \text{.....................................(21)}$$

Let θ be the angle which the principal axis of polarisation makes with X – axis or the orientation of polarisation ellipse. It represents the angle through which the coordinate axis must be rotated to bring it into coincidence with the axis of the polarisation.
It is given by

$$\tan 2\theta = \frac{2\,\mathrm{Re}\left[P_{xy}\right]}{P_{xx} - P_{yy}} \quad \text{...(22)}$$

The evaluation of the ratio of polarised power to total power provides an estimate of the mutual coherence between the orthogonal components within the frequency band being investigated, Fowler (1967).

Equations (13) to (22) were used as an alternative method of estimating coherence values for station K_3S_{10} in Sweden. The method could not be used for analysing the data from the other stations in Sweden because the data from these other stations were too scanty for any meaningful comparison purposes.

EXPERIMENTAL WORK

Asokhia (1997, 1998) showed the "Blue Road Traverse" in Sweden where the MT field work was carried out as well as the layout of the censors at the measuring site. A three component magnetometer of the variable – mu type constructed by the Electro-Mechanic Company in Austin, Texas was used for measuring the magnetic field. The telluric component was manufactured at Kiruna, north of Sweden. Asokhia (1979) gave further details on MT field work.

Once a suitable location has been found the first course of action was to measure the magnetic north and find out the east and west directions. Next the electrodes were buried. This gave electrodes as much time as possible to stabilize before commencing recording. It is essential that the influence of temperature on the potential gradient is not greater than 5×10^{-5} V/°C. Therefore it is necessary to bury the electrodes sufficiently deep in the ground. The electrode distance depends on the telluric intensity and the sensitivity of the recording instrument. In areas with sedimentary cover the distances are in the range 0.5 - 1.5km. In areas with crystalline rocks at the surface distances of a few metres are sometimes sufficient. Cables used to carry the signal from each electrode to the measuring equipment were also buried in shallow trenches. This was mainly to reduce temperature fluctuations and disturbances by animals. (The reindeer was common in the region of investigation).

The instrument incorporates digitisation of the data though analog record can always be produced with photographic developing instruments for inspection in the field.

RESULTS AND DISCUSSION

Fig. I shows the results of the coherence values obtained by method of normalised transformed coherence function and the method of polarisation analysis. The coherence values were very close for periods of about 125 seconds and below for the two methods. The mean coherence value obtained by method of normalised transformed coherence function was 0.75, Asokhia (1998) while it was about 0.73 by method of polarisation analysis. Significant differences were observed at periods longer than 125seconds and the reasons for this are uncertain yet.

CONCLUSION

The closeness in the results Obtained when coherence was computed by methods of polarisation analysis and the normalised transformed coherence function are so close that one may be tempted to substitute one for the other. Care should however be taken to carry out more research work in methods of polarisation analysis before arriving at such a conclusion. The depth of penetration of MT signals (over 100km) and the low cost of equipment both favour research work in MT survey. However, there are some constraints in MT investigations as compared with some other geophysical methods – seismic method for instance:

(1) MT survey is more time consuming – a site that can be investigated in one day by seismic methods may take a couple of weeks in MT survey.

(2) MT data are more prone to noise than seismic data and several other geophysical data.

(3) Interpretation techniques are more tedious in MT data that in many other types of geophysical data.

In spite of these constraints, MT survey methods are in their infancy and should be encouraged to grow to maturity.

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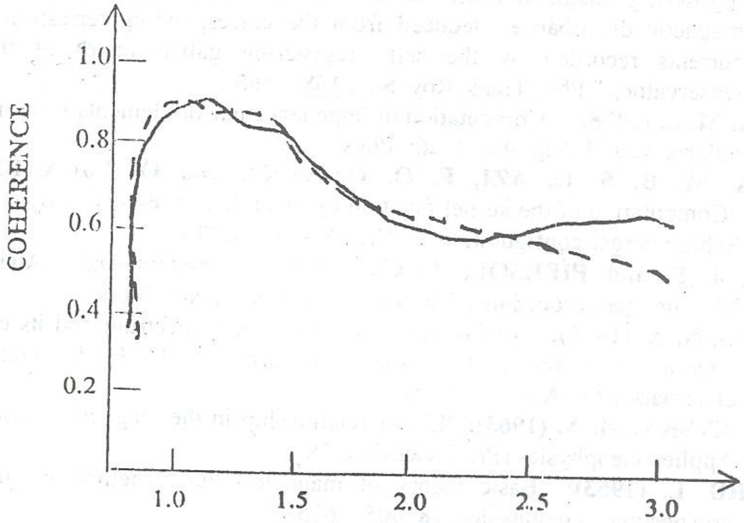

Log period in seconds

Fig.1. *Graph of log period against coherence. The unbroken curve shows coherence by method of normalized transformed functions while the other curve shows coherence by method of polarisation analysis.*

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