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EFFECTS OF HORIZONTAL OIL DRAINHOLE LENGTH ON WELLBORE PRESSURE IN STRATIFIED RESERVOIRS

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ABSTRACT

Multilateral technology has proved to be highly efficient in the production of stratified and layered reservoirs especially when the layers are not communicating with one another. Although, when there is communication between layers the tight layer is best produced by completing and producing the more permeable layer, drilling a lateral pair helps in the economic enhanced recovery of one layer if the other layer serves as a fluid injector. The length of drainhole is one of the most important factors as it has far reaching operational technical and economic consequences on the overall success. This then calls for the question: how long should the drainhole be? This question is tackled by solving an appropriate diffusivity equation relating reservoir pressure, rock and fluid properties and reservoir geometry (dimensions) with time. The final result is obtained by a numerical integration model using FORTRAN program at various well lengths to study the changes in wellbore (flowing) pressures. It is discovered that increasing the length of the drainhole does not lead to an increase in well productivity as the wellbore flowing pressures are found to be decreasing. The model enables the engineer to select an appropriate well length given reservoir characteristics and fluid properties and , of course, the variables of operational economics. It is further discovered that the model could be used to investigate the role of length in a well pair that is intended to be used for enhanced oil recovery programmes. This can be achieved by changing the mobility ratio, well length, for a given rock and fluid properties to observe the behavior of the wellbore pressure in each well.

INTRODUCTION

The concept and utilization of horizontal well technology dates back to the early 1940's. Its drilling was reported in 1941 by Renney[1]. Then, the only benefit that was thought to come from the technology was just an increase the sweep performance of oil and gas. Not much incentive could be given to such novel technology since the conventional wells then could give appreciable productivity with stimulation techniques such as fracturing and acidizing. Hence, hydraulic fracturing contented with horizontal well technology. Researches[2-8] that were later conducted on horizontal well technology showed that the increase in production or injection rates in horizontal wells was several times larger than that of vertical wells given the same formation, anisotropy or isotropy, pressure drop and choke performance. Horizontal wells have been found to be best in (1) naturally fractured reservoirs. They are drilled to connect multiple fracture systems with perpendicular wellbore to drain a larger area at higher production rates, (2)very thin reservoirs which could not be drained with vertical wells, (3)reservoirs prone to excessive gas and water production, (4)thin reservoirs with high vertical permeability and (5) reservoirs located where accessibility is difficult or impossible. The major set backs, so far are (1) high drilling costs, (2) technical difficulty in managing horizontal wells, (3) inability to produce layered non- communicating reservoirs through a single wellbore.

The performance of horizontal wells has attracted tremendous interest and favor in the recent past. Hence, since 1979 so much knowledge (and practice) has been acquired in the technology. Giger et al,[9,10] and Giger[11] have explained the reservoir aspects of horizontal well drilling. Huygen and Black[12] discovered that inspite of the unfavorable mobility ratios associated with heavy oils, horizontal wells gave a more homogeneous steam front and a much greater injectivity index than vertical wells. Gerrard and Dupuy[13] show that permeability reduction around the wellbore is less detrimental to flow in horizontal wells than in vertical wells. Islam and George[14] demonstrated in the laboratory that horizontal wells produce less sand than vertical wells. Inspite of extremely wide applicability of horizontal wells today, some efforts[15,17] have been made to understand the effects of well length on the productivity of horizontal wells in unlayered reservoirs. No such effort has, however, been made about layered reservoirs having horizontal drainholes in each layer.

The length of a horizontal well is a very important parameter in the overall success of a horizontal well in terms of well productivity. Productivity does not increase with increasing length. For a desirable productivity, the corresponding length has to be selected by the engineer. The total effective length of a horizontal well length can be varied by packing off the excess length or completely plugging off with cement.The selection of the effective length is governed by the flow dynamics of a horizontal well. This is also true for the kick-off radius of the well. If, however, the well was kicked off at any radius, the length modification could be carried out, as stated earlier, for the desired overall well performance.

This paper shows a detailed mathematical derivation of the pressure distribution in a two layered reservoir each bearing a horizontal drainhole. Although the layers may be communicating or not, this does not affect the choice of length and it is therefore not discussed. Source functions obtained from Laplace transformation of the Green's function for each flow direction were obtained which led to the derivation of the final solution using the Newman's product rule. The ensuing integration was performed numerically. It should be noted, however, that the derivation are for early time (transient) flow only. In the numerical model, the dimensionless pressures as a function of time were generated for different lengths.

RESERVOIR MODEL EQUATION

The diffusivity equation governing the flow of oil in a reservoir is obtained by a combination of (1) the law of conservation of mass, (2) the equation of state, and (3) the Darcy's law. In 3D flow, the equation is written as follows:

$$k_x \frac{\partial^2 p}{\partial x^2} + k_y \frac{\partial^2 p}{\partial y^2} + k_z \frac{\partial^2 p}{\partial z^2} = \phi \mu c_t \frac{\partial p}{\partial t} \tag{1}$$

Equation (1) is a second order, linear, heterogeneous partial differential equatioı describing the flow of oil in a horizontal well drilled into an anisotropic reservoir. In this paper the solution to equation (1) will be sought in other to understand the effect of length on wellbore pressures in drianholes drilled into a stratified and layered reservoir.

RESERVOIR MODEL DESCRIPTION

The stratified reservoir shown in Fig.2 is assumed to consist of two lateral wells, one each drilled into a different layer. In otherwords, there are two layers in the reservoirs. Each layer contains fluid of small but constant compressibility but not necessarily the same viscosities. The wells are assumed to be parallel to the top and bottom no flow boundaries and each is located in a rectangular drainage region. The wells are produced at a constant rate (uniform flux) with a unit pressure drop (infinite conductivity) across the sand face of each wellbore. The degree of stratification is represented by the mobility ratio. The reservoir pressure before production, injection or buildup is p_i . A relationship between the dimensionless pressure drop with different well positions and rate of production, injection or buildup together with time is desired to properly understand the behavior of the reservoir system. The wells have radii r_{wi} and lengths x_i (along the x-axis), heights, h_{wi} (along the vertical axis). The widths of the wells, y_{wi} are along the y-direction from y_{wi} to $y=\infty$ .Thus, the reservoir is semi-infinite in dimensions. Fluid flow direction will depend on whether (1)there is crossflow or (2)there is no crossflow. Under crossflow situation flow direction is essentially in the z-direction across each layer. Flow will be in the x-direction towards the parent well if there is no crossflow.

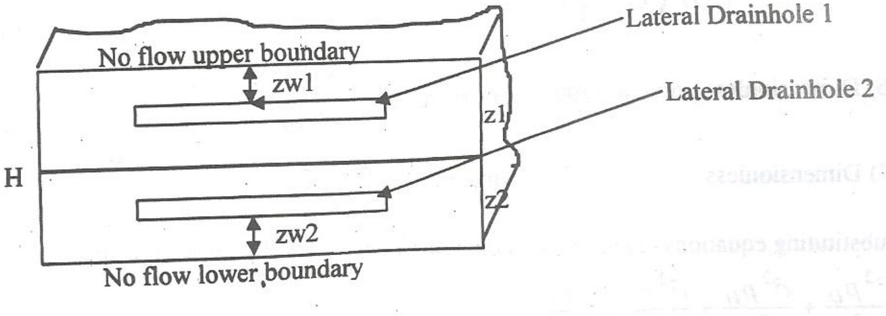


Fig.2: Reservoir Model Showing Well Positions

The reservoir is anisotropic with permeabilities k_x, k_y, and k_z in the x-,y-,and z- directions respectively. These permeabilities are considered to be independent of pressure. Production pressures are expected to be kept above the bubble point of the reservoir oil.

As there are no outer boundaries either at the bottom or on top of the reservoir, the energy required for fluid flow is strictly derived from the dissolved gas in the oil. Skin effects during these flow periods are , however, considered small and are therefore neglected.

SOLUTION TO THE DIFFUSIVITY EQUATION

We shall introduce the following dimensionless quantities:

(1) Dimensionless well length,

$$x_D = \frac{x}{H}\sqrt{\frac{k_z}{k_y}}$$

(2)

(2)Dimensionles well width,

$$y_D = \frac{y}{H}\sqrt{\frac{k_z}{k_y}}$$

(3)

(3) Dimensionless reservoir thickness,

$$z_D = \frac{z}{H}$$ (4)

(4) Dimensionless well thickness,

$$h_D = \frac{h}{H}$$

(5)

(5)Dimensionless effective wellbore radius,

$$r_{wD} = \left(\frac{r_w}{2H}\sqrt{\frac{k_z}{k_x}} + \sqrt{\frac{k_z}{k_y}}\right)$$

(6)

(6) Dimensionless pressure drop, $p_D(x_D,y_D,z_D,t_d) = \dfrac{\sqrt{k_x k_y}\, z\Delta p}{141.2 q\mu B}$

(7)

(7) Dimensionless time, $t_D = \dfrac{0.000264 k_z t}{\phi\mu c_t H^2}$

(8)

Substituting equations (2) to (8)into equation (1), we have the following equation

$$\frac{\partial^2 p_D}{\partial x^2_D} + \frac{\partial^2 p_D}{\partial^2 y_D} + \frac{\partial^2 p_D}{\partial z^2_D} = \frac{\partial p_D}{\partial t_D}$$

(9)

The final solution to equation (9) is in Appendix A as equation (A19).

To study the effects of well length, x_D, on p_{WD}, x_D is varied for a fixed z_{WD} (well stand-off) and the p_{wD} values calculated numerically.

PERIOD OF OCCURRENCE OF EARLY TIME FLOW

The early time flow period is the period when no boundary has been felt by the transient generated in the wellbore. The reservoir behaves as if it were infinite in all directions. Therefore, during this period, flow pressures increase with time. This is reflected in table1below and in the trends of the curves in fig.1. In dimensionless variables, early time period is given, for the reservoir model Fig. 2, as:

$$t_D \angle \min \begin{cases} \left(x^2{}_{D1}, x^2{}_{D2}\right)/20 \\ \left(y^2{}_{D1}, y^2{}_{D2}\right)/20 \\ \left(z^2{}_{D1} + z^2{}_{wD2}, z^2{}_{D1} + z^2{}_{wD2}\right)/20 \end{cases} \tag{10}$$

RESULTS AND DISCUSSIONS

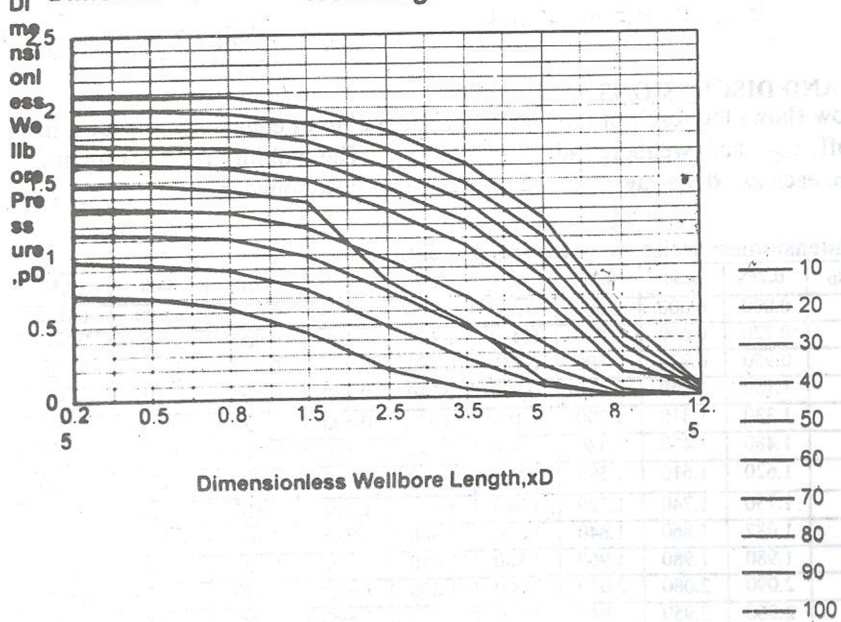
Table 1 below shows the result of the numerical integration of equation(A19) for a fixed well stand-off, z_{wDi}, and wellbore radii x_{wD}, and y_{WDi}. The mobility ratio is kept at 0.5. The wells are each z_D =0.5 away from the respective no flow boundaries.

Table 1: Dimensionless wellbore pressure, p_{wD} data.

T_D	x_D	0.25	0.50	0.80	1.50	2.50	3.50	5.00	8.00	12.50
1		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10		0.720	0.690	0.630	0.450	0.190	0.500	0.000	0.000	0.000
20		0.950	0.930	0.890	0.750	0.490	0.260	0.070	0.000	0.000
30		1.150	1.140	1.110	0.990	0.740	0.480	0.200	0.010	0.000
40		1.330	1.310	1.290	1.180	0.950	0.690	0.350	0.0400	0.000
50		1.480	1.470	1.440	1.350	1.140	0.800	0.510	0.100	0.000
60		1.620	1.610	1.590	1.500	1.300	1.050	0.670	0.170	0..100
70		1.750	1.740	1.720	1.640	1.450	1.210	0.820	0.250	0.020
80		1.087	1.860	1.840	1.760	1.580	1.350	0.960	0.340	0.030
90		1.980	1.980	1.960	1.880	1.710	1.480	1.100	0.430	0.050
100		2.090	2.080	2.070	2.000	1.830	1.610	1.230	0..530	0.070
200		2.960	2.950	2.940	2.890	2.770	2.590	2.260	1.490	0.550
300		3.620	3.620	3.610	3.570	3.470	3.320	3.030	2.290	1.190
400		4.180	4.180	4.170	4.130	4.050	3.920	3.660	2.970	1.810
500		4.680	4.670	4.670	4.630	4.550	4.440	4.200	3.550	2.390
600		5.120	5.120	5.110	5.080	5.010	4.910	4.690	4.080	2.930
700		5.540	5.530	5.530	5.500	5.430	5.330	5.130	4.550	3.430
800		5.920	5.910	5.910	5.880	5.820	5.730	5.530	4.980	3.890
900		6.280	6.270	6.270	6.240	6.180	6.100	5.910	5.390	4.320
1000		6.620	6.610	6.610	6.590	6.530	6.440	6.270	5.770	4.730

For the values of dimensionless well length, x_D, and at fixed values of dimensionless time, t_D, the dimensionless wellbore pressures are decreasing the larger the x_D. This is the expected behavior given the physics of fluid flow. At large distances from the well junction fluid velocities generated by pressure differential are attenuated with highest severity at the tip of the wellbore. Naturally, therefore, pressure will also drop the longer the wellbore in horizontal wells.In vertical wells, fluid rather converge towards the wellbore where fluid flow velocities are highest. Hence, pressure drops are higher for vertical wells in and around the wellbore. This is why vertical wells promote gas and water coning easily moreso since the breakthrough time for these fluids is shorter and the area exposed to is narrower.

Fig.1: Dimensionless Pressure versus Dimensionless Well Length



A closer look at the trends of the lines of the graph shows four distinguishable gradients describing the flow periods in a horizontal well. The first period has the highest gradient (pressure against time)and describes the infinite acting period when no boundary of any kind has been felt. Derive this period the well behaves like a fully penetrating vertical well. All wells, no matter their sizes, experience this flow period. It is called the early radial flow period. When the nearest boundary is felt, the flow behavior ceases to be infinite. This is the beginning of the transition flow period. If the transient is generated long enough, the vertical ends of the wellbore are felt if the well is thinner in height than the length. Eventually, the lateral ends of the well are felt. This is the intermediate-time linear flow period. It is similar in analogy to a vertical fracture. On the other hand, if the well thickness is larger than the length, this intermediate-time linear flow period would be masked and this leads to a substantially long flow period before the transient enters the plane of the formation.

When the transient has been generated long enough, flow lines now get highly concentrated in the plane of the reservoir formation. The well now behaves like a small point source in an infinitely large formation domain. This flow period is further pronounced if the reservoir is much larger in width than the length of the wellbore. This flow period is called the late-intermediate-time linear flow period.

If transients generated are strong enough they eventually reach the reservoir boundaries. During this period the flow is completely pseudosteady since pressures now decline linearly with time. The rate of pressure decline depends on the formation permeability and flow rate. This flow period is the last and it is called the late-time linear flow period. It is most useful in the determination of the productivity indices. For fully penetrating wells skin effect due to partial penetration is zero. Thus, the productivity index J, will now be a function of horizontal well length or pay length. A partially penetrating well may precipitate enormous reduction or a booster in the J of a horizontal well, depending on both well length and pay length. Partial entry may be due to partial drilling or incomplete completion of the drainhole. Equally important, however, are the directional permeabilities of the reservoir. A decrease in the vertical permeability by a factor of four decreases J by a factor of 1.8 for a fully penetrating well and by a factor of 1.7 for a well that penetrated just halfway[16].

In layered reservoirs with each layer having a horizontal well the more permeable layer will drain faster than the other layer if flow is allowed through the more permeable layer only. This method of production is preferred when there is cross flow between the layers. When there is no cross flow the wells have to be commingled into a common parent wellbore. Under crossflow condition individual well studies cannot be accurate unless it is performed when the farthest boundary has been felt. This may be achievable only if the horizontal permeability $\sqrt{k_x k_y}$ is high enough for quicker propagation of flow transients.

The use of mobility ratio, M, in this study will enable a feasibility study of the use of multilateral drainholes for enhanced oil recovery to be possible. The performance of the drainholes when $M < 1$, $M > 1$ or $M = 1$ can be studied. This will enable the correct choice of the displacing fluids, well orientation and size of surface equipment. Finally, the engineer can now select the correct length of his wellbore given the limit of available capital, lease size and other variables that are essential in the overall operational profit maximization.

CONCLUSION

The effect of well length on wellbore pressures has been studied mathematically. Although, no real field problem was used to validate our study, observed trends are in conformity with published works so far. It is the belief of these authors that since the solution methods are the conventional ones known for solving related problem, the size of error will be acceptably minimal. Furthermore, the use of dimensionless quantities removes the risk of large errors since only order of magnitudes are involved in the computations. Finally, it should be noted that the accuracy of our study is limited by the simplifying assumptions made in arriving at the final analytical solutions.

NOMENCLATURE

- B - Formation volume factor, rb/stb
- c_t Total compressibility, 1/psi
- H Total reservoir thickness, ft
- h Well thickness, ft
- f Interface position, fraction
- J Productivity index, stb/day/psi
- k Permeability, md
- M Mobility ratio
- p Pressure, psi
- p_i Initial reservoir pressure, psi
- p_w Wellbore pressure, psi
- q Production rate, stb/day
- s Instantaneous source function
- t Time, hours
- x,y,z Space distances, ft

Symbols

- μ Viscosity, cp
- ϕ Porosity, fraction
- Δ Change from initial value
- η Diffusivity constant
- τ Time dummy variable

Subscripts

- D Dimensionless
- i layer1 or layer 2
- w Well

Appendix A

Equation (9) is solved using source functions which are the Laplace's fundamental solution of the diffusivity equation for linear systems. For each dimension shown in figure 2 and equation (9), the source functions are:

$$s(x_D,t_D)=\frac{A}{2\sqrt{\pi t_D}}e^{-\frac{x^2{}_D}{4t_D}} \qquad \text{A1}$$

$$s(y_D,t_D)=\frac{B}{2\sqrt{\pi t_D}}e^{-\frac{y^2{}_D}{4t_D}} \qquad \text{A2}$$

and finally,

$$s(z_D,t_D)=\frac{C}{2\sqrt{\pi t_D}}e^{-\frac{z^2{}_D}{4t_D}} \qquad \text{A3}$$

where A,B, and C are constants to be determined in order to find the final size of the source functions. The instantaneous source function for each horizontal (lateral) well is obtained using the Newman product rule[16] which is a product of all the source functions as follows:

$$s(x_D,y_D,z_D,t_D)=s(x_D,t_D)s(y_D,t_D)s(z_D,t_D) \qquad \text{A4}$$

where the constant E=A*B*C.

The following conditions are used to determine the constant E:

(1) Boundary Conditions

(a) At the top of the reservoir there is no fluid flow, i.e.,

$$\frac{\partial s(x_D,y_D,f=1,t_D)}{\partial z_D}=0 \qquad \text{A5}$$

(b)Also, at the bottom of the reservoir there is no fluid flow, i.e.,

$$\frac{\partial s(x_D,y_D,f=0,t_D)}{\partial z_D}=0 \qquad \text{A6}$$

(2) Interface Conditions

(a)At the interface, f, between the two layers, the source strengths are the same, i.e.,

$$s_1(x_D,y_D,z_D=f,t_D)=s_2(x_D,y_D,z_D=f,t_D) \qquad \text{A7}$$

(b) Furthermore, the fluid velocities are assumed to be the same at the interface, i.e.,

$$-\frac{\partial s_1(x_D,y_D,z_D=f,t_D)}{\partial z_D}=M\frac{\partial s_2(x_D,y_D,z_D=f,t_D)}{\partial z_D} \qquad \text{A8}$$

where M is the mobility ratio.

(3) Initial Conditions

As production commences, the pressure drop causing flow is maintained at unity for as long as flow occurs. That is,

$$S(x_D,y_D,z_D,t_D) = 1 \qquad \text{A9}$$

Equation (A9) represents the constant terminal pressure condition. Application of the boundary conditions to equation (A4) gives $E_1 = E_2 = 0$. Application of the interface conditions gives

$$E_1 e^{-\frac{f^2}{4t_D}} = E_2 e^{-\frac{f^2}{4t_D}}$$

and

$$-E_1 e^{-\frac{f^2}{4t_D}} = ME_2 e^{-\frac{f^2}{4t_D}}$$

or

$$E_1 = E_2 \qquad \text{A10}$$

and

$$-E_1 = ME_2 \qquad \text{A11}$$

where M serves as an eigen value.

In matrix form equations (A10) and (A11) are written as

$$\begin{bmatrix} 1 & -1 \\ -1 & -M \end{bmatrix} \begin{pmatrix} E_1 \\ E_2 \end{pmatrix} = \begin{matrix} 0 \\ 0 \end{matrix} \qquad \text{A12}$$

The solution to matrix equation (A12) is $E_1 = E_2 = 0$ once again.

But this is not possible physically. Therefore, the determinant is rather set to zero. The determinant is obtained as

$$M + 1 \qquad \text{A 13}$$

That is,

$$M = -1 \qquad \text{A14}$$

Equation A4 may be represented in form of Fourier series[18] similar to the initial condition (equation A9) using a triple infinite summation as follows.

$$\left[\sum_{m=1}^{\infty}\left[\sum_{n=1}^{\infty}\left[\sum_{l=1}^{\infty} s(x_D,t_D)\right]_l s(y_D,t_D)\right]_n E s(z_D,t_D)\right]_m = 1 \qquad \text{A15}$$

Solving, we have, using orthogonality conditions

$$\sum_{m=1}^{\infty}\left[\sum_{n}^{\infty} s(x_D,t_D)\right] E s(z_D,t_D) = \frac{[1, s(y_D,t_D)]}{[s(y_D,t_D), s(y_D,t_D)]} = \frac{\int_{y_{wD}}^{y_D} [s(y_D,t_D)] dy_D}{\int_{y_{wD}}^{y_D} [s(y_D,t_D)]^2 dy_D} \qquad \text{A16}$$

Similarly, expressions for the other sources can be written. For the wellbore, $y_D=0$. We therefore obtain upon performing all the integration with indicated limits that

$$E = 2\sqrt{2}\left[\frac{erf(\frac{x_D}{2\sqrt{t_D}}) + Merf(\frac{x_{WD}}{2\sqrt{t_D}})}{erf(\frac{x_D}{\sqrt{2t_D}}) + Merf(\frac{x_{WD}}{\sqrt{2t_D}})}\right] \times \left[\frac{erf(\frac{y_{wD}}{2\sqrt{t_D}})}{erf(\frac{y_{wD}}{\sqrt{2t_D}})}\right] \times \left[\frac{erf(\frac{1}{2\sqrt{t_D}})}{erf(\frac{1}{\sqrt{2t_D}})}\right] \quad \text{A17a}$$

But there is no radius alomg the x-axis, i.e., along the well length. Therefore $x_{wD}=0$. Hence the final expression for E is written as:

$$E = 2\sqrt{2}\left[\frac{erf(\frac{x_D}{2\sqrt{t_D}})}{erf(\frac{x_D}{\sqrt{2t_D}}))}\right] \times \left[\frac{erf(\frac{y_{wD}}{2\sqrt{t_D}})}{erf(\frac{y_{wD}}{\sqrt{2t_D}})}\right] \times \left[\frac{erf(\frac{1}{2\sqrt{t_D}})}{erf(\frac{1}{\sqrt{2t_D}})}\right] \quad \text{A17b}$$

The dimensionless pressure drops for the wells are therefore given as

$$p_{Di}(x_D, y_D, z_D, t_D) = \int_0^{t_D}\int_{z_{wD}}^{z_D} s_i(x_D, y_D, z_D, t_D)\, dz_D\, d\tau \quad \text{A18}$$

The sense of integration in equation (A18) is to show that flow is predominantly vertical in each of the two layers. The wells are located at $z_{wDi} \leq z_D \leq z_{Di}$ (in the vertical direction). The final solution is obtained as:

$$p_{Di}(x_D, y_D, z_D, t_D) = \frac{1}{8\sqrt{(\pi t_D)^3}} \int_0^{t_D} \frac{e^{-\frac{x^2_D + y^2_D}{4\tau}}}{\tau} . E_i \left[erf(\frac{z_{Di}}{\sqrt{4\tau}}) + Merf(\frac{z_{wDi}}{\sqrt{4\tau}}) \right] d\tau. \quad \text{A19}$$

The integral in equations (A19) are obtained using numerical integration. Results for well 2 will show exactly the same pattern as that of well 1 in terms of pressure responses.

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