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A SIMPLIFIED COMPUTER ITERATION TECHNIQUE FOR THE INTERPRETATION OF VERTICAL ELECTRICAL SOUNDING

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ABSTRACT

A major problem in resistivity research work is interpretation of field curves. This is due to the complexities of the earth structure as a result of which it is extremely difficult to derive a suitable mathematical model that will fit the earth model perfectly. As a result, the resistivity method has fallen prey to a number of empirical procedures which are purported to give quick and simple solutions but in the final analysis fail to achieve this end. This paper proposes some acceptable assumptions to simplify the structure of the earth in such a way as to facilitate the problem of deriving simple mathematical models to fix the structure of the earth approximately. Such a technique was used in interpreting vertical electrical sounding from Umuduruokoro, near Owerri, Imo State. Within only about seven iterations a fit was found between field curve and the theoretical curve with a mean square error of less than two percent. The driller's log for the borehole drilled from this survey was in perfect agreement with the result of this geophysical investigation.

INTRODUCTION

Despite the operational simplicity of the resistivity method, " the problems of interpretation are among the most difficult in geophysics" Frohlich (1973). The difficulties follow directly from the mathematical complexities implicit in the derivatives of suitable theoretical models. Because of these mathematical complexities, the resistivity method has fallen prey to a number of empirical procedures which are purported to give quick and simple solutions, Barnes (1952), Narayan and Ramanujachary (1967). These empirical procedures have not stood the test of field data. Muskat (1945) and several other workers have debunked the methods of empirical procedures.

Barnes (1952) as well as Narayan and Ramanujachary (1967) assumed that boundaries at depth on measured curves appear as maxima, minima, inflexions or sudden breaks. These theories are not widely accepted.

Moore (1945) proposed the method of plotting cumulative values of apparent resistivities and drawn tangents to each distinct segment. According to him, the intersection of the segment gives depths to each boundary. Greenhalph (1974) subjected Moore's method to more than 100 sets of field data and concluded that the errors were intolerably too large.

Many researchers prefer curve matching interpretation techniques in order to evade the mathematical problems of theoretically generating curves that will match their field data curves. In curve matching interpretation method field curves are matched with already exiting sets of computer generated curves. A popular set of such curves for interpretation based on Schlumberger electrode configuration is given by Swets and Zeitlinger (1985). It consists of 2093 three layer curves. Mooney and Wetzel (1956) have also published 2400-three – and four – layer curves when the data are obtained by Wenner electrode configuration. Osemeikhian and Asokhia (1994) give details on how to use such curves. Unfortunately however, even when a catalogue of such curves in their thousands is available, it is possible not to get a single theoretical curve that will perfectly match a given field curve. The problem of scanning through thousands of theoretical curves for a single curve that will perfectly match a given field curve is not an easy task either.

This paper suggests that geophysicists should develop their own theoretical curves for marching their field curves. For simplicity, the earth is assumed to be horizontally stratified, homogeneous and isotropic. These assumptions are acceptable to geophysicists.

The resistivity field work described here was carried out at Umuduruokoro, about 60km north east of Owerri, the capital of Imo state of Nigeria.

THEORETICAL ANALYSIS

A differential equation which is the basis of all resistivity prospecting with direct current is given by

$\nabla\delta_{ij}\ \nabla V = 0$ ....(1)

where δ_{ij} is conductivity and V is potential. In the isotropic case the conductivity at a point in the ground is independent of direction, equation (1) reduces to Laplace's equation

$\nabla^2 V = 0$ ....(2)

Solutions to equations (1) &(2) may be developed for a particular model of the earth by selecting a coordinate system to match the geometry of the model and by imposing appropriate boundary conditions.

With a model of horizontal, homogeneous and isotropic layers, it is necessary to find the solution to Laplace s equation as expressed in equation (2) for the potential at the surface of the earth at a distance r from the current source. For an n-layer earth as shown in Fig 1., the potential at any depth z is subject to certain boundary conditions:

$$V_i(r, z_j) = V_{i+1}(r, z_j)$$

$$\ell_{i+1} V_i^1(r, z_j) = \ell_i V^1_{i+1}(r, z_j)$$

$$\lim_{r \to 0} V_1(r, z) = \frac{I\ell_1}{2\pi r}$$

$$\lim_{r \to 0} V_n(r, z) = 0$$

where ℓ is resistivity.

Ehrenburg and Watson (1932) pursued the optical analogy and developed a solution for any number of layers of fixed thickness (h). This restriction on thickness ensures that the positions of current images are readily predictable. The surface potential was formulated as:

$$V(r)=\frac{I\ell_1}{2\pi}\left[\frac{1}{r}+2\sum_{N=1}^{\infty}\frac{Q_N}{\left(r^2+4N^2h^2\right)^{1/2}}\right] \quad \text{..........(3)}$$

where the first four terms for kernel function Q_N are:

$$Q_1=K_1$$

$$Q_2=(1-K_1)^2K_2+K_1Q_1$$

$$Q_3=(1-K_1)^2(1-K_2)^2K_3+(K_1-K_1K_2)Q_2+K_2Q_1$$

$$Q_4=(1-K_1)^2(1-K_2)^2(1-K_3)^2K_4+(K_1-K_1K_2-K_2K_3)Q_3$$
$$+(K_2-K_1K_3+K_1K_2K_3)Q_2+K_3Q_1$$

The complexity increases rapidly with the number of layers. Ehrenburg and Watson included the apparent resistivity formula for a Wenner array and they presented two 2-layer and two 3-layer theoretical sounding curves.

By applying separation of variables to Laplace's equation in cylindrical co-ordinates, Stefanesco et. al.(1930) were able to arrive at a general solution for the potential at the surface of an n-layered earth having arbitrary resistivities and thickness:

$$V(r)=\frac{I\ell_1}{2\pi}\left(\frac{1}{r}+2\int_0^{\infty}\theta_n(\lambda)J_0(\lambda r)d\lambda\right) \quad \text{..........(4)}$$

where J_0 is the zero-order Bessel function of the first kind and θ_n, called the kernel function, is a function of the thickness and reflection coefficients for an assumed earth model. In particular, for 2- and 3-layer models, the kernel function is given by

$$\theta_2(\lambda)=\frac{K_1\exp(-2\lambda h_1)}{1-K_1\exp(-2\lambda h_1)}$$

$$\theta_3(\lambda)=\frac{K_1\exp(-2\lambda h_1)+K_2\exp\{-2\lambda(h_1+h_2)\}}{1+K_1K_2\exp(-2\lambda h_2)-K_1\exp(-2\lambda h_1)-K_2\exp\{-2\lambda(h_1+h_2)\}}$$

By differentiating equation (4), the Schlumberger apparent resistivity over an n-layer earth becomes:

$$\ell_a(r)=\ell_1\left[1+2r^2\int_0^{\infty}\lambda\theta_n(\lambda)J_1(\lambda r)d\lambda\right] \quad \text{..........(5)}$$

where J_1 is the first-order Bessel function of the first kind.

The evaluation of the integral in equation (5) has been done in a number of ways. Flathe (1955), solved the problem by writing the kernel function as a ratio of polynomials:

$$\theta_n(u) = \frac{P_n(u)}{H_n(u) - P_n(u)}$$

where $u = e^{-2\lambda}$ *and P,H are the coefficients.*

$$P_{j+1}(u) = P_j(u) + K_j U^{Z_j} H_j(U^{-1})$$

$$j = 1, 2, \ldots, n-1$$

$$H_{j+1}(u) = H_j(u) + K_j U^{Z_j} P_j(u^{-1})$$

$$P_1(u) = P_1(u^{-1}) = 0$$

$$H_1(u) = H_1(u^{-1}) = 1$$

Mooney et al. (1966) chose to use series computation for θ_n, P_n and H_n provided that all depths (Z_j) be restricted to integral multiples of some arbitrary reference length, h_l say:

$$\theta_n(u) = \sum_{N=1}^{\infty} Q(N) U^N$$

$$P_n(U) = \sum_{i=0}^{z_{n-1}} P(i)_n U^i$$

$$H_n(u) = \sum_{i=0}^{z_{n-1}} H(i)_n U^i$$

where the coefficients are defined recursively:

$$Q(N) = P_N \sum_{i=1}^{a} \left(P(i)_n - H(i)_n\right) Q(N-i)$$

$$P(i)_{j+1} = P(i)_j + K_j H(Z_j - i)_j$$

$$H(i)_{j+1} = H(i)_j + K_j P(Z_j - i)_j$$

$$P(0)_j = 0$$

$$H(0)_j = 1$$

where: $d = \mathrm{MIN}(Z_{n-1}, N-1)$

$i = 0, 1, \ldots, Z_{n-1}$

$j = 1, 2, \ldots, n-1$

The equation (5) becomes:

$$\ell_a(r) = \ell_1\left[1 + 2r^2\sum_{N=1}^{\infty} Q(N)\int_0^{\infty} \lambda e^{-2N\lambda} J_1(\lambda r)\, d\lambda\right] \quad \text{..................(6)}$$

where the integral is of standard form:

$$\int_0^{\infty} x e^{-ax} J_1(bx)\, dx = b(a^2 + b^2)^{-3/2}$$

Therefore, equation (6) reduces to an infinite series:

$$\ell_a(r) = \ell_1\left[1 + 2\sum_{N=1}^{\infty} Q(N)\left(1 + {4N^2}/{r^2}\right)^{-3/2}\right] \quad \text{...................7)}$$

The corresponding formula for surface potential is

$$V(r) = \frac{I\ell_1}{2\pi}\left[\frac{1}{r} + 2\sum_{N=1}^{\infty} \frac{Q(N)}{\left(r^2 + 4N^2\right)^{1/2}}\right] \quad \text{..................................(8)}$$

This equation (8) is equivalent to the image theory formula of equation (6) if and only if the thickness h is taken to be unity. It must be noted that Q(N) is easier to compute than Q_N because it is defined recursively.

Argelo (1967), developed independently an identical method for evaluating the Stefanesco integral. The difference lies in the expansion of the kernel function:

$$\theta_n(u) = \frac{P_n(u)}{Q_n(u)} = \sum_{i=1}^{\infty} P_n(u)(1 - Q_n(u))^{i-1}$$

Convergence is assured if $0<Q_n(u)<2$. In general, however, Q_n may exceed 2. Van Dam (1965, 1967) has shown that convergence will be assured if a "convergence factor" (a) is introduced into the expansion of the kernel function:

$$\theta_n(u) = \frac{aP_n(u)}{aQ_n(u)} = \sum_{i=1}^{\infty} aP_n(u)[1 - aQ_n(u)]^{i-1}$$

$$\text{where} \quad a = \left[\frac{1}{2}\{Q_n(u)_{\max} + Q_n(u)_{\min}\}\right]^{-1}$$

Mooney et al. (1966) established a criterion for terminating the infinite series in equation (7) at a preselected accuracy. If the series is truncated after M terms, then the error (E) in apparent resistivity is:

$$E < \frac{r^3}{8M^2}$$

Inversely, for a specified accuracy (E), the number of terms required in the summation is:

$$M \geq \left[\frac{r^3}{8E}\right]^{1/2}$$

In practice, the bound is too conservative because the actual error is of the order of the square root of the bound. But the fact remains that the method of Mooney et al (1966) is often intolerably slow. Run-time increases with the number of layers, maximum depth (Z_{n-1}), resistivity contrasts, current-electrode distance and level of accuracy. However Inman (1975) abandoned this approach and resorted to direct numerical integration for multilayer models.

In order to improve the rate of convergence of the series in equation (7) Nabighian (1966) used finite forward differences and found a 50-75% reduction in the number of terms. Because Q(0) = 1, he expressed equation (7) as:

$$\ell_a(r) = \ell_1\left[2\sum_{N=0}^{\infty} Q(N)\left(1+{4N^2}/{r^2}\right)^{-3/2} - 1\right] = \ell_1(S-1)$$

The infinite series S was approximated to order i+1 by:

$$S = [1+\theta(1)]f(0) + \theta^1(1)\Delta f(0) + \frac{1}{2!}\theta^{11}(1)\Delta^2 f(0) + \ldots\ldots\ldots + \frac{1}{1!}\theta^{(i)}(1)\Delta^i f(0)$$

$$+\sum_{N=0}^{M-1} B_{i+1,N}\Delta^{i+1} f(N) + \delta s$$

$$where \quad B_{i+1,N} = B_{i+1,N-1} - B_{i,N} \qquad i = 0,1,2,..$$

$$B_{0,N} = Q(N)$$

$$B_{i+1,-1} = \frac{\theta^{(i)}}{i!}$$

$$f(N) = 2(1+{4N^2}/{r^2})^{-3/2}$$

The error in S is:

$\delta s \quad < -B_{i+1,M}\Delta^i f(M)$

The kernel derivatives, which are evaluated at U=1 (or λ =0) are given recursively by:

$$\frac{\theta^{(i)}(1)}{i^!} = \frac{1}{Q(1)}\left[\frac{P^{(i)}(1)}{i^!} - \sum_{j=1}^{i}\frac{\theta^{(i-j)}(1)}{j^!(i-j)^!}Q^{(j)}(1)\right]$$

$$where \quad \theta(u) = \frac{P(u)}{Q(u)}$$

The first and the ith finite differences are defined by:

$$\Delta f(N) = f(N+1) - f(N)$$

$$\Delta^i f(N) = \Delta^{i-1} f(N+1) - \Delta^{i-1} f(N)$$

Nabighian obtained sufficient accuracy with order 3(i=2). However he did not give recursion formulae for the derivatives of P(u) and Q(u) but suggested instead that they be obtained directly from polynomial expressions.

Ghosh (1971) introduced a novel approach to the problem of computing sounding curves for stratified models by starting with the integral formula of Stefanesco et al. (1930); equation (5), and expressed it as

$$\ell_a(r) = r^2 \int_0^\infty \lambda T(\lambda) J.(\lambda r) d\lambda \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

$$where \quad T(\lambda) = \ell_1[1 + 2\theta_n(\lambda)]$$

The function T(λ) is called the resistivity transform function because it is defined by a Hankel transformation.

$$T(\lambda) = \int_0^\infty r^{-1} \ell_a(r) J.(\lambda r) dr$$

Equation (9) is a convolution integral. Therefore, it is possible to determine a linear digital filter {b_i} which converts resistivity transform samples into apparent resistivity values for theoretical models:

$$\ell_a(i) = \sum_i b_i T_{m-i}$$

The method is accurate, fast, simple in operation and has small computer-storage requirements. In addition, depths are no longer restricted to integral multiples and may take any arbitrary values.

EXPERIMENTAL WORK

The experimetal work was done at Umuduruokoro, about 60km north east of Owerri, the capital of Imo state of Nigeria. The Schlumberger array system was used, Osemeikhian and Asokhia (1994). The direction of expansion of the cables was constrained by topography though it is desirable that array should be expanded parallel to probable strike so as to minimize the effect of non - horizontal beddings.

RESULTS AND DISCUSSION

The sounding curve consists of three segments overlapping at two points at each end as the inner electrodes were expanded thrice when the voltage drop became too small to be accurately measured. A curve of best fit was then drawn and the raw data were obtained from this curve, Osemeikhian and Asokhia (1994). Model resistivity values were estimated from the data while model thicknesses were estimated from the fact that depth is approximately 0.167 to 0.125 of current electrode separation, AB. The model parameters were then used for iterative operation with the computer to interpret the data.The simple computer programme was based on the equations in the theory. VES 1 shows the result of the computer curve and the field curve. The result was a four - layer - curve with model resistivities of 214, 875, 594 and 34.4 Ω-m for the respective layers. The thickness of the respective layers were 1.1, 8.56 and 21.33m. The number of iterations for this result was only about seven

CONCLUSION

This proposed simplified computer iteration technique for the interpretation of vertical electrical sounding is very economical in terms of time, labour and cost. It is also very reliable. Within only about seven iterations a fit was found between field curve and theoretical curve with a root mean square error of less than two percent. The accuracy of interpretation was confirmed by the driller's log from the borehole drilled as a result of this geophysical investigation.

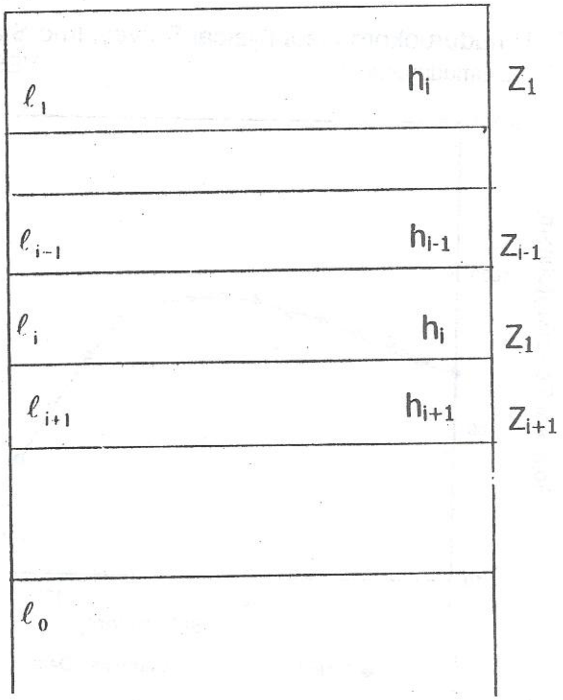
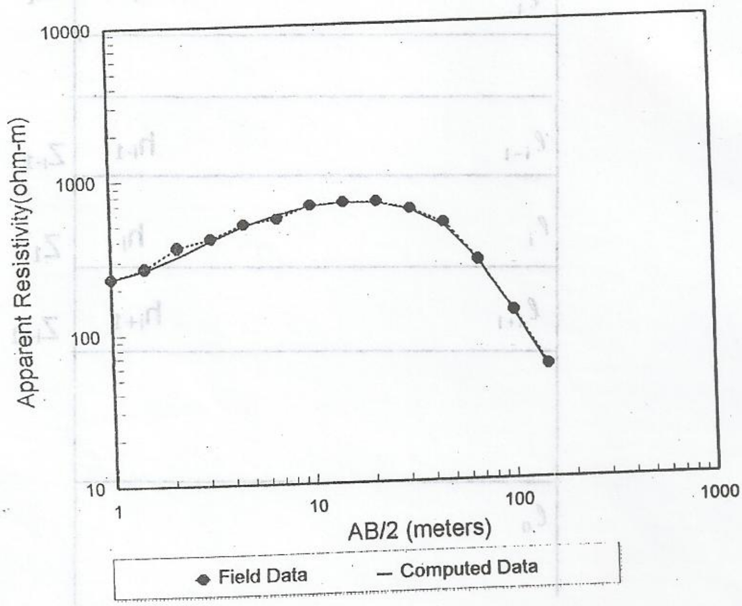


Fig.1

Horizontally-Stratified Earth Model of Homogeneous and Isotropic Layers.

PROJECT: Umuduruokoro Geophysical Survey, Imo State, Nigeria.

SITE: Umuduruokoro VES-1



Resistivity Sounding Interpretation

Field and Theoretical Data

AB/2 values (metres)	Observed values (ohm-metre)	Computed values (ohm-metre)	Log difference
1	235.00	234.73	-0.01
1.47	270.00	264.31	0.00
2.15	361.00	318.26	0.01
3.16	409.00	396.43	0.01
4.64	500.00	488.10	0.00
6.81	544.00	579.54	-0.01
10	665.00	653.87	0.00
14.7	688.00	691.68	-0.02
21.5	689.00	677.66	0.00
31.6	608.00	601.05	0.00
46.4	486.00	456.51	0.01
68.1	276.00	274.17	0.07
100	128.00	125.55	-0.04
147	56.00	53.83	-0.01

Model Parameters

Geoelectric Layer	Specific Resistivity (ohm-metre)	Thickness (metres)	Cumulative Thickness (metres)
1	214.00	1.10	0.53
2	875.00	8.56	1.97
3	594.00	21.33	8.98
4	34.40	infinity	infinity

RMS error (%) 1.96

Field Measurments by: M. B. Asokhia & Tunbosun.

Computer Interpretations by: M. B. Asokhia.

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