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NUMERICAL SOLUTION OF A REACTING VISCOUS PROBLEM

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ABSTRACT

A viscous fluid flowing through a cylinder is studied, of particular interest, a steady state of a reacting viscous fluid. The flow is reacting and heat is generated through reaction and viscosity. To ensure that the problem represents a physical problem, the numerical technique of solving the problem is discussed. It is discovered that, the problem exists and the results show that the temperature of the reacting system increases as viscosity and the heat release increase. The importance of this research to our society is discussed.

Keywords: Viscosity, Temperature, Density, Pressure, Diffusion Coefficient, Thermal Conductivity and Exponential Factor.

Nomenclature

Y	=	Pre-mixed reactants
ρ	=	Density
c	=	Specific heat
T	=	Temperature
μ	=	Dynamic viscosity
Q	=	Heat release/unit mass
D	=	Diffusion coefficient
K	=	Thermal conductivity
R	=	Universal gas constant
β	=	Pre-exponential factor
P	=	Pressure
U	=	Velocity Component along Z-axis
t	=	Time.
E	=	Reactivation energy

ϕ = heat release per unit mass.

1. INTRODUCTION

The physical situation to be studied in this paper deals with viscous reacting fluid, flowing in a tube of uniform, but arbitrary cross-section. The particular area of interest of this research is the bounded steady state of the problem. Internal fluid flow through pipe in which the presence of frictional forces acting on the flow is being taken into account. Almost all viscous fluids are exothermal, that is, generates heats when flowing through a medium. The analysis of such flows is important in the many situations in which fluid must be transported from one place to another. When a fluid flows through a pipe, the layers of fluid at the wall has zero velocity. The fluid velocity increases progressively as we move from the wall to the source of the tube. A velocity distribution in a pipe is built up as shown in Figure 1.

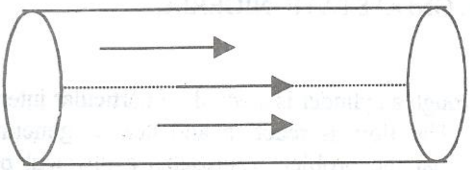


Fig. 1: Velocity distribution in pipe flow.

The velocity distribution in the pipe is a function of the type of flow in the pipe. This means that type of flow also play an important part in the determination of the magnitude of the frictional forces acting on the fluid.

In the existing literature, Campo and Lacoa (6) examined the meanbulk and wall-temperature distribution of hot fluids flowing inside horizontal tube, particularly the thermal responses of the in-tube flows. They critically examined the thermal responses of this kind of in-tube flows using two different mathematical models -

(a) a complete two-dimensional differential model.

(b) a largely simplified one-dimensional lumped model.

They assumed that the temperature affects the thermophysical properties of the fluid. They used the finite volume method for the numerical calculations of the temperatures field of the moving fluid.

In his contribution, Ayeni (1982) considered a viscous flow through a cylinder whose motion was unsteady. He used asymptotic techniques to determine the criteria for thermal runaway. Also, Okoya and Ayeni (1994) worked on the reacting flows (not viscous). From the above, we can observe that, previous researches on fluid are either on fluids that are viscous (not reacting) or reacting (not viscous). None investigated on fluid that possesses these two characteristics- viscous and at the same time reacting. Hence, the motivation for this research. The viscosity depends on temperature and the form of dependence is new.

In this paper, we formulate the mathematical equations for steady viscous reacting fluid, flowing in a tube. We examine the existence of the solution. In the process of examining the properties of the solution, we employed numerical techniques using FORTRAN 77 program implement the techniques.

2. MATHEMATICS EQUATIONS

The mathematical equations are centred on the continuity equation applied to a control volume. The continuity equation is based on Navier-Strokes' equations for viscous fluid. These equations are derived on the basis of the conservation of mass which states that fluid can neither be created nor destroyed. Hence, Mass going in (m) = Mass inside + mass out. It is assumed that, the tube is parallel to the x-axis along which $\partial u/\partial x = 0$. Thus, the flow has only one non-vanishing velocity components are $q = (0,0,U(r))$, so that, the continuity equation is written in differential form as,

$\frac{\partial \rho}{\partial t} + \nabla(\rho q) = 0$ where, ρ is as defined above and $\rho = \rho\,(x,y,z,t)$ is the density. Since the fluid be understudied is viscous and reacting, hence, the species and energy equation s are modified to give equations of this type of fluid. The resulting momentum, species and energy equations respectively are

$$-\frac{\partial P}{\partial t} + \frac{\mu}{r}\frac{\partial(r\partial u)}{\partial r\partial r} = 0 \tag{2}$$

$$\rho\frac{\partial y}{\partial t} = \frac{\rho D}{r}\frac{\partial}{\partial r}\frac{(r\partial y)}{\partial r} - \beta y e^{-E/RT} \tag{3}$$

$$c\rho\frac{\partial T}{\partial t} = \frac{k}{r}\frac{(\partial(\partial T))}{\partial r\partial r} + \mu\frac{(\partial u)^2}{\partial r} + \phi\beta e^{-E/RT} \tag{4}$$

where all the relevant terms are as defined in the nomenclature.

Consider equation (2)

$$-\frac{\partial P}{\partial z} + \frac{\mu\partial}{r\partial r}\frac{(r\partial u)}{\partial r} = 0 \tag{5}$$

where, P is a function of Z alone and U is a function of r alone. If we consider $\partial P/\partial z$ = constant. That is, the pressure gradient is a constant along the pipe axis. Since equation (5) involves only one variable r, it could be written as total differentiation to give;

$$\frac{\mu}{r}\frac{d}{dr}\frac{(rdu)}{dr}=\frac{dP}{dz} \qquad (6)$$

Integrating, we obtain

$$u = r^2/4\mu\ P' + C_1 \ln r + C_2 \qquad (7)$$

where,

$$P' = \frac{dP}{dz},\ C_1 = \frac{k_1}{\mu} \text{ and } C_1 = \frac{k_2}{\mu}$$

are integrating constants, whose values are determined by the boundary conditions. The boundary conditions are, U = 0 (i.e, no slip condition) and the other condition is the requirement of a finite velocity at r = 0, where r =R, for finiteness, $C_1 = 0$, this gives rise to equation $U = P'(P^2 - r^2)/4_{\mu}$, which is parabolic in shape, with the maximum velocity at the axis.

3. NON DIMENSIONALIZATION

Equations (3) and (4) are nondimensionalised by introducing the following nondimensional variables.

$$U = \frac{u}{V},\ y = \frac{Y}{y_0},\theta = \frac{E(T - T_0)}{RT_0^2},x = r/R \text{ and } \tau = tv/R$$

where, E is the activation energy
Putting these into equations (3) and (4), hence, equations (3) reduces to,

$$\frac{\partial y}{\partial \tau} + (1 - x^2)\frac{\partial y}{\partial x} = \frac{D}{vRx}\frac{\partial}{\partial x}\frac{(x\partial y)}{\partial x} - \frac{\beta}{R\rho v} ye^{-\varepsilon/RT} e^{\theta/1+\varepsilon\theta} \qquad (8)$$

Let $\varepsilon \dashrightarrow 0$, so that $e^{-\varepsilon/RT} e^{\theta/1+\varepsilon\theta} \dashrightarrow 1$

Let $a = \frac{D}{vR}, b = \frac{\beta}{R\rho V}$ and

Define t = τ, therefore,

$$\frac{\partial}{\partial \tau} = \frac{\partial}{\partial t}.\frac{\partial t}{\partial \tau} = \frac{\partial}{\partial t}$$

Equation (8) can be written as

$$\frac{\partial y}{\partial t} + (1 - x^2)\frac{\partial y}{\partial x} = \frac{a}{x}\frac{\partial}{\partial x}\frac{(x\partial y)}{\partial x} - by \qquad (9)$$

Similarly, Equation (4) gives

$$\frac{\partial\theta}{\partial\tau}+\left(1-x^2\right)\frac{\partial\theta}{\partial x}=\frac{k}{\rho CVRx}\frac{\partial}{\partial x}\frac{\left(x\partial\theta\right)}{\partial x}+\frac{4\mu vEx^2}{R^2T_0^2}+\frac{Q\beta E}{vT_0^2}e^{-\varepsilon/R_0T}e^{\theta/1+\varepsilon\theta} \quad (10)$$

Let $\varepsilon=\frac{R_0T_0}{E}$ where, $E\neq 0$

and

Let $d_1=\frac{k}{\rho CVR}, d_2=\frac{4\mu VE}{RR_0T_0^2}$ and $d_3=\frac{REQ\beta}{VR_0T_0^2}$

Hence, equation (10) can be written as

$$\frac{\partial\theta}{\partial t}+\left(1-x^2\right)\frac{\partial\theta}{\partial x}=\frac{d_1}{x}\frac{\partial}{\partial x}\frac{\left(x\partial\theta\right)}{\partial x}+d_2x^2+d_3 \quad (11)$$

The problems are considered on bounded regions and the bounded

conditions are, for equation (9), y(1,x) = 0, y(L ,x) = 1, with initial condition y(x,0) = x - 1 and for equation (10), θ(0,t) = 0 θ(L,t) = 1, Initial condition, θ(x,0) = x – 1.

For a steady state flow, $\frac{\partial y}{\partial t}=0$ and $\frac{\partial\theta}{\partial t}=0$, hence, equations (9) and (11) reduces to:

$$\left(1-x^2\right)\frac{\partial y}{\partial t}=\frac{a}{x}\frac{\partial}{\partial x}\frac{\left(x\partial y\right)}{\partial x}-by \quad (12)$$

Boundary conditions

y(0) = 0, y(1) = 1

and

$$\left(1-x^2\right)\frac{\partial\theta}{\partial x}=\frac{d_1}{x}\frac{\partial}{\partial x}\frac{\left(x\partial\theta\right)}{\partial x}+d_2x^2+d_3$$

Boundary conditions

θ (0) = 0, θ(1) = 1

Equations (12) and (13) can only be solved for all values of x, except zero, because we shall run into problem of singularity at x = 0. In order to avoid this problem, we adapt the method of Hicks and Weize (7) to remove the singularity with approximation

$$\lim_{x\to 0}\frac{1}{x}\frac{(dy)}{dx}=\frac{\left(d^2y\right)}{dx^2/x}=0 \quad (14)$$

Hence, equations (12) and (13) reduce to,

$$\frac{d^2y}{dx^2} = \frac{b}{a}y \tag{15}$$

y(0) = 0, y(1) = 1

and

$$\frac{d^2\theta}{dx^2} = -\frac{d_3}{2d_1^2} - \left(x - x^3 - d_1\right) - \frac{d_2x^2}{d_1} - \frac{d_3}{d_1} \tag{16}$$

θ(0) = 0, θ(1) = 1, respectively.

Equations (15) and (16) are now solvable at the neighbourhood of zero.

4. NUMERICAL SOLUTIONS

Different numerical techniques are available to solve this type of Boundary-Value problem. In this work, the Finite difference method is considered.

We shall consider the numerical method of solutions of equation (12) for various values of parameters a and b and equation (13) for various values of d_1, d_2 and d_3.

According to Burden and Faires (1993), the differential quotients y' and y" are approximated by the difference quotients substituting into equations (12), (13), (15) and (16). This can be done by partitioning interval (0,1) into n subintervals of length 1/n, such that $x_i = ih$, $i = 0(1)n$. x_i and ih are used interchangeably, x_i is adopted. The difference quotients used are,

$$y' = \frac{y_{i+1} - y_{i-1}}{2h} \tag{17}$$

and

$$y'' = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} \tag{18}$$

Putting equations (17) and (18)into equations (12) and (15), we obtain

$$(2a+h)\frac{\left(x_i - x_i^3 - a\right)}{x_i}y_{i-1} - \left(4a + 2h^2b\right)y_i + (2a-h)\frac{\left(x_i - x_i^3 - a\right)}{x_i}y_{i+1} = 0 \tag{19}$$

and

$$y_{i-1} - (2+bh^2)\, y_i + y_{i+1} = 0,\ i = 1(1)n \tag{20}$$

respectively.

Similarly, equations (13) and (16) are now,

$$(2d_1+h)\frac{\left(x_i - x_i^3 - d_1\right)}{x_i}\theta_{i-1} - 4d_1\theta_1 + 2d_1 - h\frac{\left(x_i - x_i^3 - d_1\right)}{x_i}\theta_{i+1} =$$

$$- 2h^2(d_2x_i^2 - d_3) \qquad (21)$$

and

$$2d_1\theta_{i+1} - 4d_i\theta_i + 2d_i\theta_{i+1} = - h^2d_3x^3 + h^2d_2x^2 + h^2d_3x - h^2d_3d_1 \qquad (22)$$

respectively. These methods are called Finite difference methods with truncation error of order $0(h^2)$ and equations (19) to (22) are tridiagonal, of the form

$$Ay = b^*$$

Where, in equation (19),

$$A = \begin{bmatrix} -(4a+2h^2b) & 2a-h(x_i-x_i^3-a)/x_i & \\ 2a+h(x_i-x_i^3-a)/x_i & -(4a+2h^2b) & 2a-h(x_i-x_i^3-a)/x_i \\ ------- & ------- & ------- \\ ------- & ------- & ------- \\ ------- & ------- & ------- \\ & 2a+h(x_i-x_i^3-a)/x_1 & (4a+2h^2b) \end{bmatrix}$$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ . \\ . \\ y_{n-1} \end{bmatrix}$$

and

$$b^* = \begin{bmatrix} -(2a+h(x_i-x_i^3-a)/x_i) \\ 0 \\ 0 \\ 0 \\ -(2a - h(x_i-x_i^3-a)/x_i) \end{bmatrix}$$

and in equation (21),

$$A = \begin{bmatrix} -4d_1 & 2d_1 - h\left(x_i - x_i^3 - d_1\right)/x_i & & \\ 2d_1 + h\left(x_i - x_i^3 - d_1\right)/x_i & -4d_1 & 2d_1 - h\left(x_i - x_i^3 - d_1\right)/x_i & \\ ------ & ------ & ------ & \\ ------ & ------ & ------ & \\ ------ & ------ & ------ & \\ & & 2d_1 + h\left(x_i - x_i^3 - d_1\right)/x_1 & -4d_1 \end{bmatrix}$$

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ . \\ . \\ \theta_{n-1} \end{bmatrix}$$

and

$$b^* = \begin{bmatrix} -2h^2\left(d_2 x_i^2 + d_3\right) - \left(2d_1 + h\left(x_i - x_i^3 - d_1\right)/x_i\right)\theta_0 \\ 2h^2\left(d_2 + x_i^2 + d^3\right) \\ \\ \\ -2h^2\left(d_2 x_i^2 + d_3\right) - \left(2d_1 - h\left(x_i - x_i^3 - d_1\right)/x_i\right)\theta_0 \end{bmatrix}$$

Alao (1997) established that the solution is feasible, if for all values of parameters a and d_1 greater than zero, we have

$$h < \frac{2}{\left|(1/a) - 3a\left(\frac{1}{2a}\right)^{2/3}\right|} \qquad (25a)$$

and

$$h < \frac{2}{\left|1/d_1 - 3d_1\left(\frac{1}{2}d_1\right)^{2/3}\right|} \qquad (25b)$$

for equations (12) and (13) respectively.

Having established that the tridiagonal linear systems (23) and (24) have unique solutions and the choice of h determined. We can now solve equations (12) and (13) together with the boundary conditions y(0) = 0 and y(0) = 0, and θ(0) = 0 and θ(1) = 1 respectively. The algorithms and Fortran 77 programming language are used to implement this method to solve the problems.

5.0 RESULTS AND DISCUSSION.

The results and their physical implication are discussed. Recall the key parameters for equation (10) are

$$a = \frac{D}{VR} \text{ – modified diffusion coefficient}$$

and $$b = \frac{\beta}{RRV} \text{ – modified pre-exponential factor,}$$

and that of equation (12) are

$$d_1 = \frac{k}{\rho CVR} \text{ – modified thermal conductivity}$$

$$d_2 = \frac{wVE\mu}{RR_0T_0^2} \text{ – modified viscosity}$$

and $$d_3 = \frac{REQ\beta}{VR_0T_0^2} \text{ – modified heat release}$$

From the investigation of the existence and uniqueness of the solutions, it is discovered that solutions of equations (13) exist for all times, when parameters a > o and equation (25a) holds and also solutions of equation (14) occur for all times when parameter d_1 > o and equation (25b) holds. Figures (2) and (3) represent the graphs of the results of equation (13) for various values of parameters a and b, while figures (4), (5) and (6) represent the graphs of the results of equation (14) for various values of parameters d_1,d_2 and d_3..

From the graphs, the effects of the parameters a (modified diffusion coefficient) and b (modified pre-exponential factor) on temperature. For a constant value of the parameter a, there is a decrease in temperature as parameter b increases (see Figs. 2) and likewise, in Fig. 3, it can be seen that, for constant values of the parameter b, there is a decrease in temperature as parameter a increases.

The effects of the parameters d_1(modified thermal conductivity), d_2 (modified viscosity) and d_3 (modified heat release) on temperature: for constant values of d_1 and d_2, (i.e. d_1,d_2 = 2.00), there is an increase in temperature for an increase in parameter d_3 (see Figs. 4). Also in Fig. 5, temperature decreases as parameter d_1 increases (i.e d_1 from 1.00 to 2.00) for constant d_2 and d_3 (i.e d_2,d_3 = 1.00).

Lastly, increase in parameter d_2 (d_2 changes form 1.00 to 2.00) causes increase in temperature when parameters d_1 and d_2 are constants (i,e d_2,d_3 = 1.00) (See Figs. 6.

The effect of viscosity on temperature: since viscosity (μ) is directly proportional to parameter d_2, therefore, Figs.(6) shows that increase in viscosity causes increase in temperature and verse visa.

Conclusively, the above results, most especially the relationships, between the heat release and temperature, and between the viscosity and temperature would be found useful in the medical and engineering fields such as flow of blood through the arteries and causes of high blood pressure.

Fig.ii: Effect of change in parameter b on temperature

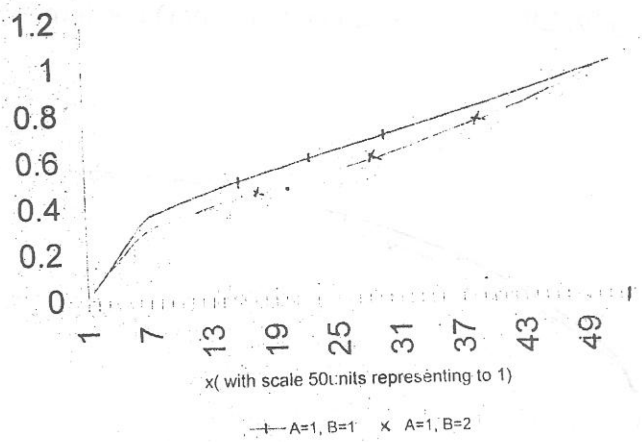


Fig.iii: Effect of change in parameter a on temperature

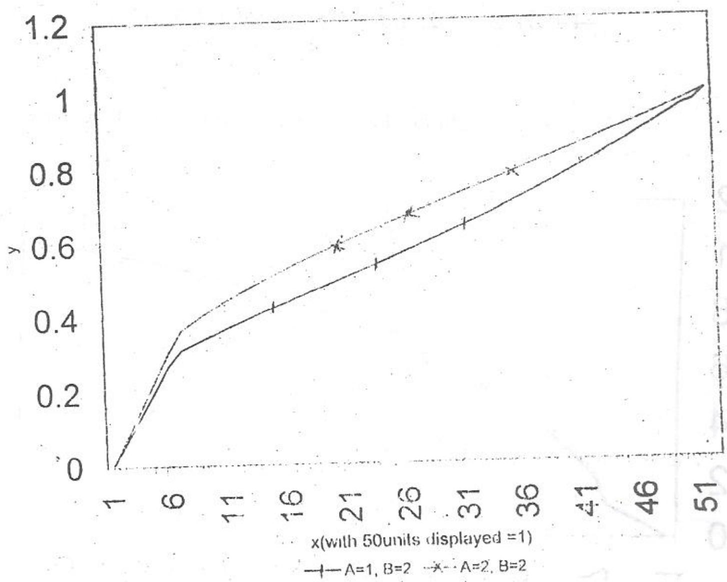


Fig.IV: Effect of change in parameter D3 on temperature

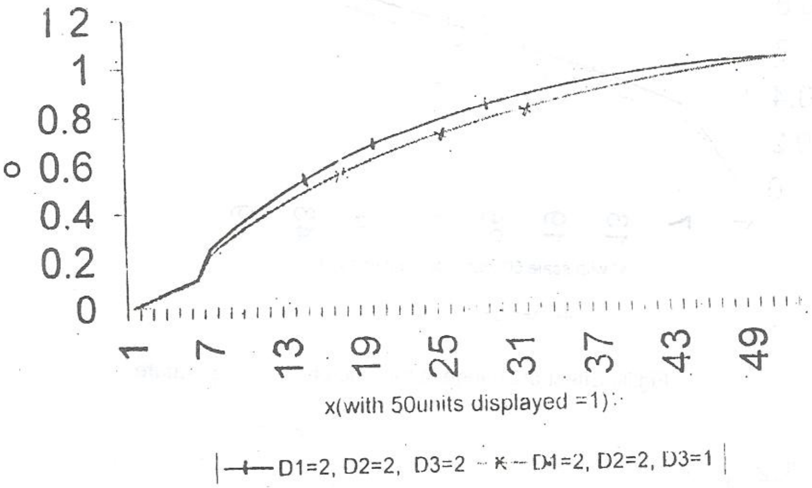


Fig. V: Effect of change in parameter D1 on temperature

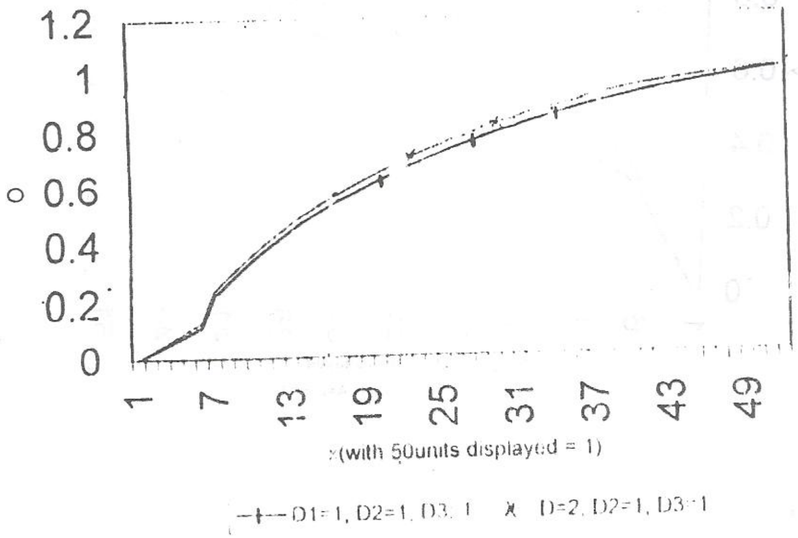
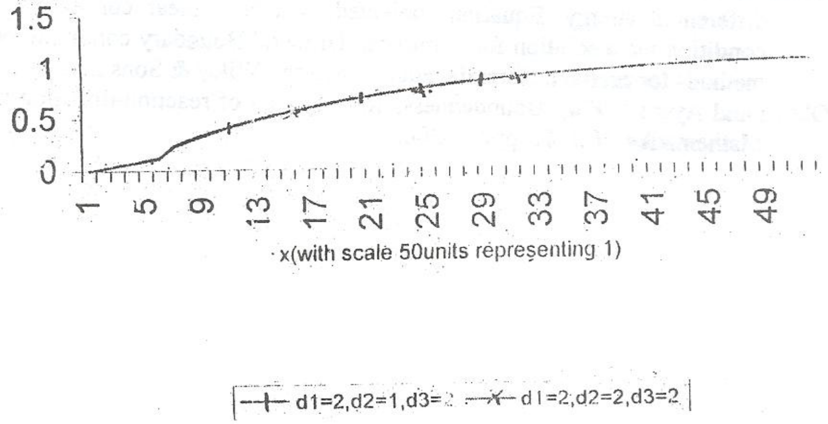


Fig.VI: Effect of change in parameter D2 on temperature.



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