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LIFETIMES OF PRECIPITATING IONOSPHERIC ELECTRON BEAMS

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ABSTRACT

Using, on the one hand, the binary collision approximation, we have calculated the life times of ionospheric electron beams. On the other hand, treating space plasma as a particle distribution with an enhanced high energy tail, in the (Vlasov) kinetic equation, the growth times of the type of beam-plasma instability excited by the incoming beams are calculated. We find that the value of growth times, as indicators of the lifetimes of the electron beams are about 10^{-9} the characteristics lifetimes from the binary-collision approximation. This suggests, as expected, that anomalous transport due to the instability, is the dominant process in the decay of the electron beams.

1 INTRODUCTION

Electron beams, which precipitate into the upper ionosphere, from the tail-side of the earth's magnetosphere, have received considerable attention [1,2,3]. These beams are responsible for auroral phenomena in the upper latitudes [2,4], are associated with very low frequency (VLF) hiss [1], and may be involved in the evolution of observed ion conic distributions [2] and lower hybrid bursts [5]. Part of the population of the beams is thought to originate from unstable processes in the earth's tail, possibly magnetic reconnection processes which could feed electrons into near-auroral regions [6]. But the acceleration of this part of the beams, in the final pre-precipitation stages, is most likely due to field-aligned electron fields [4]. Another part of the beam populations, the so-called collimated precipitation, comes from ambient electrons which diffuse across field lines into the acceleration region [7]. What is observed is an incoming plane flux of electrons with a narrow spread of energies centered on 2keV [2] which, with the background electrons, can be modelled, as in section 3, as a "slideway" electron distribution. It is of interest to speculate on the possible mechanisms by which the electron beams decay, specifically, to examine the relative effects on beam decay of discrete collisions, compared with the collective modes due to unstable processes to which the slideway distribution is subject. Fainberg [8] found that energy loss per particle through Coulomb collisions is a factor of the order 10^{-6} to 10^{-9} compared with energy loss by the electrons through the growth of unstable waves, suggesting that unstable collective modes are dominant processes in beam decay. To confirm this, we propose, in this paper, to calculate the instability build-up time (the inverse of the mode growth rate)and compare this with the beam lifetime which should be obtained if decay is exclusively due to discrete collisions, which is, the shortest of the three characteristic lifetimes from discrete collisions; momentum transfer time, particle deflection time, and energy exchange time [9].

The paper is organised as follows: In section2, we obtain values of beam lifetime at various precipitation heights, based on discrete particle momentum transfer times; in section3, the growth times of the ionospheric lower hybrid instability are calculated for various precipitation heights. In section 4, the results are compared and discussed.

2. BINARY COLLISION THEORY AND CHARACTERISTIC TIMES

Coulòmb collisions, on their own, should relax a nonthermal plasma to thermodynamic equilibrium. We consider momemtum, $\vec{p}$; the square of the change in perpendicular velocity, $(\Delta v_\perp)^2$; and the square of the change in energy $(\Delta\varepsilon)^2$. The evolution of these quantities, for a test particle of type α which enters a Maxwellian plasma, of type β, are [9]:

$$\frac{d\vec{p}}{dt} = -\frac{m_\alpha \vec{v}}{\tau_s^{\alpha/\beta}},$$

$$\frac{d(\Delta v_\perp)_\alpha^2}{dt} = \frac{v^2}{\tau_d^{\alpha/\beta}},$$

and

$$\frac{d(\Delta\varepsilon)_\alpha^2}{dt} = \frac{\varepsilon^2}{\tau_\varepsilon^{\alpha/\beta}}$$

The field particles described by $f_\beta = \left(n_\beta \Pi^{-\frac{3}{2}} / v_{t\beta}{}^2 \right) \exp\left(-v^2/v_{t\beta}{}^2\right)$, where $v_{t\beta} = \left(2T_\beta/m_\beta\right)^{1/2}$, m is mass, T is temperature and n is particle density.

$$\tau_s^{\alpha/\beta} = \tau_1^{\alpha/\beta} \Big/ \left[1 + (m_\alpha/m_\beta)\mu\right] \quad \text{(1a)}$$

$$\tau_d^{\alpha/\beta} = \tau_1^{\alpha/\beta} \Big/ \left[2(\mu + \mu' - (\mu/2x_\beta)\right] \quad \text{(1b)}$$

$$\tau_\varepsilon^{\alpha/\beta} = \tau_1^{\alpha/\beta} \Big/ \left[4\mu/x_\beta\right] \quad \text{(1c)}$$

where

$$\tau_1^{\alpha/\beta} = \frac{\left(m_\alpha \varepsilon_\alpha^3\right)^{1/2}}{\pi 2^{1/2} e_\alpha^2 e_\beta^2 \lambda n_\beta}, \quad x = \frac{v_\alpha^2}{v_{t\beta}^2}$$

e is charge, λ is the Coulomb logarithm, $\varepsilon_\alpha = \frac{1}{2} m_\alpha v_\alpha^2$ is kinetic energy of the incoming test particle,

$$\mu \equiv \mu(x_\beta) = \left(\frac{2}{\pi^{1/2}}\right)\int_0^{x_\beta} t^{1/2}\exp(-t)dt\,,$$

and

$$\mu' \equiv \mu'(x_\beta) = \left(\frac{d\mu}{dx_\beta}\right) = \left(\frac{2}{\pi^{1/2}}\right)x_\beta^{1/2}\exp(-x_\beta)\,.$$

For the background electrons, from table 1, $464 < x_e < 5797$, for the values $5000 > T_e >$ 400k and incoming electron kinetic energy $KE_e = 2$keV. Thus $x_e >> 1$, and so, $\rho(x_e) \approx 1$ and $\mu'(x_e) \approx 0$. In fact, we need only take a value $kE \approx 4.32$eV for these limiting values of μ and μ' to apply, far lower than the measured energies ~ keV for electrons in precipitating electron beams [7]. If relaxation is due exclusively to collisions, the electron beam will be destroyed in the shortest of the times $\tau_s^{e/e}$, $\tau_s^{e/i}$, $\tau_d^{e/e}$, $\tau_\varepsilon^{e/e}$ and $\tau_\varepsilon^{e/i}$. For $x_\beta >> 1$ we have that $\tau^{e/i} >> \tau^{e/e} \sim \left(m_e/m_i\right)\tau^{e/i}$. Thus we need only consider the electron-electron collision times. Then, from equation (1), with $\mu(x_\beta)\approx 1$, $\mu'(x_\beta) \approx 0$, the beam is destroyed in a time of the order of $\tau_s^{e/e}$. Calculations of the lifetimes, presented in table 1 gives, for the range of precipitation heights considered, that 2.2hrs$<\tau_s<$181.3hrs.

3. LIFETIMES FROM WAVE-PARTICLE RESONANCE

Rockets and satellite observations [1] show that the velocity space distribution functions for precipitating electrons have regions of positive slope, and allow us to model the electron population as the superposition of a background Maxwellian and a drifting Maxwellian or, in order words, the "slideway" distribution [2,10], which is motivated by observation [7] that part of the incoming electron population can be trapped in the potential wells associated with localized electric fluctuations. The slideway distribution is unstable, and the inverse of its growth rate is a measure of the lifetime of the beam. Adopting the notation in[3].

altitude	*n_e(cm^{-3})	*n_b(cm^{-3})	V_b(keV)	τ_s (hr)	γ^{-1}(s)
300	10^5	0.42	220	2.18	1.8×10^{-5}
500	4.2×10^4	0.44	137	5.18	3×10^{-5}
750	2×10^4	0.83	62.4	10.88	5.3×10^{-5}
1000	7×10^3	2.72	15.5	31.08	1.55×10^{-4}
1500	3×10^3	2.48	5.29	72.52	5.2×10^{-4}
2000	1.2×10^3	2.86	2.36	181.3	2.4×10^{-3}

From ref [10]

Table 1. calculated lifetimes and growth times of electron beams at different altitudes from the earth.

$$f_e = (n/v_{te})\pi^{-\frac{1}{2}} \exp\left(-v_\parallel^2/v_{te}\right)^2 - (n_b/v_{tb})\pi^{-\frac{1}{2}} \exp\left(-(v_\parallel - v_b)^2/v_{tb}^2\right)$$

and for the ions,

$$f_i = (n_i/v_{ti})\pi^{-\frac{1}{2}} \exp\left(-v^2/v_{ti}^2\right),$$

where the subscript b refers to the beam electrons. Solving the collisionless Vlasov equation, we find that the first order perturbed particle densities are

$$\begin{aligned} n_{e1} &= \int f_{e1}\, dv = (ei/m_e)\int dv E_\parallel \nabla_{v\parallel} f_e/(\omega = k_\parallel v_\parallel) \\ &= (iE_\parallel/k_\parallel)[(n_e/T_e)W(\xi_e) + (n_b/T_b)W(\xi_b)] \end{aligned} \tag{2}$$

and

$$\begin{aligned} n_{i1} &= \int f_{i1}\, dv = -(ie/m_i)\int dv \bar{E}\cdot\nabla_v f_i/(\omega = kv) \\ &= (iE/k)[(n_i/T_{i0})W(\xi_i)] \end{aligned} \tag{3}$$

where $\xi_i = \frac{\omega}{kv_{ti}}$, $\xi_e = \frac{\omega}{k_\parallel}$, $\xi_b = \frac{\omega - k_\parallel v_b}{k_\parallel v_{tb}}$, and

$$W(\xi) = -\pi^{-1/2}\int \frac{s\exp(-s^2)ds}{s-\xi}$$

is Landau integral. Equations (2) and (3), with Poisson's equation, $\nabla.E = 4\pi e(n_{i1} - n_{e1})$ give the following dielectric function:

$$\in = 1 - (k\lambda_{Di})^{-2}\left\{W(\xi_i) + \left(\frac{T_i}{T_b}\right)\left[\left(\frac{n_b}{n}\right)W(\xi_b) + \left(\frac{n_e T_b}{nT_e}\right)W(\xi_e)\right]\right\}$$

where $\lambda_{Di} = \left(\frac{kT_i}{4\pi ne^2}\right)^{\frac{1}{2}}$ is the ion Debye length. With $\xi_i >> 0$, $\xi_e >> 0$, setting $\in \approx 0$, the dispersion relation is

$$\omega^2 = \omega_{pi}^2 + \left(\frac{k_\parallel}{k}\right)^2 \omega_{pe}^2 + 2\omega^2\left(\frac{n_b}{n_e}\right)\left(\frac{\omega_{pe}}{kv_b}\right)W(\xi_b)$$

if we set $\omega = \omega_r + i\gamma$ with $\gamma << \omega_r$, we find

$$\omega_r = \omega_{pi} + \left(\frac{k_\parallel}{k}\right)^2 \omega_{pe}^2$$

and

$$\gamma = -\omega_r\left[\pi^{\frac{1}{2}}\left(\frac{n_b}{n}\right)\left(\frac{\omega_{pe}^2}{k^2 v_{tb}^2}\right)\xi\exp\left(-\xi^2\right)\right] \tag{4}$$

where $\xi = \dfrac{\omega_r - k_{\|} v_b}{k_{\|} v_{tb}}$. We observe that for $k\,\|/k \approx (m_e/m_i)^{\frac{1}{2}}$, $\omega_r^2 \approx 2\omega_{pi}$, showing that the unstable mode is the lower hybrid mode. The beam "lifetime" is the inverse of the growth rate, or γ^{-1}. With $\dfrac{\omega_r}{k_{\|}} \approx v_b$, we have $k_{\|}^2 \approx (\omega_r/2v_b)^2 \approx 2\omega_{pi}^2/v_b^2$. Then $k^2 \approx 2\omega_{pe}^2/v_b^2$ and $\xi = 1$. Then,

$$\tau = \left(\frac{8}{\pi}\right)^{1/2}\left(\frac{n}{n_b}\right)\left(\frac{1}{\omega_{pi}}\right)\left(\frac{v_{tb}}{v_b}\right)^2$$

The results for τ are given in table 1 for various precipitation altitudes and corresponding values of background plasma densities and temperatures.

4. CONCLUSION

There is experimental confirmation of the existence of beam-induced ionospheric lower hybrid modes [2]. This implies that the growth time τ calculated in section 3, which are many orders of magnitude greater than lifetimes τ_s from discrete particle collisions, should be used to obtain altitude-dependent estimates of the transport coefficients.

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