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SOME IDENTITIES IN SPINOR CALCULUS

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ABSTRACT

Using the methods of spinor calculus we prove two identities and point out that a third identity can be similarly proved.

INTRODUCTION

If GL(2,C) is the general linear group in two dimensions, then the special linear group in two dimensions [1]

$SL(2,C) = \{M \in GL(2,C) \mid \det M = +1\}$

If F denotes the complex representation space of SL(2,C), then a lineir representation D(M), $M \in SL(2,C)$ of SL(2,C) is a homomorphism from SL(2,C) into the automorphism group of F, i.e.

$M \in SL(2,C) \rightarrow D(M)$

There exist in F two inequivalent representations of SL(2,C):

(1) The self-representation defined by
$D(M) = M \; \forall \, M \in SL(2,C)$

(2) The self-conjugate representation defined by
$D(M) = M^* \; \forall \, M \in SL(2,C)$

Let ψ be an element of the representation space F, then ψ transforms under the self-representation as

$$\psi'_{A} = M_A^{\;B} \psi_B \qquad A.B = 1,2 \tag{1}$$

If F represents the complex representation space of $D(M) = M^*$, with $\bar{\psi}$ being an element of $\dot{F}$ then $\bar{\psi}$ transforms as follows:

$$\bar{\psi}'_{\dot{A}} = M^*{}_{\dot{A}}{}^{\dot{B}} \bar{\psi}_{\dot{B}} \qquad \dot{A},\ \dot{B} = 1,2 \tag{2}$$

An equivalent representation to $D(M) = M$ is the representation

$D(M) = M^{-1T} \; \forall \, M \in SL(2,C)$

The corresponding complex representation space is denoted by F^* with elements ψ^A, and it transforms as follows:

$$\psi'^{A} = (M^{-IT})^A{}_\beta \psi^\beta \tag{3}$$

On the other hand, the representation $D(M) = M^{*-IT}$ is equivalent to $D(M) = M^*$. If the complex representation space of M^{*-IT} is denoted by $\dot{F}^*$, then $\bar{\psi}^{\dot{A}} \in \dot{F}^*$ transforms as follows:

$$\bar{\psi}'^{\dot{A}} = (M^{*-IT})^{\dot{A}}{}_{\dot{\beta}} \bar{\psi}^{\dot{\beta}} \qquad \dot{A},\dot{B} = 1,2 \tag{4}$$

The elements ψ_A and $\overline{\psi}_{\dot{A}}$ are called left- and right-handed spinors respectively and are denoted by (½,0) and (0, ½). ψ_A and ψ^A are also called undotted spinors while $\overline{\psi}_{\dot{A}}$ and $\overline{\psi}^{\dot{A}}$ are called dotted spinors. They are all two-component spinors and are called **Weyl spinors.**

Since M and M^{-IT} are equivalent, there exists a non-singular 2 x 2 matrix $\varepsilon = (\varepsilon^{AB})$ such that

$$M^{-IT} = \varepsilon M \varepsilon^{-1} \qquad (5)$$

Here $\varepsilon^{-1} = (\varepsilon_{AB})$

Similarly, there exists a nonsingular 2 x 2 matrix such that

$$M^{*-IT} = \overline{\varepsilon} M^{*} \overline{\varepsilon}^{-1} \qquad (6)$$

where

$$\varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \overline{\varepsilon} \text{ and } \varepsilon^{-1} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \overline{\varepsilon}^{-1}$$

The ε-matrices serve as metrics. For example, ε and $\overline{\varepsilon}$ are used for raising indices:

$$\psi^{A} = \varepsilon^{AB} \psi B$$

Similarly, ε^{-1} and $\overline{\varepsilon}^{-1}$ are used for lowering indices:

$$\psi_{\dot{A}} = \varepsilon_{\dot{A}\dot{B}} \psi^{\dot{B}}$$

In order to link spinor calculus to the Lorentz four-component formalism, we introduce the following two sets of four matrices:

$$\sigma^{\mu} = (\sigma^{0}, \sigma) = (1, \sigma) \quad \mu = 0,1,2,3$$

$$\overline{\sigma}^{\mu} = (\overline{\sigma}^{0}, \sigma) = (1, -\sigma)$$

where $\sigma = (\sigma^{1}, \sigma^{2}, \sigma^{3})$

with $\sigma^{1}, \sigma^{2}, \sigma^{3}$ being the three Pauli matrices and $1_{2 \times 2}$ is the unit matrix in two dimensions. Recall that

$$\sigma^{1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^{2} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^{3} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

the index structures of σ^{μ} and $\overline{\sigma}^{\mu}$ are respectively $\sigma^{\mu}_{A\dot{A}}$ and $\overline{\sigma}^{\mu}_{\dot{A}A}$ where

$$\overline{\sigma}^{\mu \dot{A} A} = \varepsilon^{AB} \varepsilon^{\dot{A}\dot{B}} \sigma^{\mu}_{B\dot{B}} \qquad (7)$$

The relation

$$\sigma^{\mu}_{A\dot{A}} \overline{\sigma}^{\mu \dot{B} B} = 2 \delta^{B}_{A} \delta^{\dot{B}}_{\dot{A}} \qquad (8)$$

expresses the completeness of σ^{μ} and $\overline{\sigma}^{\mu}$.

With the introduction of these two four matrices, it can be shown that the Lorentz transformation, Λ^{μ}_{ν}, which transforms four vectors linked by the restricted Lorentz group (synonymously, the proper orthochronous Lorentz group), is given by

$$\Lambda^{\mu}_{\nu} = \frac{1}{2} Tr[\bar{\sigma}^{\mu} M \sigma_{\nu} M^{+}] \tag{9}$$

The generators of the Lorentz group in the spinor representations (½, 0) and (0,½) are

$$\sigma^{\mu\nu} = \tfrac{i}{4}\left(\sigma^{\mu}\bar{\sigma}^{\nu} - \sigma^{\nu}\bar{\sigma}^{\mu}\right)$$

and

$$\bar{\sigma}^{\mu\nu} = \tfrac{i}{4}\left(\bar{\sigma}^{\mu}\sigma^{\nu} - \bar{\sigma}^{\nu}\sigma^{\mu}\right)$$

where $\bar{\sigma}^{\mu\nu} = \sigma^{\mu\nu+}$

Since the operation of complex conjugation interchanges the (½, 0) and (0, ½) representations, and the operation of parity interchanges N_i and N_i^{+}, an operation identical with complex conjugation, a theory invariant under the parity transformation must involve the Dirac four-component spinor

$$\psi_a = \begin{pmatrix} \phi_A \\ \bar{\psi}^{\dot{A}} \end{pmatrix} \qquad a = 1,2,3,4 \tag{10}$$

THE IDENTITIES IN SPINOR CALCULUS

We shall now prove the following two identities:

$$\bar{\psi}'(x')\psi'(x') = \bar{\psi}(x)\psi(x) \tag{11}$$

$$\bar{\psi}'(x')\gamma^{\mu}\psi'(x') = \Lambda^{\mu}_{\nu}\bar{\psi}(x)\gamma^{\nu}\psi(x) \tag{12}$$

We shall start with the first identity. Let us note that $\psi(x)$ is given by equation (10) and that in the Weyl (chiral) representation the $\gamma-matrices$ are defined by

$$\gamma^{\mu} = \begin{pmatrix} 0 & \sigma^{\mu} \\ \bar{\sigma}^{\mu} & 0 \end{pmatrix} \qquad \mu = 0,1,2,3 \tag{13}$$

From equations (1) and (4) we have

$$\psi(x) = \begin{pmatrix} \phi_A \\ \bar{\psi}^{\dot{A}} \end{pmatrix} \text{ and } \bar{\psi}(x) = \psi + (x)\gamma^{0} = \left(\phi^{A*}\,\bar{\psi}^{*}_{\dot{A}}\right)\begin{pmatrix} 0 & \sigma^{0}_{A\dot{B}} \\ \bar{\sigma}^{0}_{\dot{A}B} & 0 \end{pmatrix}$$

$$\left(\bar{\psi}^{*}_{\dot{A}} \quad \bar{\sigma}^{0\dot{A}B}, \phi^{A*}\sigma^{0}_{A\dot{B}}\right)$$

$$\bar{\psi}(x)\psi(x) = \bar{\psi}^{*}_{\dot{A}}\,\bar{\sigma}^{0\dot{A}B}\phi_{B} + \phi^{A*}\sigma^{0}_{A\dot{B}}\bar{\psi}^{\dot{B}} \tag{14}$$

We now use

$$\phi^{B} = \bar{\psi}^{*}_{\dot{A}} \quad \bar{\sigma}^{0\dot{A}B} \tag{15}$$

obtaining $\phi^{A}\sigma^{0}_{A\dot{C}} = \psi^{*}_{\dot{B}} \quad \sigma^{0\dot{A}B'}\sigma^{0}_{A\dot{C}} = \psi^{*}_{\dot{B}}\delta_{\dot{C}}{}^{\dot{B}} = \psi^{*}_{\dot{C}}$

i.e. $\phi^{A*}\sigma^0_{A\dot{C}} = \psi^*_{\dot{C}}$ (16)

Eq. (14) then becomes

$$\phi^B\phi_B + \overline{\psi}_{\dot{B}}\overline{\psi}^{\dot{B}} \quad (17)$$

$$\text{Now, } \psi'(x') = \begin{pmatrix} \phi'_A \\ \overline{\psi}'^{\dot{A}} \end{pmatrix} = \begin{pmatrix} M_A^B\phi_B \\ (M^{*-IT})^{\dot{A}}{}_{\dot{B}}\overline{\psi}^{\dot{B}} \end{pmatrix}$$

$$\text{Thus } \overline{\psi}'(x') = \psi^+\gamma^0 = \left(\phi^{1A*}\overline{\psi}^*_{\dot{A}}\right)\begin{pmatrix} 0 & \sigma^0_{A\dot{C}} \\ \overline{\sigma}^{0\dot{A}C} & 0 \end{pmatrix}$$

$$\left(\left(\phi^B M_B^A\right)^*, \left(\overline{\psi}_{\dot{B}}(M^{*-IT})^{\dot{B}}_{\dot{A}}\right)^*\right)\begin{pmatrix} 0 & \sigma^0_{A\dot{C}} \\ \overline{\sigma}^{0\dot{A}C} & 0 \end{pmatrix}$$

$$= \left(\left(\overline{\psi}_{\dot{B}}\left(M^{*-IT}\right)^{\dot{B}}_{\dot{A}}\right)^*\overline{\sigma}^{0\dot{A}C}, \left(\phi^B M_B^A\right)^*\sigma^0_{A\dot{C}}\right)$$

$$\therefore \overline{\psi}'(x')\psi'(x') = \left(\overline{\psi}_{\dot{B}}\left(M^{*-IT}\right)^{\dot{B}}_{\dot{A}}\right)^*\overline{\sigma}^{0\dot{A}A}M_A^B\phi_B + (\phi^B M_B^A)^*\sigma^0_{A\dot{C}}(M^{*-IT})^{\dot{C}}{}_{\dot{B}}\overline{\psi}^{\dot{B}}$$

(using Eq. (16) i.e. $\overline{\psi}^*_{\dot{B}} = \phi^D\sigma^0_{D\dot{B}}$ and Eq. (15) i.e. $\phi^{B*} = \overline{\psi}_{\dot{A}}\overline{\sigma}^{0\dot{A}B}$)

$$= \phi^D\left(\left(M^{*-IT}\right)^{\dot{B}}_{\dot{A}}\right)^*\overline{\sigma}^{0\dot{A}A}\sigma^0_{D\dot{B}}M_A^B\phi_B + \overline{\psi}_{\dot{A}}\left(M_B^A\right)^*\sigma^0_{A\dot{C}}\overline{\sigma}^{0\dot{A}B}(M^{*-IT})^{\dot{C}}{}_{\dot{B}}\overline{\psi}^{\dot{B}}$$

$$= \phi^D(M^{-IT})^A_D M_A^B\phi_B + \overline{\psi}_{\dot{A}}(M^{*A}_{\dot{C}})(M^{*-I})^{\dot{C}}{}_{\dot{B}}\overline{\psi}^{\dot{B}}$$

$$= \phi^D(M^{-I})^A_D M_A^B\phi_B + \overline{\psi}_{\dot{A}}\left[(M^{*A}_{\dot{C}})(M^{*-I})^{\dot{C}}{}_{\dot{B}}\right]\overline{\psi}^{\dot{B}}$$

$$= \phi^D\delta_D{}^B\phi_B + \overline{\psi}_{\dot{A}}\delta^A_{\dot{B}}\overline{\psi}^{\dot{B}}$$

$$= \phi^B\phi_B + \overline{\psi}^{\dot{B}}\overline{\psi}_{\dot{B}} \quad (18)$$

which is the same as Eq. (17)

We now show that

$$\overline{\psi}'(x')\gamma^\mu\psi'(x') = \Lambda^\mu_\nu\overline{\psi}(x)\gamma^\nu\psi(x)$$

$$\text{By Eq. (10), } \overline{\psi}(x) = \psi'^+\gamma^0 = \left(\phi^{A*}\overline{\psi}^*_{\dot{A}}\right)\begin{pmatrix} 0 & \sigma^0_{A\dot{A}} \\ \overline{\sigma}^{0\dot{A}A} & 0 \end{pmatrix}$$

$$=\left(\overline{\psi}^{*}_{\dot{A}}\ \overline{\sigma}^{0\dot{A}A}, \phi^{A*}\sigma^{0}_{A\dot{A}}\right)$$

$$\therefore \overline{\psi}(x)\gamma^{v}\psi(x)=\left(\overline{\psi}^{*}_{\dot{A}}\ \overline{\sigma}^{0\dot{A}A}, \phi^{A*}\sigma^{0}_{A\dot{A}}\right)\begin{pmatrix}0 & \sigma^{v}_{A\dot{A}}\\ \overline{\sigma}^{v\dot{A}A} & 0\end{pmatrix}$$

$$=\left(\phi^{A*}\ \sigma^{0}_{A\dot{A}}\ \overline{\sigma}^{v\dot{A}B}, \overline{\psi}^{*}_{\dot{A}}\ \overline{\sigma}^{0\dot{A}A*}\sigma^{v}_{A\dot{B}}\right)\begin{pmatrix}\phi_{B}\\ \overline{\psi}^{\dot{B}}\end{pmatrix}$$

$$=\phi^{A*}\ \sigma^{0}_{A\dot{A}}\ \overline{\sigma}^{v\dot{A}B}\phi_{B}+\overline{\psi}^{*}_{\dot{A}}\ \overline{\sigma}^{0\dot{A}A*}\sigma^{v}_{A\dot{B}}\overline{\psi}^{\dot{B}}$$

With $\psi'(x')=\begin{bmatrix}\phi'_{A}\\ \overline{\psi}^{1A*}\end{bmatrix}$

$$\overline{\psi}'(x')=\begin{pmatrix}\phi'_{A}\\ \overline{\psi}^{1\dot{A}}\end{pmatrix}^{+}\gamma^{0}=\left(\phi^{1A*}, \overline{\psi}^{1*}_{\dot{A}}\right)\begin{pmatrix}0 & \sigma^{0}_{A\dot{B}}\\ \overline{\sigma}^{0\dot{A}B} & 0\end{pmatrix}$$

$$=\left(\overline{\psi}^{!*}_{\dot{A}}\ \overline{\sigma}^{0\dot{A}B}, \phi^{!A*}\sigma^{0}_{A\dot{B}}\right)$$

$$\text{Thus } \overline{\psi}'(x')\gamma^{\mu}\psi'(x')=\left(\overline{\psi}'^{*}_{\dot{A}}\ \overline{\sigma}^{0\dot{A}B}, \phi'^{A*}\sigma^{0}_{A\dot{B}}\right)\begin{pmatrix}0 & \sigma^{\mu B\dot{B}}\\ \overline{\sigma}^{\mu\dot{B}B} & 0\end{pmatrix}\begin{pmatrix}\phi'_{B}\\ \overline{\psi}^{!\dot{B}}\end{pmatrix}$$

$$=\phi'^{A*}\ \sigma^{0}_{A\dot{B}}\ \overline{\sigma}^{\mu\dot{B}B}\phi'_{B}+\overline{\psi}'^{*}_{\dot{A}}\ \overline{\sigma}^{0\dot{A}B}\sigma^{\mu}_{B\dot{B}}\overline{\psi}'^{\dot{B}} \tag{19}$$

We first consider the first term of the sum, Eq. (19)

$$\phi'^{A*}\ \sigma^{0}_{A\dot{A}}\ \overline{\sigma}^{\mu'AB}\phi'_{B}$$

$$=\left[(M^{-IT})^{A}_{C}\right]^{*}\sigma^{0}_{A\dot{A}}\overline{\sigma}^{\mu\dot{A}B}M^{D}_{B}\phi_{D}$$

$$=\phi^{C*}(M^{-IT})^{*A}_{C}\sigma^{0}_{A\dot{A}}\overline{\sigma}^{\mu\dot{A}B}M^{D}_{B}\phi_{D} \tag{20}$$

From Eq. (15) we have

$$\phi^{C*}=\overline{\psi}_{\dot{B}}\overline{\sigma}^{0\dot{B}C}$$

Hence, Eq. (20) becomes

$$\overline{\psi}_{\dot{B}}\left[(M^{-IT})^{*A}_{C}\right]\overline{\sigma}^{0\dot{B}C}\sigma^{0}_{A\dot{A}}\overline{\sigma}^{\mu\dot{A}B}M^{D}_{B}\phi_{D}$$

(21)

Using $\varepsilon_{CF}M^{F}_{E}\varepsilon^{EA}=(M^{-IT})c^{A}$, Eq. (21) becomes

$$\overline{\psi}_{\dot{B}}\varepsilon_{CF}\varepsilon^{EA}(M^{F}_{E})^{*}\overline{\sigma}^{O\dot{B}C}\sigma^{O}_{A\dot{A}}\overline{\sigma}^{\mu\dot{A}B}M^{D}_{B}\phi_{D}$$

$$=\overline{\psi}_{\dot{C}}\delta^{\dot{C}}_{\dot{B}}\varepsilon_{CF}\varepsilon^{EA}(M^{F}_{E})^{*}\overline{\sigma}^{O\dot{B}C}\sigma^{O}_{A\dot{A}}\overline{\sigma}^{\mu\dot{A}B}M^{D}_{B}\delta^{G}_{D}\phi_{G} \tag{22}$$

we now use $= 2\delta_{\dot{B}}^{\dot{C}}\delta_D{}^G = \bar{\sigma}^{\nu\dot{C}G}\sigma_{\nu D\dot{B}}$ (23)

Equation (22) now becomes

$$\tfrac{1}{2}\bar{\psi}_{\dot{C}}\bar{\sigma}^{\nu\dot{C}G}\sigma_{\nu D\dot{B}}(M_C^A)^*\bar{\sigma}^{O\dot{B}C}\sigma^O_{A\dot{A}}\bar{\sigma}^{\mu\dot{A}B}M_B^D\phi_G$$

(using $\bar{\psi}_{\dot{C}} = \phi^{H^*}\sigma^0_{H\dot{C}}$)

$$= \tfrac{1}{2}\phi^{H^*}\bar{\sigma}^{\nu\dot{C}G}\sigma_{\nu D\dot{B}}(M_C^A)^*\bar{\sigma}^{OBC}\sigma^O_{A\dot{A}}\sigma^0_{H\dot{C}}\bar{\sigma}^{\mu\dot{A}B}M_B^D\phi_G$$

$$= \tfrac{1}{2}\phi^{H^*}\bar{\sigma}^{\nu\dot{C}G}\sigma_{\nu D\dot{B}}M^{*B}_{\dot{A}}\bar{\sigma}^{\mu\dot{A}B}M_B^D\phi_G$$

$$= \tfrac{1}{2}\left[(M^+)^{\dot{B}}_{\dot{A}}\bar{\sigma}^{\mu\dot{A}B}M_B^D\sigma_{\nu D\dot{B}}\right]\left[\phi^{H^*}\sigma^0_{H\dot{C}}\bar{\sigma}^{\nu\dot{C}G}\phi_G\right]$$

$$= \tfrac{1}{2}Tr\left[M^+\bar{\sigma}^\mu M\sigma_\nu\right]\left[\phi^{H^*}\sigma^0_{H\dot{C}}\bar{\sigma}^{\nu\dot{C}G}\phi_G\right. \tag{24}$$

We now use $\Lambda_\nu^\mu = \tfrac{1}{2}Tr\left[M^+\bar{\sigma}^\mu M\sigma_\nu\right]$ (25)

Equation (24) then becomes

$$\phi'^{A^*}\sigma^0_{A\dot{A}}\bar{\sigma}^{\mu\dot{A}B}\phi'_B = \Lambda_V^\mu\phi^{H^*}\sigma^0_{H\dot{C}}\bar{\sigma}^{\nu\dot{C}G}\phi_G \tag{26}$$

which proves the first term of the sum given in equation (19).

We now prove the second term in equation (19), i.e.

$$\bar{\psi}'^*_{\dot{A}}\sigma^{0\dot{A}A}_{A\dot{A}}\bar{\sigma}^\mu_{A\dot{B}}\bar{\psi}'^{\dot{B}} = \Lambda_V^\mu\bar{\psi}^*_{\dot{A}}\bar{\sigma}^{0\dot{A}A}\sigma^\nu_{A\dot{B}}\bar{\psi}^{\dot{B}} \tag{27}$$

By using equations (4), (2) and the identity

$$\sigma^\mu_{A\dot{B}} = \varepsilon_{AB}\varepsilon_{\dot{B}\dot{E}}\bar{\sigma}^{\mu\dot{E}B} \tag{28}$$

the left hand side of equation (27) becomes

$$\left[(M^*)^{\dot{C}}_{\dot{A}}\bar{\psi}_{\dot{C}}\right]^*\bar{\sigma}^{0\dot{A}A}\varepsilon_{AB}\varepsilon_{\dot{B}\dot{E}}\bar{\sigma}^{\mu\dot{E}B}(M^{*-1T})^{\dot{B}}_{\dot{D}}\bar{\psi}^{\dot{D}} \tag{29}$$

Upon using $(M^{*-1T})^{\dot{B}}_{\dot{D}} = \varepsilon^{\dot{B}\dot{F}}(M^*)^{\dot{G}}_{\dot{F}}\varepsilon_{\dot{G}\dot{D}}$ and $\bar{\psi}^*_{\dot{C}} = \phi^D\sigma^0_{D\dot{C}}$.

The expression (29) becomes

$$\phi^D\left[(M^*)^{\dot{C}}_{\dot{A}}\right]^*\sigma^0_{D\dot{C}}\bar{\sigma}^{O\dot{A}A}\varepsilon_{AB}\varepsilon_{\dot{B}\dot{E}}\bar{\sigma}^{\mu\dot{E}B}\varepsilon^{\dot{B}\dot{F}}(M^*)^{\dot{G}}_{\dot{F}}\varepsilon_{\dot{G}D}\bar{\psi}^{\dot{D}}$$

$$= \phi^D M_D^A\varepsilon_{AB}\varepsilon_{\dot{B}\dot{E}}\bar{\sigma}^{\mu\dot{E}B}\varepsilon^{\dot{B}\dot{F}}\varepsilon_{GD}(M^+)^{\dot{G}}_{\dot{F}}\bar{\psi}^{\dot{D}}$$

$$= \phi^E\delta_E^D M_D^A\varepsilon_{AB}\varepsilon_{\dot{B}\dot{E}}\varepsilon^{\dot{B}\dot{F}}\varepsilon_{\dot{G}\dot{D}}\bar{\sigma}^{\mu\dot{E}B}(M^+)^{\dot{G}}_{\dot{F}}\delta^{\dot{D}}_{\dot{H}}\bar{\psi}^{\dot{H}}$$

(using equation (23))

$$= \tfrac{1}{2}\bar{\sigma}^{\nu\dot{D}D}\sigma_{\nu E\dot{H}}\phi^E M_D^A\varepsilon_{AB}\varepsilon_{\dot{G}\dot{D}}\bar{\sigma}^{\mu\dot{E}B}(M^+)^{\dot{G}}_{\dot{F}}\bar{\psi}^{\dot{H}}$$

(Using $\bar{\sigma}^{\nu\dot{D}D}\varepsilon_{AB}\varepsilon_{\dot{G}\dot{D}} = \sigma^\nu_{A\dot{G}}\delta_B^D$ and $\phi^E = \bar{\psi}^*_{\dot{C}}\bar{\sigma}^{0\dot{C}E}$)

$$= \tfrac{1}{2}\left[(M^+)^{\dot{G}}_{\dot{F}}\bar{\sigma}^{\mu\dot{E}B}M_D^A\sigma_{\nu E\dot{H}}\right]\bar{\psi}^*_{\dot{C}}\bar{\sigma}^{0\dot{C}E}\sigma^\nu_{A\dot{G}}\bar{\psi}^{\dot{H}}\delta_B^D$$

$$= \tfrac{1}{2}Tr\left[M^+\sigma^\mu M\sigma_\nu\right]\delta^{\dot{E}}_{\dot{F}}\delta^B_D\delta^A_E\delta^{\dot{G}}_{\dot{H}}\psi'_{\dot{C}}\sigma^{0\dot{C}E}\sigma^\nu_{A\dot{G}}\psi^{\dot{H}}$$

(using $\Lambda^{\mu}_{\nu}(M) = \frac{1}{2}Tr\left[M^{+}\overline{\sigma}^{\mu}M\sigma_{\nu}\right]$)

$$= \Lambda^{\mu}_{V}\overline{\psi}^{*}_{\dot{C}}\overline{\sigma}^{0\dot{C}A}\sigma^{\nu}_{A\dot{G}}\overline{\psi}^{\dot{G}}$$

which is to be proved.
Combining equations (26) and (27) yields the second identity, equation (12).
It is clear that the identity

$$\overline{\psi}'(x')\sigma^{\mu\nu}\psi'(x') = \Lambda^{\mu}_{\alpha}\Lambda^{\nu}_{\beta}\overline{\psi}(x)\sigma^{\alpha\beta}\psi(x)$$

can be proved in a similar fashion.

It should be pointed out that the two-component Weyl formalism can be used to obtain the above formulae faster.

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REFERENCE

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