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SENSITIVITY AND ERROR ANALYSIS OF BLANEY-MORIN-NIGERIA EVAPOTRANSPIRATION MODEL

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ABSTRACT

The response of Blaney-Morin-Nigeria model to errors in estimation of the input parameters of temperature, relative humidity and radiation ratio, is examined. Direct response approach and application of partial derivatives method were used for the purpose. The results showed that a $\pm 4^{\circ}C$ change in temperature caused about 10% change in evapotranspiration (E_t). The response of the model to a $\pm 15\%$ variation in Humidity was 28% and a $\pm 15\%$ change in radiation ratio alters E_t by 15%. Based on the uncertainty of each measuring instrument, the overall performance of BMN model was affected by 7.5% for temperature, 11.4% for relative humidity and 7.5% radiation ratio. The implications of these results are discussed.

1 INTRODUCTION

Evapotranspiration (E_t), the atmospheric water demand, is important to many subject areas. The ability to accurately measure or estimate Evapotranspiration (E_t) will enhance water resources and environmental management of a given catchment or basin. In arid regions, full irrigation is needed for survival while irrigation is supplemental in the humid regions (Merva and Fernandez, (1985)). The cost of obtaining water is high, therefore the irrigation should aim at applying only that water which is necessary. To do this effectively, it is necessary to be able to predict crop water requirement and the exact time when irrigation becomes necessary. Many formulae exist to estimate

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(E_t), ranging from a simple empirical equations to well physically based complex equations, such as, Penman and Penman-Monteith equations (Itier (1996)). Penman and Penman-Monteith are generally used to estimate crop water requirement because of their better performances over other equations. Under Nigeria condition, Duru (1984) proposed an (E_t) equation which has a better accuracy and consistency than Penman model. The model was designated as the Blaney-Morin-Nigeria (BMN) evapotranspiration model. Oguntunde (1998) compared BMN with other (E_t) equations in humid and subhumid stations of Nigeria and found BMN performing better than Penman as claimed by Duru (1984). Hence, BMN equation was selected for this study.

There are various types of errors that can affect the performance of a given model. Two forms considered in this study are:

i. errors due to the inherent sensitivity of the instruments used to measure the input parameters.
ii. errors due to fluctuation in input parameters (effect of climate change).

The objectives of this research work are to examine;

a. the response of BMN model to its input parameters and
b. the impact of error of each instrument on the overall performance of the equation.

2.0 THEORETICAL ANALYSIS: Partial derivatives method

The underlying principles of analysis is contained in the solution to the following questions

i. If the inaccuracy of each instrument is known, how is the inaccuracy of the model estimated ?
ii. If there must be a certain accuracy in a computed result, what errors are allowable in the individual instruments ?

To answer these questions, we consider a problem of computing a quality N, where N is a known function of the n independent variables, $U_1, U_2, U_3, \ldots\ldots U_n$. That is,

$$N = f(U_1, U_2, U_3, \ldots\ldots U_n) \quad (1)$$

The U's are measured quantities and are in error by $\pm\Delta U_1, \pm\Delta U_2, \pm\Delta U_3, \ldots, \pm\Delta U_n$ respectively. These errors will cause an error ΔN in the computed result N. If the ΔU's are considered as absolute limits on the individual errors and we wish to calculate similar absolute limits on the error in N, we could calculate.

$$N \pm \Delta N = f(U_1 \pm \Delta U_1, U_2 \pm \Delta U_2, U_3 \pm \Delta U_3, \ldots U_n \pm \Delta U_n) \quad (2)$$

By subtracting equation (1) from equation (2), we obtain $\pm\Delta N$. This procedure is time-consuming, but an approximate solution is obtained by application of the Taylor series, we get

$$f\left(U_1 \pm \Delta U_1, U_2 \pm \Delta U_2, U_3 \pm \Delta U_3, \ldots U_n \pm \Delta U_n\right) = f\left(U_1, U_2, U_3 \quad \ldots, U_n\right) + \Delta U_1 \frac{\partial f}{\partial U_1} + \Delta U_2 \frac{\partial f}{\partial U_2} + \ldots\ldots\ldots + \Delta U_n \frac{\partial f}{\partial U_n} + \frac{1}{2}\left\{(\Delta U_1)^2 \frac{\partial^2 f}{\partial U_1} + \ldots\ldots\ldots\right\} + \quad (3)$$

where all the partial derivatives are to be evaluated at the known values of U_1, U_2, U_3, ...U_n. In actual practice, the ΔU's will be small quantities, and thus terms such as $(\Delta U)^2$ and higher terms be negligible. Then equation (3) may be given approximately as,

$$N \pm \Delta N = f\left(U_1, U_2, U_3 \quad \ldots, U_n\right) + \Delta U_1 \frac{\partial f}{\partial U_1} + \Delta U_2 \frac{\partial f}{\partial U_{21}} + \ldots + \Delta U_n \frac{\partial f}{\partial U_n} \quad (4)$$

So the absolute error E_a is given by

$$E_a = \Delta N \quad \left|\Delta U_1 \frac{\partial f}{\partial U_1}\right| + \left|\Delta U_2 \frac{\partial f}{\partial U_2}\right| + \ldots\ldots\ldots + \left|\Delta U_n \frac{\partial f}{\partial U_n}\right| \quad (5)$$

If the relative or percentage error E_r is desired, it may therefore be expressed as

$$E_r = \frac{\Delta N}{N}.100 = \frac{100.E_a}{N}. \quad (6)$$

While the computed result may be given as either

$$N \pm E_a \quad \text{or} \quad N \pm E_a$$

Such problem in which a certain overall accuracy is acquired, and we may wish to know what the component accuracy are needed, is (apparently) mathematically indeterminate since an infinite number of combination of individual accuracy could result in the same overall accuracy. The means of resolving this difficulty is found in the "method of equal effects" reported by Doebelin (1990). This principle merely assumes that each source of error contribute an equal amount to the total error.

Mathematically, from equation (5), if each term is assumed to be equal, we may write

$$\left|\frac{\partial f}{\partial U_1}\Delta U_1\right| = \left|\frac{\partial f}{\partial U_2}\Delta U_2\right| = \ldots\ldots\ldots = \left|\frac{\partial f}{\partial U_n}\Delta U_n\right| = \frac{\Delta N}{N} \quad (7)$$

Now the allowable overall error ΔN are known, and so are n, U_1, U_2, U_3, ...U_n .

Thus,

$$\frac{\partial f}{\partial U_i}\Delta U_i = \frac{\Delta N}{N}, \; where, \Delta U_1 = \frac{\Delta N}{n(\partial f / \partial U_i)} \quad (8)$$

and the allowable error ΔU_i in each measurement may be estimated.

When ΔU's are considered not as absolute limits of error, but rather as statistical bounds such as $\pm 3s$ limits, probable errors or uncertainties, the formulae for computing overall errors must be modified. Scarborough (1955) showed that the proper method of combining such errors is according to the root-sum square (rss) formula

$$\sqrt{\left[\left(\Delta U_1 \frac{\partial f}{\partial U_1}\right)^2 + \left(\Delta U_2 \frac{\partial f}{\partial U_2}\right)^2 + \ldots\ldots\ldots + \left(\Delta U_n \frac{\partial f}{\partial U_n}\right)^2\right]} \quad (9)$$

The overall error E_{arss} represents a $\pm 3s$ limit on N, (where s is the standard deviation of the sample data of the measurements), and 99.7 percent of the values of N can be expected to fall within this limits. Following this viewpoint, equation (8) also must be modified as

$$\Delta U_i = \frac{\Delta N}{\sqrt{(n(\partial f / \partial U_i))}} \quad (10)$$

2.1 THE BMN EVAPOTRANSPIRATION MODEL AND ITS DERIVATIVES

The equation is given as;

$$E_t = r(0.45T + 8)(520 - R^{1.31})/100 \quad (11)$$

where, E_t is the potential evapo-transpiration (mm/day);
T is the mean monthly temperature in ^{o}c;
R is the mean monthly relative humidity and r is the radiation ratio which Duru (1) assumed to be constant over a month
The derivatives of equation (11) with respect to r, T and R lead to;

$$\frac{\partial E_t}{\partial T} = 0.45r\left(520 - R^{1.31}\right)/100 \qquad (12)$$

$$\frac{\partial E_t}{\partial R} = -1.31rR^{0.31}(0.45T + 8)/100 \qquad (13)$$

$$\frac{\partial E_t}{\partial r} = (0.45T + 8)\left(520 - R^{1.31}\right)/100 \qquad (14)$$

3 RESPONSE ANALYSIS OF BMN MODEL

Researchers have carried out sensitivity analysis on some evapotranspiration models using different approaches, and the results appear in work by Tanner and Pelton (1960), Mckenny and Rosenberg (1993). In the present work, we are interested in the direct response of the predicted value as affected by a change in a given parameter as well as using partial derivative technique to perform a sensitivity analysis of the effect of errors in input data on predicted evapotranspiration (Doebelin, (1990)).

The input parameters in BMN model are temperature (T), relative humidity (R) and radiation ratio (r). These parameters were varied following the procedures outlined by Oguntunde (1998). The meteorological data used in this study were obtained from IITA database at Ibadan. The base values of the parameters were the average of 10 years (1985-94) data collected. Finally, computer programs were written in FORTRAN77 to implement the simulation, which generate results in Tables 1-3.

4 RESULTS AND DISCUSSION

The response of Blaney-Morin-Nigeria (BMN) E_t model to fluctuation in input parameter is given by the line graphs in Figure 1. Based on instrument sensitivities, relative errors in BMN model as functions of indicated input parameters are shown in Figure 2. Error analyses of the model to each of the three input variables are presented in Tables (1-3).

The BMN model responded least to a variation in temperature, a variation of -4°c to +6°c of the temperature caused between 10% and 15% variation in the result (see Figure 1(a)) Error analysis of the model for instrument accuracy, varied between 7.45% to 7.55% over the same range of temperature change. The actual values of E_t, absolute error (E_a) and root-mean-square error (E_{rss}) are also presented in Table 1. The combine effect of the two types of errors analysed, indicated that variation in the model result could probably be as high as 17.5% for a change of ±4°C. This value may be undesirable in some applications hence the need to minimize the error. Error in temperature will also have a multiplier effect on humidity measurement.

The variation in radiation ratio (r), Figure.1(c), appears to have relatively higher effect on the results obtained by applying BMN equation. Evapotranspiration (E_t) changes directly with the same percent change in radiation ratio. For example, a 50% variation in the input parameter alters the model result by 50%. Consequently, the relative error in the calculation of E_t by equation (11) due to the measuring instrument is about 7.5% and remains almost constant over the range tested, Figure 2 (c). Although, the value corresponds to that of temperature input, the slope of its graph is near zero as against a positively rising Slope of temperature graphs, Figure 2(a). By inspection, the position of radiation ratio (r) in equation (11), clearly explains the former result while the low value of r itself, been a ratio, answers the near constant response of the later results. This indicates that moderate accuracy in r is required.

The most important parameter in the BMN model in term of its effects on the predicted response appear to be the relative humidity (R), see Figure 1(b) and Figure 2 (b). Humidity causes about 28% change when it is varied by 15%, while a range of 5.8% to 11.4% relative error, due to instrument interest sensitivity, was recorded. Combination of the two types of error could yield a value up to 40% at 15% variation of the parameter. By close examination of Figure 1(b) and equation (11), it appears there is an inverse relationship between E_t by BMN and relative humidity. The result of this analysis explains why it was the component of humidity that has its degree greater than unity as compared to the two other parameters. The relatively higher sensitivity of E_t to errors in humidity calls for a considerable precision in its measurement.

5. CONCLUSION

This study aimed at unravel the impact of fluctuations in climatic parameters, due to global warning, ozone layer depletion, e.t.c., to an indigenous evapotranspiration model proposed by Duru (1984). The effects of measuring instruments inherent sensitivities to the input variables were also examined. The result showed that while BMN model responded moderately to radiation ratio and temperature, its sensitivity to humidity is relatively higher. Also since any error in temperature will have a multiplier effect on humidity, both input parameters must be measured to a considerable high precision whereas, a moderate accuracy is required for radiation ratio estimation.

TABLE 1: ERROR ANALYSIS OF BMN MODEL TO TEMPERATURE

T	DT	Ea	Earnms	ET	ER
22.0100	.2201	.2231	.1482	2.9907	7.458
24.0100	.2401	.2349	.1557	3.1410	7.480
26.0100	.2601	.2468	.1633	3.2913	7.499
28.0100	.2801	.2587	.1708	3.4417	7.517
30.0100	.3001	.2706	.1733	3.5920	7.533
32.0100	.3201	.2825	.1859	3.7423	7.548

TABLE 2: ERROR ANALYSIS OF BMN MODEL TO HUMIDITY

T	DT	Ea	Earms	ET	ER
35.0000	1.3000	.2582	.1679	4.4597	5.790
70.0000	1.4000	.2545	.1649	4.0786	6.240
75.0000	1.5000	.2507	.1633	3.6891	6.796
80.0000	1.6000	.2468	.1633	3.2913	7.499
85.0000	1.7000	.2429	.1648	2.8859	8.416
90.0000	1.8000	.2388	.1680	2.4729	9.659
95.0000	1.9000	2348	.1728	2.0528	-11.436

TABLE 3: ERROR ANALYSIS OF BMN MODEL TO RADIATION RATIO

SR	DSR	Ea	Earms	ET	ER
.0400	.0012	.1234	.0816	1.6457	7.499
.0600	.0018	.1851	.1224	2.4685	7.499
.0800	.0024	.2468	.1633	3.2913	7.499
.1000	.0030	.3085	.2041	4.1142	7.499
.1200	.0036	.3702	.2449	4.9370	7.499
.1400	.0042	.4319	.2857	5.7599	7.499

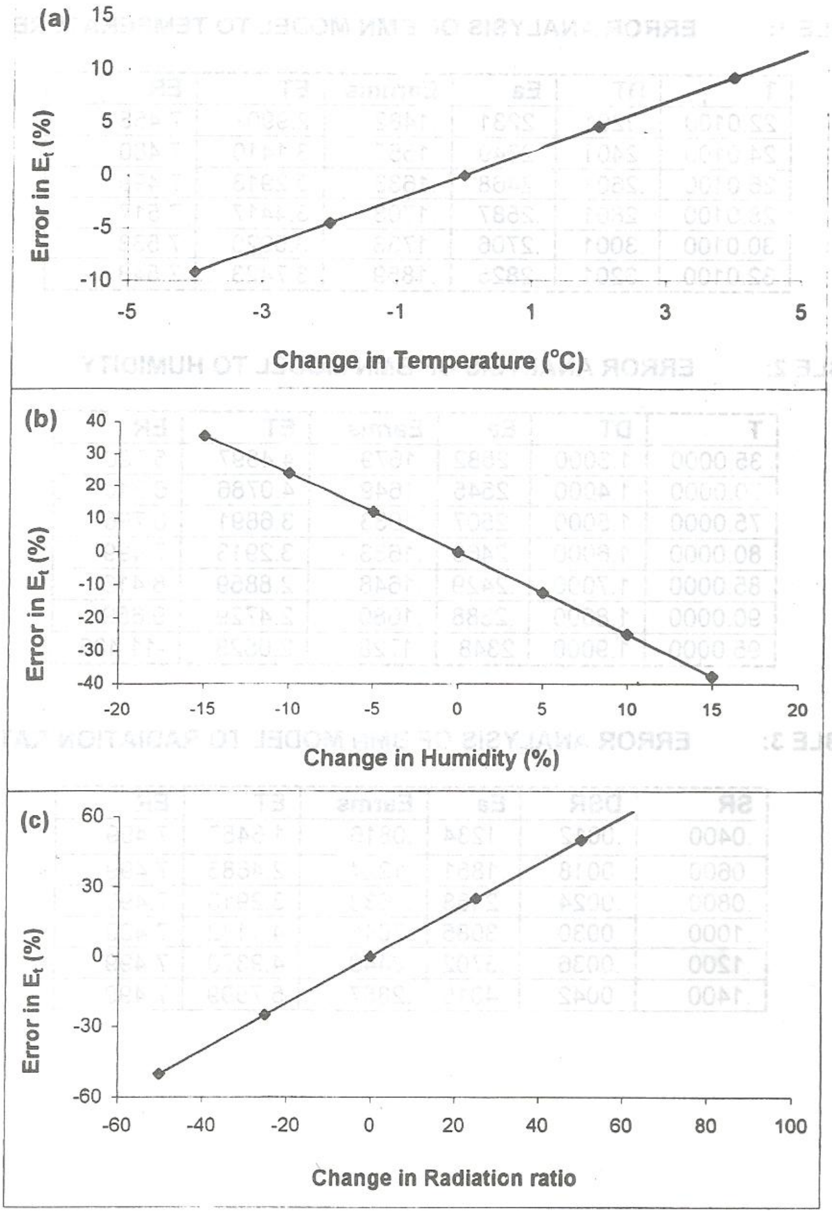

Fig.1: Sensitivity of Blaney-Morin-Nigeria(BMN) model to flunctuations in input parameters of (a) temperature (b) humidity and (c) radiation ratio

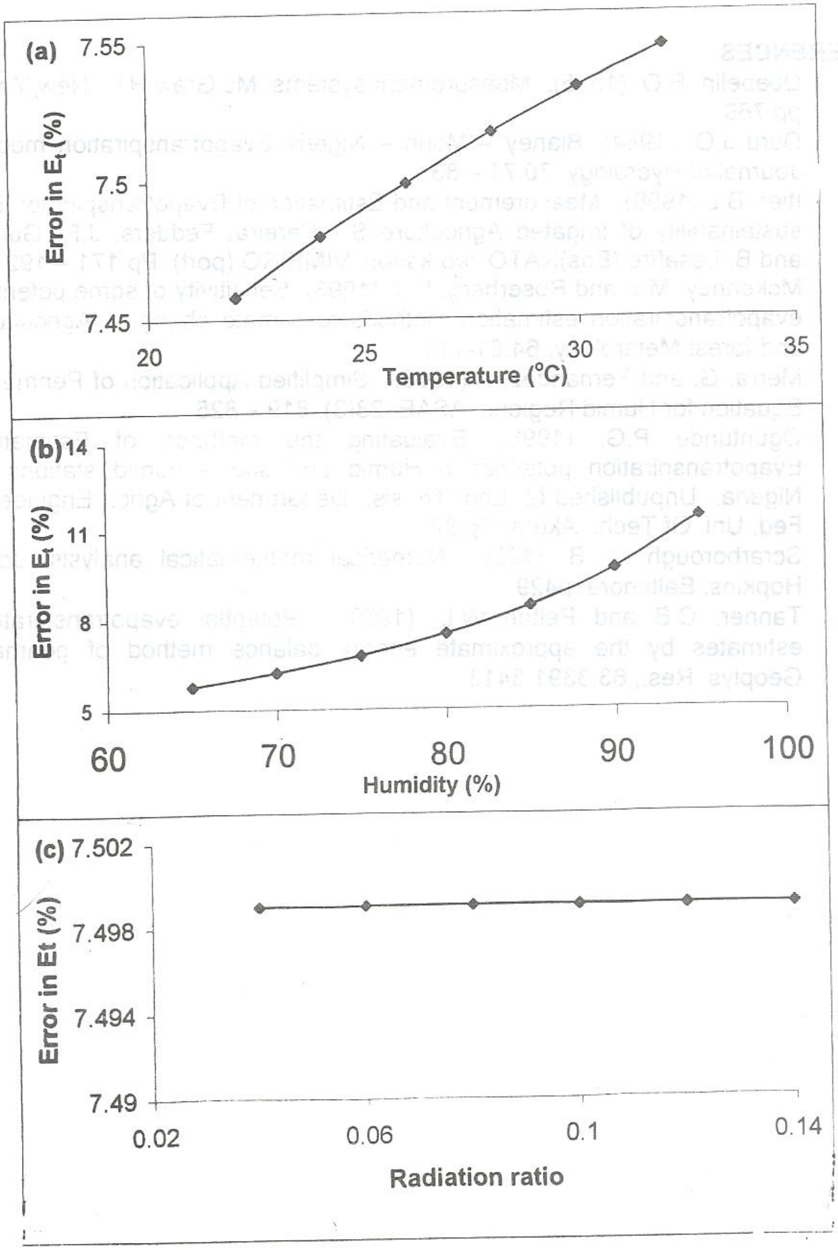

Fig. 2: Relative errors in Blaney-Morin-Nigeria(BMN) model as a function of the indicated input parameters

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