J. Nig. Ass. Math. Phys.
Vol. 3 (1999)

THE SATURATION OF BEAM – EXCITED IONOSPHERIC LOWER HYPRID RESONANCE THROUGH ION ACCELERATION

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ABSTRACT

Electron beams which precipitate into the aurora along the earth's magnetic field lines are responsible for the excitation of broadband electrostatic waves at heights ≤ 1500km, with an intensity peak near the lower hybrid frequency. It is argued that the lower hybrid wave become saturated by transferring energy to ambient ions, to produce an ion conic, which is a distribution with the ions concentrated on a cone in velocity space. The electron and ion populations are modelled, at the outset, as displaced and ordinary Maxwellians, respectively. The relaxation time of the electron beam and the rate of energy transfer from it are estimated from the nonlinear diffusion coefficient, in the limits $v_d/v_e << 1$ and $v_d/v_e >> 1$, where v_d and v_e are the drift and the thermal speeds of the electrons.

1 INTRODUCTION

Plasma phenomenon, such as the precipitation of energetic electrons, the generation of broadband electrostatic waves, and upward streaming ions, seem to occur in this sequence at altitudes about 1500km (Chang & C Coppi 1981, Chike–Obi 1992). Broadband radio noise is produced by the precipitating electron beams (Hoffman & Laaspere 1978), and has an intensity peak near the lower hybrid frequency. There is evidence (Arnoldy et al 1992, Vago et al 1992, Chang 1993) of the presence of upward flowing ions in regions in which electron beams and lower hybrid waves are observed. The ions in the upward fluxes have pitch angles between 90° and 130° forming a distribution known as an ion conic. Efforts to explain the ion acceleration mechanism seem to have settled on two competing theories: that the ions are accelerated by either energy transfer from lower hybrid waves (Chang & Coppi 1981) or from ion cyclotron waves (Andre and Chang 1992). Lower hybrid waves are, however, more prominent at lower heights (< 1500km) (Gurnett and Frank 1972), which lends support to a theory based on lower hybrid energization. This paper is organized as follows: In section 2, we model the electron and ion populations as Maxwellian, with the entire electron distribution displaced by the drift speed of the electron beam. The resulting dispersion relation shows that excited waves near the lower hybrid frequency have the most prominent growth rates. An intensity peak at lower hybrid frequencies is also obtained in a model in which the electron distribution is a superposition of a displaced Maxwellian, for the electron beam, and ordinary Maxwellian for background electrons. In section 3, we propose a calculation, based on a quasilinear diffusion equation, of an energy relaxation time, which we define as the time interval over which the incoming electrons should lose most of their energy excess (over the background particles) to the growing waves. A resonance condition is found, due to which, the lower hybrid waves transfer energy to the ions, resulting in ion acceleration and lower hybrid wave saturation. The rate of ion heating is calculated in the limits of low and high electron beam drift speeds. A discussion of the results is given in section 4.

2 EXCITATION FO THE UNSTABLE LOWER HYBRID WAVES

We consider a warm electron beam with thermal velocity v_e, drifting with velocity v_d, relative to the background ions. For electrostatic modes, $E = -\nabla\phi$, where $\phi = \phi_k \exp\left(-i\omega t + ik_\parallel X_\parallel + ik_\perp X_\perp\right)$, the relevant dispersion relation can be written in analytic form in which $\Omega_i << \omega << \Omega_e, k_\perp >> k_\parallel$, and $\rho_1^{-2} << \rho_e^{-2}$. $\Omega_i = eB/m_iC$ and $\Omega_e = eBm_eC$ are the ion and electron gyrofrequencies, ρ_i and ρ_e are the gyroradii, m and T are mass and temperature respectively, and the subscripts $\parallel$ and $\perp$ stand for the parallel and perpendicular components of the wave vector $\underline{k}$ relative to the local geomagnetic field $\underline{B}$. we assume a Maxwellian distribution for the ions:

$$f_i = n_i \left(2\pi T_i / m_i\right)^{-1/2} \exp\left(-v^2 / v_i^2\right) \tag{1}$$

and a displaced Maxwellian for the electrons:

$$f_e = n_e \left(2\pi T_e / m_e\right)^{-1/2} \exp-\left(v_\parallel - v_d\right)^2 / v_e^2 \tag{2}$$

where v_i is the ion thermal speed, and n_i and n_e are the ion and electron densities. Solving the linearized Vlasov equation for the perturbed ion and electron densities, and substituting in Poisson's equation, the dispersion relation is

$$k^2\lambda_1^2 = W_i + \left(T_i n_e / T_e n_i\right) W_e$$

where

$$W(x) = -\frac{1}{\sqrt{\pi}} \int d\xi \frac{\xi \exp\left(-\xi^2\right)}{(\xi - x)}$$

is the Landau integral, $W_i = W(\omega/kv_i)$, $W_e = W(y) = W\left(\omega - k_{\|} v_d\right)/k_{\|} v_{e'}$ and $\lambda_i^2 = T_i n_i e^2/4\pi$. It is convenient to write the real part of the dispersion relation as

$$\frac{\omega^2}{\omega_i^2} = 2x^2 \operatorname{Re}\left[w(x) + \frac{W(y)}{\theta}\right] \tag{3}$$

Its imaginary part is

$$\operatorname{Im}W(X) + (1/\theta)W(y) = 0 \tag{4}$$

Where $x = \omega/KV_i$ and $\theta = T_e/T_i$. Writing y as a function of x, we now plot, from equations (3) and (4), graphs of ω^2/ω_i^2 and v_c / v_e (figure 1 as functions of x, where v_c is the value of v_d for the onset of the instability. We notice ω^2/ω_i^2 (x) has two branches, as is the case with $v_c / v_e(x)$. The lower branch of $\omega^2/\omega_i^2(x)$ is more unstable than the upper branch since the growth rate of the instability decreases as the threshold velocity increases. Therefore, the amplitude spectrum should be peaked in the region $\omega^2 \leq \omega_i^2 \approx \omega_{1h'}$ where ω_{1h} is lower hybrid frequency. Also, the critical velocity also decreases with increasing T_e / T_i (Duwa 1997)

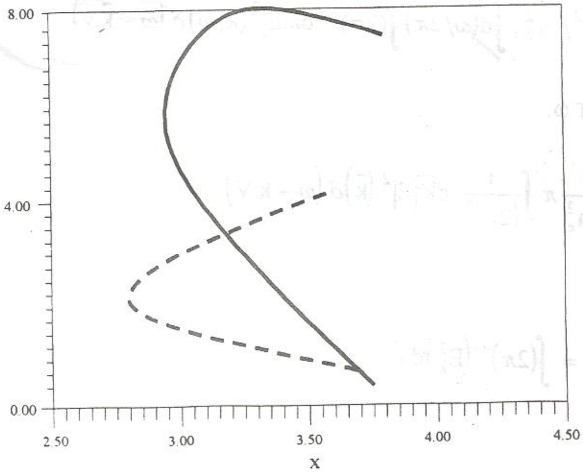

Fig. 1. Plots of $(\omega/\omega_i)^2$ (solid) and v_c/v_e (dashed) against x.

3. ION ENERGIZATION BY UNSTABLE LOWER HYBRID WAVES.

Unstable lower hybrid waves, excited by precipitating electron beams whose drift speeds exceed the critical speed, can heat the background ions. We consider a model in which the source region of the drifting electrons, in the upper ionosphere, is located at some x = L, and the incoming electrons travel earthward along the earth's magnetic field lines. As the electrons stream through the background ions, the electron distribution develops a "plateau" at x = 0 by exciting lower hybrid waves and transferring energy to the waves. The waves can resonate with ions transferring momentum and energy to the ions. In other words, the unstable 1.h. wave mediate to effectively transfer energy and momentum from the incoming electron beams to the background ions. The time interval over which the ions are heated is equal to the time interval over which the waves reach the marginal stability point, which is equal to the time interval required for the electron distribution to develop a plateau, or, the travel time from x = L to x = 0. This time interval τ_D, can be calculated from the perpendicular diffusion coefficient D_e from quasilinear theory, using the following equation for the evolution of electron distribution (Retterer et al 1994):

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial \bar{v}}\left(\bar{D}e\frac{\partial f}{\partial \bar{v}}\right) \qquad (5)$$

The diffusion coefficient is

$$\bar{D}_e = \left(e^2/m_e^2\right)\int d(\omega/2\pi)\int (2\pi)^{-3} d\bar{k}|E|^2(k,\omega)\,\delta\left(\omega - \bar{k}.\bar{v}\right)$$

Integrating over ω,

$$\bar{D}_e \approx \frac{e^2}{m_e^2}\pi\int \frac{1}{(2\pi)^3}\, d\bar{k}\,|E|^2\left(\bar{k}\right)\delta\left(\omega - \bar{k}.\bar{V}\right)$$

Using

$$\left\langle E^2\right\rangle k = \int (2\pi)^{-3}\left(E_k^2\right) d\,\bar{k}$$

and

$$\delta\left(\omega - \bar{k}.\bar{v}\right) \approx \frac{1}{k\,\Delta\,v},$$

we find that

$$D_e \approx \frac{e^2}{m_e^2}\frac{<E^2>}{k\Delta v}$$

where Δv is the drift velocity of electrons, v_d. The diffusion time τ_D may be estimated from $D_e \sim v_d^2/\tau_D$, or,

$$\tau_D = \frac{v_d^3 k m^2}{e^2 < E_{ih}^2 >} \tag{6}$$

Then we can estimate the $L \approx <\tau_D v_d>$. For typical incoming beams and auroral plasma, $v_d \approx 10^7\, m/s$, , $v_\parallel \approx 10^8 \approx$ E_{rms} $50mV/m$, $\omega_{1h} \approx 5\Pi \times 10^3$ rad/s, $k_\parallel \approx \omega_{1h}/v_d$, and $k = 40k_\parallel$. The distance travelled is therefore about 8600km. Note that by using $\tau_D = \gamma^{-1}$, and the expression for growth rate (Papadopoulus and Palmadesso 1976):

$$\gamma \approx \omega_{1h}\left(\frac{V_d}{V_e}\right)^2 \frac{\Pi^{1/2}}{4}$$

with $V_d/V_e = 3$, the same result for L is obtained. The energy lost from the electrons during the time interval τ_p is calculated from

$$\Delta E_e = \frac{1}{2} mLS \int (f_o - f_\infty) V_\parallel^2 dv_\parallel \tag{7}$$

where $f_o = (n/V_e)\Pi^{-1/2} \exp\left[-\left(v_\parallel - v_d\right)^2 \big/ v_e^2\right]$ is the initial displace Maxwellian $f_\infty = (n/V_o)\Pi^{-1/2} \exp\left(-v_d^2/v_e^2\right)$ is the final electron distribution, and S is the cross – sectional area of the beam. We find

$$\Delta E_e = \frac{1}{2} m_e n L S v_e^2 \left[\frac{1}{4}\mu\left(x^2\right) + \frac{1}{2}x^2\phi(x) - \frac{1}{3}\Pi^{-1/2}x^3 \exp\left(-x^3\right) - \Pi^{-1/2}x \exp\left(-x^2\right) - 1\right] \tag{8}$$

where $X = v_d/v_e$, $\mu(x^2) = 4(\Pi)^{-1/2}\int t^2 \exp(-t^2)dt$ is the Maxwell integral (Trubnikov 1965) and $\phi = 2(\Pi)^{-1/2}\int \exp(-t^2)dt$ is the error function (Abramowitz and Stegun 1970). For low drift speeds, x << 1, equation (7) gives

$$\Delta E_e \approx \left(\frac{m_e nLS}{6\sqrt{\Pi}}\right)\left[\left(v_e\, v_d^3\right)^{\frac{1}{2}} - \frac{3}{5}\left(\frac{v_d^5}{v_e}\right)^{\frac{1}{2}}\right] \quad (9)$$

and for x >> 1

$$\Delta E_e \approx \left(\frac{m_e nLS}{4\sqrt{\Pi}}\right)\left(v_e - v_d\right)^2 \quad (10)$$

Neglecting energy losses, most of the electron energy goes into ion perpendicular energy due to the following resonance condition (Coppi et al 1976).

$$\left.K_{\parallel} V_{\parallel}\right)_e \approx \omega \approx \left.k_i v_i\right)_i$$

Hence, the perpendicular energy of an ion increases, on the average, during the diffusion time τ_D, by $\Delta E_{i\perp} = \Delta E_e/nLS$, or,

$$\Delta E_{i\perp} = \frac{1}{2} m e v_e^2 F(x) \quad (11)$$

where

$$F(x) = \frac{1}{4}\mu\left(x^2\right) + \frac{1}{2}x^2\phi(x) - \frac{1}{3\sqrt{\Pi}}x^3 \exp\left(-x^2\right) - \frac{1}{\sqrt{\Pi}}x\left(\exp\left(-x^2\right) - 1\right) \quad . (12)$$

Realistic values $x \equiv V_d/V_e$ are 1.83 (Maggs 1976) and 6.11 (Lotko and Maggs 1981). With these, $\Delta E_{i\perp,1.83}$ 240. 2eV and $\Delta E_{i\perp,6.11}$ = 170.6ev. We may compute the average rate of ion heating, $\Delta E_{i\perp}/\Delta t$, from $\Delta E_{i\perp}/\Delta t = \gamma \Delta E_{i\perp}$ and equation (10). Then, for $1.83 \le v_d/v_e \le 6.11$, we have 5.6Mev/s ≤ $\Delta E_{i\perp}/\Delta t$ ≤ 44.3 Mev/s.

4. DISCUSSION

We have analyzed electron – beam excitation of ionospheric lower hybrid waves, the saturation of the waves, and the consequent energization of ions, using a model which represents the electrons as a displaced Maxwellian and thermal ion background. The resulting dispersion relation shows that most unstable of the excited frequencies are in the lower hybrid region. In section 2, using a quasilinear equation for the evolution of the electron distribution function, we found a diffusion time τ_D ~ 1s and the distance traveled by the electron beam before thermalization to be 8600km. By considering two limits, $v_d << v_e$ and $v_d >> v_e$, are treated analytically, we have estimated energy transfer from the electrons to the ions. The rate of ion heating was found to be about 44MeV/s, which was about the same results obtained elsewhere (see, for example, Chang & Coppi 1981).

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