J. Nig. Ass. Math. Phys.
Vol. 3 (1999)

POTENTIAL DISTRIBUTION AND FOCUSING PROPERTIES OF AN EQUIDIAMETER CO-AXIAL CYLINDRICAL ELECTROSTATIC LENS.

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ABSTRACT

An alternative method of derivation of the results of Read *et al* (1970) is presented. The values were obtained using a different parametrisation of the potential function, and using a numerical approach. The results were used to plot the equipotential lines, trace the electron ray path and determine the cardinal points of an equidiameter co-axial cylindrical electrostatic lens over a range of voltage ratio and lens separation. The cardinal points (f_1 and f_2) thus measured from the electron ray path showed a close approximation to the calculated value.

INTRODUCTION

An electrostatic lens is a device for focusing charged particles, such as electrons, in space. The action of an electrostatic lens is analogous to that of the equivalent optical lens. The focusing of electrons in a potential field is similar to that of light in a medium. It occurs through a gradual deflection of the particles' trajectory.

In this paper, the focusing properties of an equidiameter co-axial cylindrical electrostatic lens is determined by studying the potential distribution around the electrode arrangement constituting the lens and the motion of electron across the potential region. The lens geometry to be considered consists of two long thin co-axial cylinders of the same diameter D separated by a distance g. The two cylinders are at potentials V_1 and V_2 and the electrons are assumed to be travelling from the region held at V_1 to the region held at V_2.

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Practically, lens systems have been made from both thick-walled cylinders and thin-walled cylinders but all data in the literature (Bernard, 1967; Read,1969 and Read *et al*, 1970) including that in this paper, deal only with thin-walled lens. The region of the lens that has to be thin is near the gap, and a method of constructing thick-walled lenses which should behave like the equivalent thin-walled lens as reported by Read *et al*, 1970 is as shown in Figure 1.

This work is an alternative derivation of the results of Read *et al* (1970) using a different parametrisation of the potential function, and using a numerical approach.

THEORETICAL ANALYSIS

The focusing properties of an electrostatic cylindrical lens can be determined by studying the potential distribution around the arrangement. The potential distribution U(r, θ, z) can be obtained by solving the Laplace equation in cylindrical co-ordinates,

$$\nabla^2 U(r,\theta,z) = 0 \qquad (1)$$

given the appropriate boundary conditions.

There are several analytical and numerical methods of solving the Laplace equation (1) to obtain U(r, θ, z). Read et al, (1970), have described an accurate analytic iterative method, based on the assumption that the thickness of the material, from which a two-dimensional electrostatic lens is constructed is very less than the radii of the cylinders. He obtained the potential at point (R,z), due to a charge q, uniformly distributed around a circle of radius R lying in plane $z = z_o$ and having its centre on the z-axis is given by

$$U(R,z) = \left(\frac{q\,k}{4\pi^2 \varepsilon_o R}\right).G(k) \qquad (2)$$

where

$$K^2 = \frac{4R^2}{4R^2 + (z - z_o)^2}, \qquad R = \frac{1}{2}D$$

in which G(k) is the complete elliptical integral of the first kind. And D is the diameter of the lens. From the calculated potential distribution U(R,z), it is then possible to obtain the axial potential function

$$\phi_o(z) = U(0,z) \qquad (3)$$

The axial potential distribution $\phi(z)$ is related to the axial potential $\phi_o(z)$ as

$$\phi(z)=\left(\frac{V_1+V_2}{2}\right)+\left(\frac{V_2-V_1}{2}\right)\phi_0(z) \tag{4}$$

for a two-cylinder lens having voltages V_1 and V_2 on the left-hand and right-hand cylinders respectively.

However, the axial potential function for this lens configuration have been parametrised (Grivet, 1965) by the expression

$$\phi_0(z)=\frac{1}{w`g}\ln\frac{\operatorname{Cosh}\left(wz+\frac{1}{2}w'g\right)}{\operatorname{Cosh}\left(wz-\frac{1}{2}w'g\right)} \tag{5}$$

which is a modification of an expression (Equation 6) by Betram (1940)

$$\phi_0(z)=\frac{V_1+V_2}{2}+\frac{V_2-V_1}{2wg}\ln\frac{\operatorname{Cosh}w\left(z+\frac{wg}{2}\right)}{\operatorname{Cosh}w\left(z-\frac{wg}{2}\right)} \tag{6}$$

Betram (1940) arrived at this expression by considering two co-axial cylinders of equal diameter, separated by a distance g, and by making the approximation that the potentials at the radii of the cylinders vary linearly across the gap between the cylinders, where $w=\frac{1.32}{R}$, $w=\frac{1.67}{R}$ and D is the diameter of the lens. For a lens with g = 0, equation (6) reduces to

$$\phi(z)=\left[\frac{V_1+V_2}{2}\right]+\left[\frac{V_2-V_1}{2}\right]\tanh(wz) \tag{7}$$

as earlier suggested by Gray (1939), which is correct to a few percent.

In terms of Bessel functions, ϕ (z) is expressed as

$$\phi(z)=\frac{1}{2}(V_1+V_2)+\frac{1}{2}(V_2-V_1)\left\{1-2\sum_{n=1}^{\infty}\frac{e^{-\left(\frac{\mu}{|z|}\right)}}{\mu J_1(\mu)}\right\} \quad (8)$$

Hence, with the applied voltages V_1 and V_2, the potential distribution in the space of the lens is

$$U(r,z)=\frac{1}{2}(V_1+V_2)+\frac{1}{2}(V_2-V_1)\left\{1-2\sum_{n=1}^{\infty}\frac{e^{-\left(\mu/|z|\right)}}{\mu_n \; J_1(\mu_n)}\right\} \quad (9)$$

where μ_n is the nth root of $J_o(\mu) = 0$, J_o and J_1 are Bessel functions of the first kind.

Having obtained ϕ (z) for a cylindrical electrostatic lens, the paraxial ray path through the lens can be traced by integrating the ray equation

$$\frac{d^2\rho}{dz^2}+\frac{3}{16}\left(\frac{\phi'}{\phi}\right)^2\rho=0 \quad (10)$$

where $\rho = r\phi^{1/4}$ (Grivet, 1965). Thereafter, the cardinal points of the lens can be obtained from the traced ray path. Conversely, with

$$\frac{f_2}{f_1}=\sqrt{\frac{V_2}{V_1}} \quad (11)$$

it was shown (Grivet, 1965) that

$$f_2=\frac{3.030\gamma^{1/4}}{2\varepsilon^2}, \quad \text{where} \quad \varepsilon=\sqrt{\gamma-1} \quad (12)$$

The mid focal points, F_1 and F_2 are related to the focal lengths by the expressions

$$f_1 = F_1\left(\frac{V_2}{V_1}\right)^{-1/4} \tag{13}$$

and

$$f_2 = F_2\left(\frac{V_2}{V_1}\right)^{1/4} \tag{14}$$

Thus, the cardinal points of an electrostatic lens can be calculated using equations (11) - (14).

NUMERICAL COMPUTATION

The general procedure for computing lens data discussed in the previous section will now be applied to the determination of the parameters of a double-cylinder electrostatic lens shown in figure 1. Lens of diameter 2cm, lens separations g= 0.1cm, 0.5 and 1.0cm between cylinders maintained at voltage ratios V_2/V_1 ranging from 1.5 to 50.0 are considered. In finding the cardinal points, the axial potential ϕ (z) is first computed using the parametrised function, equation (5).

In plotting the parametrised axial potential function against the axial points z, a computer program was written in FORTRAN to evaluate the expression at different values of z. The values of $w\left[=\frac{1.32}{R}\right], w\left[=\frac{1.67}{R}\right]$ and g are input and a choice of z is made in order to evaluate the parametrised axial potential function $\phi_o(z)$ corresponding to the axial points. A plot of the axial potential $\phi_o(z)$ against the axial points shows that the potential does not differ significantly for the three lens separation considered. Also, in comparing the values of the parametrised axial potential with the axial potential, obtained by Read *et al*, figure (2), it was observed that the difference in two axial potential functions was not very significant for the lens separations.

The potential distribution U(r,z) in the space of cylindrical electrostatic lens having applied voltages V_1 and V_2 was used to plot the equipotential lines. This was done by solving the equation resulting from setting U(r,z) to an initial values. Thus yielding the radial points ± r corresponding to the axial point z. Based on this method, the equipotential lines (Figure 3) for a co-axial cylindrical lens diameter D = 2 cm and lens separation 0.1 cm was computed and plotted. This was found to vary as g varied (Figure 4).

In tracing the electron ray path in the electrostatic lens, it was considered that the incident electron moved in the paraxial region, such that equation (10) reduces to

$$\frac{d^2r}{dz^2} + \frac{\phi'}{2\phi}\frac{dr}{dz} + \frac{\phi''}{4\phi} r = 0 \qquad (15)$$

Therefore, equation (15) was solved by the quartic Runge-Kutta method (Appendix 1) with the following choice of the initial values 0.1, 0.00 and 0.001 for r, z and $\frac{dr}{dz}$ respectively. The computation was iterated by a DO LOOP, to obtain different values of r, z and $\frac{dr}{dz}$. The output values of r and the corresponding values of z gave the points used in plotting the electron ray path (Figure 5) from where the focal length, f_1 and f_2 were measured. The measured values of the cardinal points were thereafter compared with the values obtained by calculation.

RESULTS AND DISCUSSION

The comparison of the parametrised axial potential function, with the potential function obtained by an iterative method of solving Laplace equation (table 1) shows that the differences in the values of the axial potential function is about 3% on the average. This determination is for lens gap, g = 0.1 cm to 1.0 cm. The insignificant difference noticed thus justifies the choice of the parametrised axial potential function as an approximation to the analytical axial potential function obtained by an iterative method.

The cardinal points (f_1 and f_2) measured from the electron - ray path (figure 5) shows a close approximation to the values obtained by calculation (table 2). The small discrepancy between the analytical values of Read *et al* and values obtained from the plotted ray path can be attributed to the round off errors in the numerical method.

CONCLUSION

The potential distribution in the space of a co-axial cylindrical electrostatic lens considered, yielded the axial potential function along the axis of the lens configuration. This facilitated the description of the expressions for calculating the cardinal points of the lens.

Also, the axial potential function made it possible to integrate the ray equation which generated the points (r, z) used for the tracing of the electron path. The separation of the cylinders constituting the lens resulted in the equipotential lines which served as a refractive medium for the passage of electrons. Since the comparison of the values of the calculated focal lengths, with the ones obtained in the electron path showed a close approximation, it can be concluded that the focusing of electrons, moving in the paraxial region of an electrostatic lens is accurately determined by the axial potential function and the separation between the lens configuration.

Table 1 **Axial Potential Function** $(\phi_0(z))$

Z	Read *et al*	Parametrised Expression (This work)
0.1	0.120	0.147
0.2	0.250	0.226
0.3	0.360	0.334
0.4	0.460	0.434
0.5	0.560	0.525
0.6	0.640	0.606
0.7	0.710	0.710
0.8	0.770	0.740
0.9	0.810	0.800
1.0	0.850	0.850

Table 2 Lens Parameters for Electrostatic Cylindrical Lens with g = 0.1 cm

V_2/V_1	$\gamma = V_1/V_2$	f_1 (cm)	f_2 (cm)	F_1 (cm)	F_2 (cm)
1.5	0.667	16.67	20.42	18.45	18.43
2.0	0.500	5.26	7.44	6.26	6.26
3.0	0.333	1.86	3.22	2.45	2.45
5.0	0.200	0.74	0.83	0.55	0.56
10.0	0.100	0.29	0.91	0.51	0.51
12.0	0.083	0.23	0.80	0.43	0.43
15.0	0.067	0.18	0.70	0.36	0.36
20.0	0.050	0.13	0.60	0.28	0.28
50.0	0.020	0.05	0.15	0.15	0.15

ACKNOWLEDGEMENTS

The authors are grateful to Prof. Awele Maduemezia, Dr.I. P. Farai, and others whose comments have helped improve the manuscript.

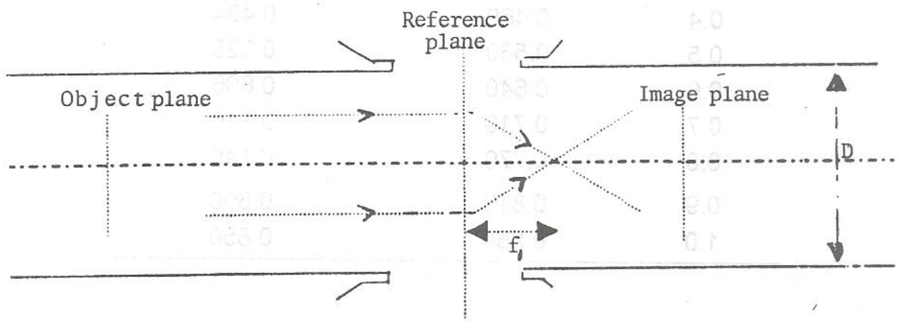

Fig 1: Co-axial Cylindrical Electrostatic lens showing electric path.

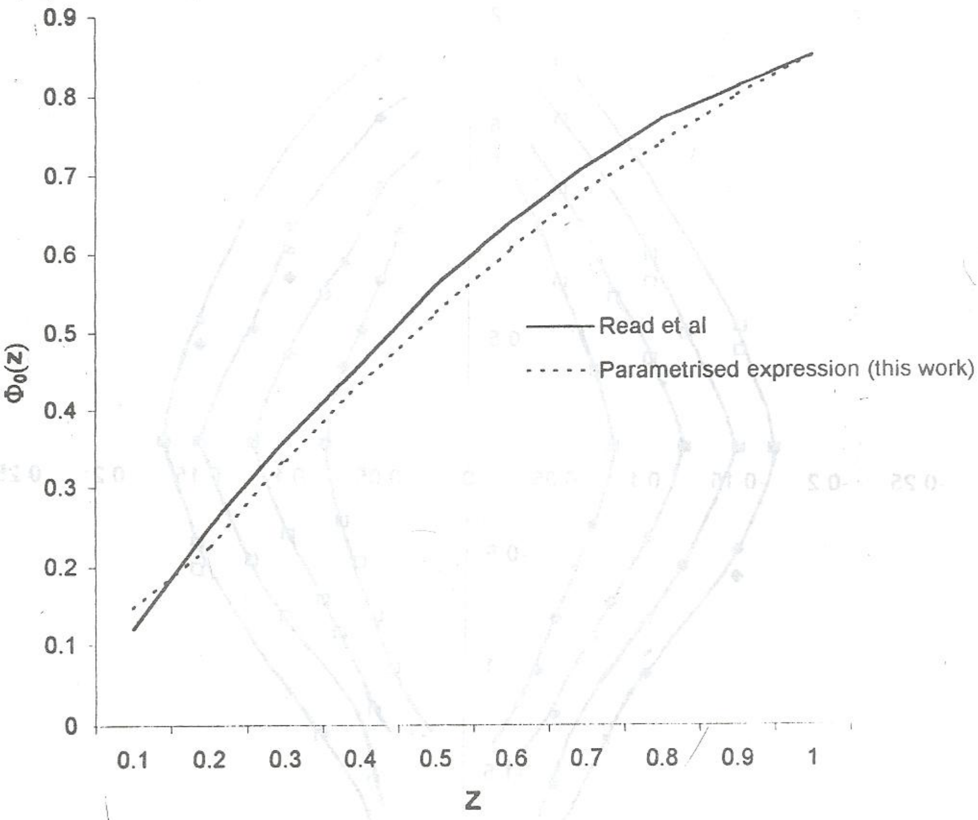

Fig. 2: Comparison of Axial Potential

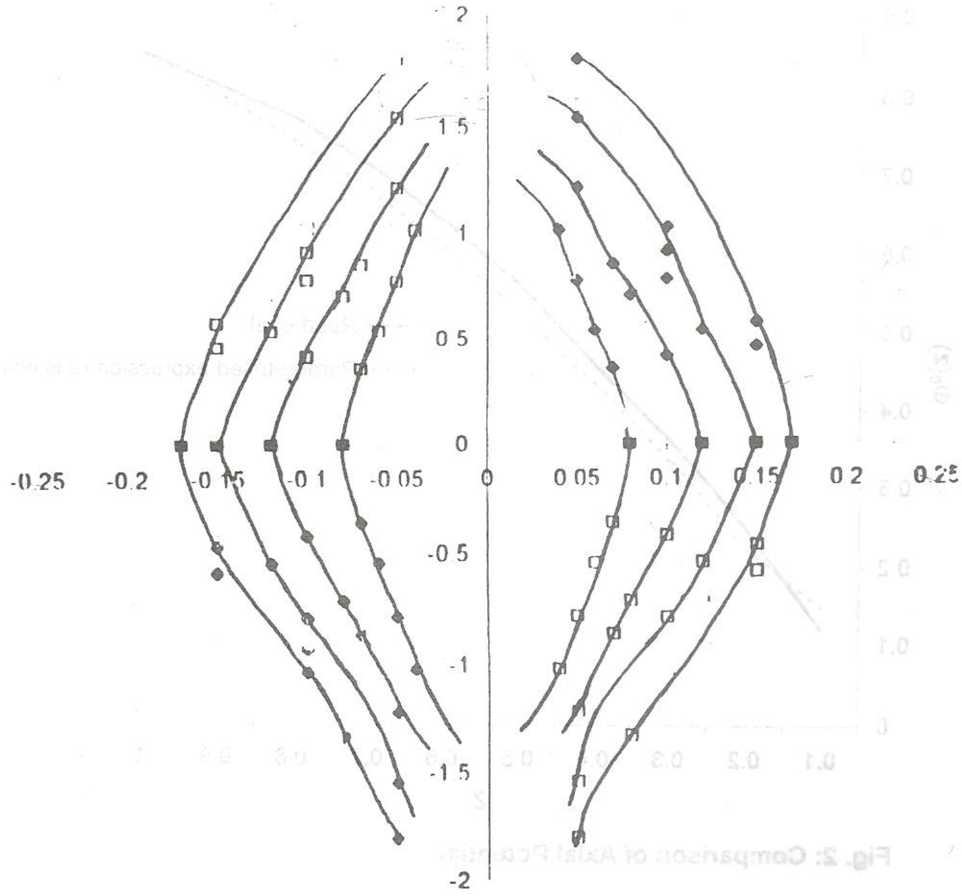

Fig. 3: Equipotential lines when g = 0.1 cm

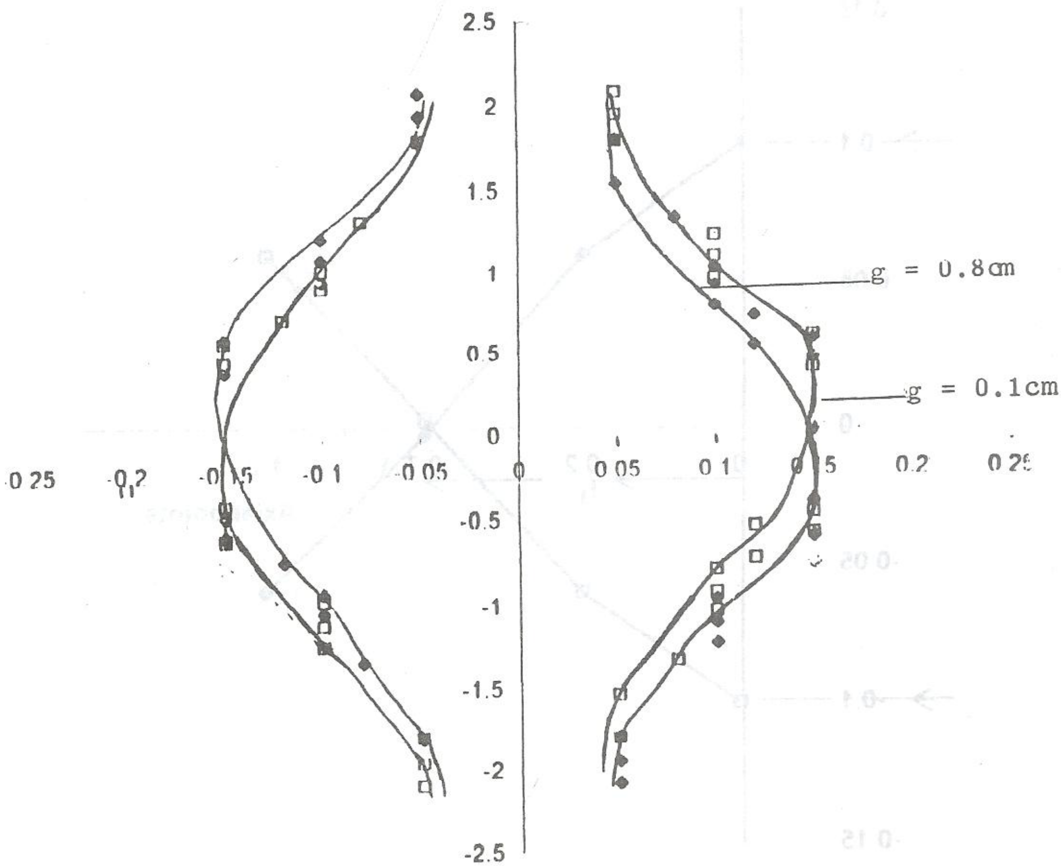

Fig. 4: Comparison of Equipotential lines (for g = 0.1 cm and g = 0.8 cm)

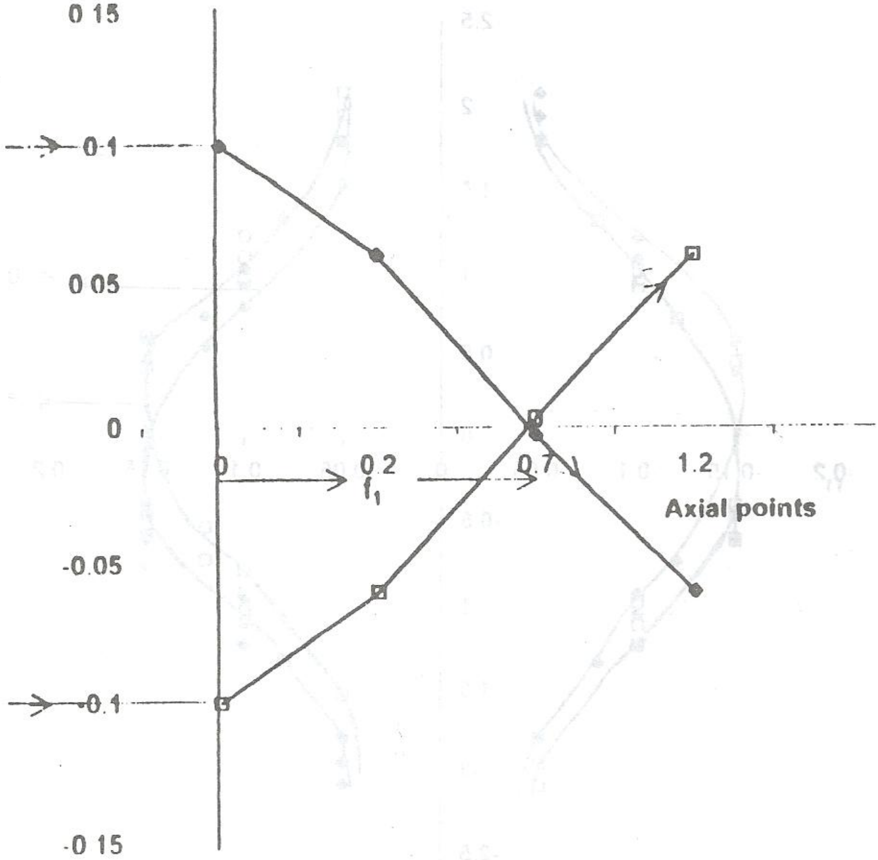

Fig 5 Electron ray path

Appendix 1

```
      DIMENSION R(1001),Z(1001),V(1001)
      OPEN(UNIT=4,FILE='TRACE.OUT',STATUS='NEW')
      WRITE(*,*)'ENTER NUMBER OF ITERATION'
      READ(*,*)ITERATION
C     INITIALIZING PARAMETERS
      V(1) = 0.001
      R(1) = 0.1
      Z(1) = 0.0
      H = 0.2
      V1 = 10
      V2 = 50
      W = 1.3160952
C     CALCULATION STARTS

      DO 10 I = 1, ITERATION
      Q =(EXP(W*Z(I))-EXP(-W*Z(I)))/(EXP(W*Z(I))+EXP(-W*Z(I)))
      P = ((V1+V2)/2)+((V2-V1)/2)*Q
      P1=W*((V2-V1)/2)+(1/(EXP(W*Z(I))+EXP(-W*Z(I))))**2
      P11=2*W*P1*P
      C=P1/(2*P)
      D=P11/(4*P)
      A1=(-C*V(I))-(D*R(I))
      B1=V(I)
      T1=V(I)+((H/2)*A1)
      T11=R(I)+((H/2)*B1)
      A2 = (-C*T1)-(D*T11)
      B2 = T1
      T2=V(I)+((H/2)*A2)
      T22=R(I)+((H/2)*B2)
      A3=(-C*T2)-(D*T22)
      B3=T2
      T3=V(I)+(H*A3)
      T33=R(I)+(H*B3)
      A4=(-C*T3)-(D*T33)
      B4=T3
      V(I+1)=V(I)+(H/6)*(A1+(2*A2)+(2*A3)+A4)
      R(I+1)=R(I)+(H/6)*(B1+(2*B2)+(2*B3)+B4)
      Z(I+1)=Z(I)+H
      WRITE(4,*)R(I),Z(I),V(I)
10    CONTINUE
      STOP
      END
```

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