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PHASE PLANE ANALYSIS OF A LIQUID FRONT MOVING THROUGH A HOT POROUS ROCK

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ABSTRACT

It is known that hot geothermal reservoirs may have temperatures as high as 200 – 300°C and in such reservoirs the water vapour may have pressures in the range of 1- 30 atmospheres. In this work we show analytically how pressure runaway occurs.

INTRODUCTION

Geothermal reservoirs are of interest because of their potential to generate power by drawing off vapour along the wells in order to drive turbines. The reservoirs are typically located near regions of volcanic or tectonic activity. These reservoirs may therefore have temperatures as high as 200 – 300°C (see (21)). In several geothermal reservoirs, cold water is actively injected into the reservoir through well bores and as the injected liquid moves into the hot rock and a fraction of the liquid vaporizes. The mechanism by which relatively cold liquid vaporizes as it invades a hot permeable rock is of much interest and therefore a subject of research. Also of research interest is the high rise in the pressure.

Woods and Fitzgerald (2) examined the underlying physical processes which control such vaporization. In particular, they investigated how the heat transfer between the rock and the water is coupled to the dynamics by which the newly formed vapour can migrate ahead of the liquid – vapour interface. They also investigated how the vaporization process depends upon the rate of supply of water, the geometry of the source of water and the intrinsic properties of the reservoir. The purpose of the present paper is to investigate the phenomenon of both pressure and thermal runaways.

MATHEMATICAL FORMULATION

Here we are interested in the vapour region and following (2), we have Darcy's law

$$\mu \underline{v} = -k\nabla p \tag{2.1}$$

where $\underline{v}$ is the velocity of vapour and p is the pressure. Here μ represents the dynamic viscosity of the vapour and k the permeability.

The conservation of mass within the vapour region is

$$\phi \frac{\partial \rho}{\partial t} + \nabla.\left(\underline{v}\,p\right) = 0 \tag{2.2}$$

where ρ is the vapour density

The equation of state relates the density ρ, temperature θ and the pressure p of the vapour

$$p = \rho R \theta \tag{2.3}$$

where R is the gas constant.

Combining (2.1) - (2.3) we obtain

$$\left(\frac{p}{\theta}\right)_t - \frac{k}{\phi\mu}\nabla.\left(\frac{p}{\theta}(\nabla p)\right) = 0 \tag{2.4}$$

According to (2.4), the similarity length scale L over which changes occur in time t is given by L~(kp/ϕμ)t). The similarity lengthscale of thermal diffusion, L, is $(\lambda\tau)^{\frac{1}{2}}$. Hence L is larger than L, and the temperature field within the vapour region adjusts to the far – field vapour temperature through thermal conduction across a very narrow thermal boundary layer near the interface. Beyond this narrow boundary layer, the vapour attains the far – field temperature and for this isothermal situation

$$\frac{\partial p}{\partial t} - \frac{k}{\phi\mu}\nabla.(p\nabla p) = 0 \tag{2.5}$$

METHOD OF SOLUTION

Let $t = \frac{\phi u}{k}\tau$

We obtain

$$\frac{\partial p}{\partial \tau} = \nabla.(p\nabla p) \tag{3.1}$$

i.e

$$\frac{\partial p}{\partial \tau} = \frac{1}{r^n} \quad \frac{\partial}{\partial r}\left(r^n p \frac{\partial}{\partial r} p\right) \tag{3.2}$$

Let P = $\left(\tau - \tau_o\right)^{-1}\ f(r)$ (3.3)

We obtain

$$ff'' + \frac{n}{r} ff' + (f')^2 + f = 0 \tag{3.4}$$

following Tayler [1], we let

$f = r^2 g,\ f' = rh$ (3.5)

so that $f'' = h + rh'$ (3.6)

and

$$\frac{dh}{dg} = \frac{n\,gh + h^2 + g + gh}{g(2g - h)},\ g \geq 0 \tag{3.7}$$

Thus

$$\frac{dr}{r} = \int_0^{f/r^2} \frac{dg}{h(g) - 2g} \tag{3.8}$$

we observe that there are two critical points in the (g,h(g) plane namely (0,0) and (g, 2g) where

$$g = \frac{-1}{2n+6} \tag{3.9}$$

Hence

$$f = \frac{-r^2}{2n+6} \tag{3.10}$$

and

$$p = \frac{-r^2}{(2n+6)(\tau - \tau_0)} \tag{3.11}$$

i.e

$$p = \frac{-r^2}{\frac{k}{\phi\mu}(2n+6)(\tau - \tau_0)} = \frac{-\phi\mu r^2}{k(2n+6)(\tau - \tau_o)}$$

$$p = \frac{\phi\mu r^2}{k(2n+6)(\tau_0 - \tau)}, (kt)^{\frac{1}{2}} < r < (kp/\phi\mu)t)^{\frac{1}{2}} \qquad (3.12)$$

DISCUSSION OF RESULTS

(i) We see clearly form (3.12) that pressure becomes very large ($p \to \infty$), i.e pressure run away occurs as $t \to t_o$. Also (3.12) and (3.1) show that velocity becomes very large as $t \to t_o$.

(ii) Also (3.12) depicts the effect of geometry on pressure, for a plain-air source, n = 0, cylinder, n = 1 and sphere, n = 2.

Also (3.12) shows the relationship between the key parameters, ϕ, μ and k. Moreover (3.12) shows that newly formed vapour migrates ahead of the liquid – vapour interface because its pressure is very high

REFERENCES

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