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AN EXPRESSION FOR E_Z COMPONENT OF SPATIAL NON UNIFORM ELECTROMAGNETIC SOURCE FIELDS IN LOW LATITUDE.

ABAJINGIN D.D AND OBODI G.N
ONDO STATE UNIVERSITY, ADO - EKITI.

ABSTRACT

In this presentation we obtained the expression for Ez component of Non-uniform source fields in low latitude. The plots obtained both in space-frequency and space - time domains give a more accurate form than our earlier expression. The non-homogenous differential equation was reduced to cauchy initial boundary values specified by functions defined as

$$E_Z(z_o,t)\big|_{t=0} = g(z) \text{and} G(z) = \frac{\partial E_Z(z_o,t)}{\partial t}\Big|_{t=0}$$

These functions are specified at plane $t = 0$.

INTRODUCTION

The directional homogeneous Maxwell equation; $\nabla \cdot E = 0$, has greatly hindered proposed theoretical frame works for obtaining the exact field patterns of electromagnetic field sources in low latitudes. By it, five components of the source field could only be obtained up to now; when the plane wave model is used to analyse the wave propagation in low latitude region. This restriction is usually met when the equation above is decomposed and evaluated term by term. The result of the decomposition shows that each electrical component of the source field here does not vary along its corresponding Cartesian co-ordinate. This suggests that $E_Z = 0$. This trivial solution was used by different authors to obtain the five components of the source fields in low latitude. For an accurate electromagnetic induction study an exact field pattern is necessary, as a result the E_z component of the inducing source field in low latitude must necessarily have a field pattern. For this to become a reality the trivial solution E_z must be abandoned.

Recently[1] an attempt was made to obtain the six components of the source field by disemphassising the imposed restriction on E_Z component by proposing a time variation solution as against the trivial solution earlier assumed for E_Z. On the plots is shown a marked discontinuity region at the centre of the low latitude region. With this feature the suitability of our derived equation to induction works across the specified low latitude region is quite doubtful. This is why in this paper we propose a new analytical model to obtain expressions for the E_Z component of the source field in low latitude region. In our derivations we focus on waves prescribed on the initial surface $z = 0$, where z is prescribed as the vertical direction, the direction of our proposed plane wave. This initial prescription has been used extensively to obtain the (E_x, E_y, H_x, H_y, H_z) components of electromagnetic source field [2]. This approach marked the distinction between the deviation in reference onewhere the focus was on the waves prescribed on horizontal plane which moves in a vertical direction z. With this back ground electric field component E_Z is now culled from the Maxwell's equation and expressed as

$$E_Z = \left(\sigma + \varepsilon \frac{\partial}{\partial t}\right)^{-1} \left\{ \frac{\partial H_Y}{\partial x} - \frac{\partial H_X}{\partial y} \right\} \qquad (1)$$

2. DERIVATIONS.

The Electromagnetic field satisfies the following partial differential equation in space and time:

$$\frac{\partial^2 E_z}{\partial t^2} + 2b \frac{\partial E_z}{\partial t} - a_0^2 \frac{\partial^2 E_z}{\partial z^2} = 0 \qquad (2)$$

b = $\frac{\sigma}{2\varepsilon}$, $a_0 = 1 \left| \sqrt{\mu\varepsilon} \right.$ and σ, μ, ε are the usual electrical parameters of space and a_0 is the velocity of propagation of wave in space.

There are a lot of experimental measurements of parameters of electromagnetic fields that confirms both space and time variation of the various component of e. m fields [3] This has made us to adopt these variations in putting up our analytical model for the E_Z component.

There are two ways to pose the initial boundary problems namely initial prescription in space and initial prescription in time. We limit ourselves to the initial prescriptions in space. This allows for a straight forward comparison of the space - time with the space - frequency domain expressions

3 INITIAL PRESCRIBED DISTRIBUTION IN SPACE.

The initial conditions prescribed in this case are that at the instant t = 0 (initial time) the field, and it's time derivative $\frac{\partial}{\partial t}$ are specified as functions of the space

Coordinate, Z, where Z is the vertical direction . That is, $E_z(z,0) = g(z)$, and

$$\left.\frac{\partial E_Z}{\partial t}\right|_{t=0} = G(z) \qquad (3)$$

A harmonic analysis in space rather than in time is pursued and the general particular solution used in

$$E_z = \frac{\gamma}{\sigma\mu\omega} e^{-bt} \left\{\left(1 - b + w^2 - b^2\right) + i\left(w - 2wb\right)\right\}\left\{A_\gamma e^{igt} - B_\gamma e^{-iqt}\right\} e^{i\gamma z} \qquad (4)$$

where γ is a wave constant.
Using the expression describing the field component E_z for a case when the initial distribution of the field at an instant t = 0 is prescribed in space, the equation for propagation of electromagnetic waves in a conducting medium is

$$E_Z(z,t) = \tfrac{1}{2} g\left(z_0 - a_0 t\right) + \tfrac{1}{2} g\left(z_0 + a_0 t\right) + \tfrac{1}{2} h\left(z_0 - a_0 t\right) - \tfrac{1}{2} h\left(z_0 + a_0 t\right) \qquad (5)$$

where z_0 is the value of z when t=0 on z axis. Equation (5) above reduces to g(z) and h(z) at t = 0 therefore the prescribed function and it's time derivative at an initial time t = 0 are g(z) and h(z)respectively. Also

$$h(z) = -\frac{1}{a_0} \int_0^Z G\left(z^1\right) dz^1$$

4. [A] PRESCRIBED DISTRIBUTION IN SPACE: SPACE -TIME DOMAIN

We generate the function g(z) from the initial prescription conditions, then

$$g(Z) = E_Z(z,0) = \frac{\gamma}{\sigma\mu w} \left\{\left(1 - b + w^2 - b^2\right) + i\left(w - 2wb\right)\right\}\left\{A_\gamma + B_\gamma\right\} e^{i\gamma z} \quad *$$

We proposed a model where the field amplitude reduces to 1/100 of its value at $z_0 = H_1$ at time t. This require that $\exp(-aH_0) = 100\exp(-aH_1)$. Taking the logarithm to base e the above reduces to $-aH_0 Log_e e = Log_e 100 - aH_1 Log_e e$. Then

$$a = \frac{1}{H_1 - H_0} Log_e 100$$

Our model justifies that $\left\{A_\gamma + B_\gamma\right\} = E_z \exp(-az)\left\{A_\gamma + B_\gamma\right\}$ are functions that depend on γ.
With this equation (*) reduces to

$$g\left(z_0 - a_0 t\right) = \frac{\gamma}{\sigma\mu\omega} \left\{\left(1 - b + w^2 - b^2\right) + i\left(w - 2wb\right)\right\} e^{-(a - i\gamma)(z_0 - a_0 t)} \qquad (6)$$

and

$$g\left(z_0 + a_0 t\right) = \frac{\gamma}{\sigma\mu\omega}\left\{\left(1 - b + w^2 - b^2\right) + i\left(w - 2wb\right)\right\} e^{-(a-i\gamma)(z_0 + a_0 t)} \qquad (7)$$

From the second initial conditions in eqn (3), we obtain

$$G(z) = \left.\frac{\partial E_z}{\partial t}\right|_{t=o} = \left\{i\left(w s_3 s_1 - 2b s_3 s_3\right) - \left(2b s_3 s_1 - w s_3 s_2\right)\right\} e^{-(a-i\gamma)Z_0}$$

where the parameters s_1, s_2, ... are defined in the appendix. Thus the function h(z) as defined earlier is given as

$$h(z) = -\frac{1}{a_0}\int_0^z a\left(z^1\right)dz^1 = -\frac{1}{a_0}\int_0^z \left(is_4 - si\right) e^{-(a-i\gamma)z^1} dz^1$$

Integrating between 0 and $z_0 - a_0 t$, we get

$$h\left(z_0 - a_0 t\right) = \frac{-\left(s_5 - is_4\right)}{a_0\left(a - i\gamma\right)} e^{-(a-i\gamma)z^1} \Big|_0^{z_0 - a_0 t}$$

$$= \frac{i\left(a s_4 - \gamma a_5\right) - \left(\gamma s_4 + as_5\right)}{a_0\left(a^2 - i\gamma^2\right)}\left\{e^{-(a-i\gamma)(z_0 - a_0 t)} - 1\right\} \qquad (8)$$

Similarly $h(z_0 + a_0 t)$ becomes

$$= \frac{i\left(a s_4 - \gamma a_5\right) - \left(\gamma s_4 + as_5\right)}{a_0\left(a^2 - i\gamma^2\right)}\left\{e^{-(a-i\gamma)(z_0 + a_0 t)} - 1\right. \qquad (9)$$

Substitute equation (6,7, 8 and 9) in equation (5) simplify, we have that the Real part of

$$Ez(z_1 t) = e^{-a(z_0)}\left\{c_1 s_1 s_3 - \frac{\gamma c_1 s_4}{T} - \frac{a c_1 s_5}{T} - \frac{c_2 s_3 s_2}{T} - \frac{c_2 a s_4}{T} + \frac{\gamma c_2 s_5}{T}\right\} +$$

$$e^{-a(z_0)}\left\{s_1 c_3 s_3 + \frac{\gamma s_3 s_4}{T} + \frac{a c3 s_5}{T} - \frac{c_4 c_2 s_2}{T} + \frac{a c_4 s_4}{T} - \frac{\gamma c_4 s_5}{T}\right\} \qquad (10)$$

and the imaginary part is

$$E_z(z_1 t) = e^{-a(z_0)}\left\{s_1 s_3 c_2 + \frac{a c_1 s_4}{T} - \frac{\gamma\ c_1 s_5}{T} - \frac{\gamma\ c_2\ s_4}{T} - \frac{a c_2 s_5}{T} + c_1 s_2 s_3\right\} +$$

$$e^{-a(z_0)}\left\{s_1 c_4 s_3 + c_3 s_2 s_3 + \frac{\gamma\ c_4 s_4}{T} + \frac{a c_4 s_5}{T} - \frac{a c_3 s_4}{T} + \frac{\gamma\ c_3 s_5}{T}\right\} \quad (11)$$

Again parameters c_1, c_2, … are defined in the Appendix.

3. PRESCRIBED DISTRIBUTION IN SPACE: SPACE - FREQUENCY DOMAIN

Again substitute equations (6), (7), (8) and (9) in (5) and take the Fourier Transform of the result to obtain

$$E_z(z_1 w) = \left\{\frac{(s_3 s_1\ T - s_5)}{T} + i\frac{(s_3 s_2\ \ T + s_4)}{T}\right\}\int_{-\infty}^{\infty} e^{-(a-i\gamma)(z_0 - a_0 t)}\ e^{-iwt}\quad dt +$$

$$\left\{\frac{(s_3 s_1\ T + s_5)}{T} + i\frac{(s_3\ s_2\ \ T - s_4)}{T}\right\}\int_{-\infty}^{\infty} e^{-(a-i\gamma)(z_0 + a_0 t)}\ e^{-iwt}\quad dt + \quad (12)$$

Integrating within the limit $0< t< H_1/a_0$ and expressing the complex exponential as cosine and sine functions, the real and imaginary parts of (12) are respectively

$$E_z(z_1 w) = e^{-az_0}\left\{e^{-a H_1} - e^{+aH_1}\right\}\{Q_7 + Q_6\} + \frac{Q_2 - M_1 - M_2 Q_1}{Q_5} \quad (13)$$

and

$$E_z(z_1 w) = e^{-az_0}\left\{e^{-a H_1} - e^{aH_1}\right\}\{Q_8 + Q_9\} + \frac{Q_3 + Q_4 - M_5 Q_6}{Q_5} \quad (14)$$

where m_1, m_2, … and Q_1, Q_2, … are defined in the appendix

5. RESULTS AND DISCUSSION

Results of test on Equations (10) , (11), (13) and (14) were generated using initial prescription from experimental results as input parameters. The characteristic representing these equations measures spatial distributions of the electric fields E_z prescribed in space and specified in space - time and space - frequency domains. The profiles for both real and imaginary parts of E_z component are as shown in figures 1 (a, b) for time variation, while frequency variations are shown in figure 2 (a, b).

For the sake of brevity in this work, we have considered only the case of fields prescribed on an initial plane. The space –time expression gives a smooth curves, Fig1(a b) for both imaginary and real parts for the Ez components. Both parts of Fig1(a, b), of our expressions give an adequate formulation expected when time was zero. The two sets of curves as obtained here are plotted for $w=0.001H_z$.

The space frequency expressions show same characteristic curves, Fig2(a b) which are symmetric about the vertical axis. The variation adopted here are that which specifies the amplitude of E_z across the transverse span of the low latitude region at w= $0.001H_3$.

The results so far obtained with our approach here gives a better picture of the exact field pattern suggested by Oni,[5,6] and they equally specify a finite value of the wave amplitude even at the edges of field. This shows a remarkable success in predicting the exact field distribution even at the field edges, since these values were predicted to be infinite in earlier works[5]. We have deliberately used same frequency ($w=0.001H_3$) for our plots.

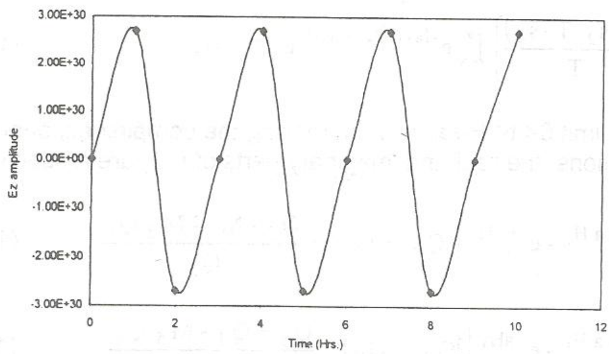

Fig 1a. Plot of Imaginary Ez Amplitude vs Time(Hrs.) (w=.001 Hz)

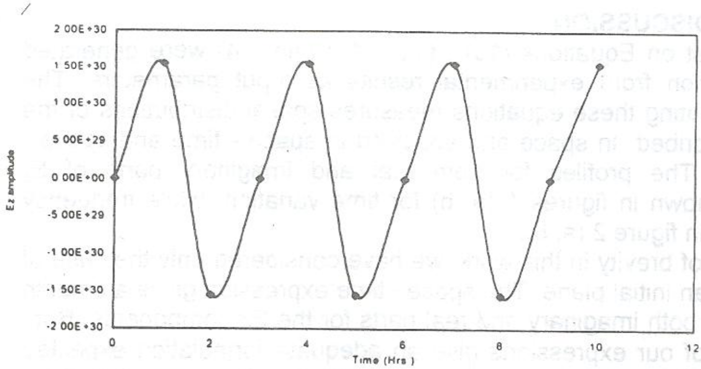

Fig 1b. Plot of Real Ez Amplitude vs Time(Hrs.) (W=.001Hz)

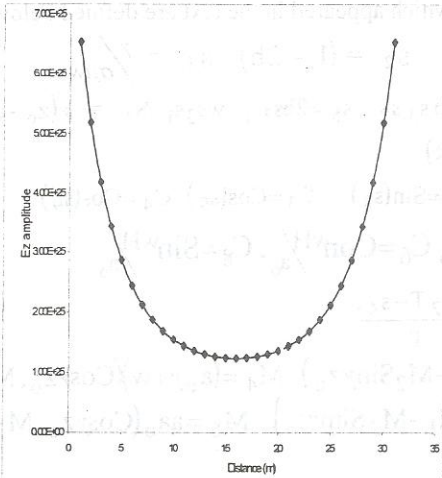

Fig 2b. Plot of Imaginary Ez Amplitude vs Distance (m) (w=0.001Hz

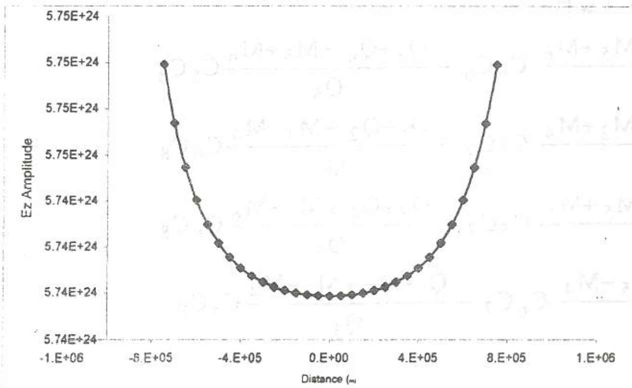

Fig 2a. Plot of Real Ez Amplitude vs Distance (m) (W=0.001Hz)

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APPENDIX

The following parameters which appeared in the text are defined below:

$$S_1 = 1 - b + w^2 - b^2 \quad s_2 = (1 - 2b) \quad s_3 = \gamma / \sigma\mu w$$

$$S_4 = w s_1 s_3 - 2b s_3 s_2 \quad s_5 = 2bs_3 s_1 - ws_3 s_1 \quad S_8 = \gamma(z_o - a_o t)$$

$$s_9 = \gamma(z_o + a_o t)$$

$$C_1 = Cos(s_8) \quad C_2 = Sin(s_8) \quad C_3 = Cos(s_9) \quad C_4 = Cos(s_9)$$

$$C_5 = Cos\gamma z_o, C_7 = Sin\gamma z_o \; C_6 = Cos^{wH}/_{a_o}, C_8 = Sin^{wH}/_{a_o}$$

$$Ml = \frac{s_3 s_1 T - s_5,}{T} \; M_2 = \frac{s_3 s_2 T - s_4,}{T}$$

$$M_3 = aa_o(Cos\gamma z_o M_1 - M_2 Sin\gamma z_o) \quad M_4 = (a_o\gamma + w)(Cos\gamma z_o, M_2, + M_1 Sin\gamma z_o),$$

$$M_5 = (a_o\gamma + w)(Cos\gamma z_o M_1 - M_2 Sin\gamma z_o), \quad M_6 = aa_o(Cos\gamma z_o, M_2, + M_1 Sin\gamma z_o),$$

$$T = 2ao(a^2 + \gamma^2)$$

$$Q_1 = aa_o(Cos\gamma z_o P_1 - P_2 Sin\gamma z_o) \quad Q_2 = (a_o\gamma - w)(Cos\gamma z_o P_2 + P_1 Sin\gamma z_o),$$

$$Q_3 = aa_o(Cos\gamma z_o P_2 + P_1 Sin\gamma z_o) \quad Q_4 = (a_o\gamma - w)(Cos\gamma z_o P_2 + P_1 Sin\gamma z_o),$$

$$Q_5 = a^2 a^2{}_o + (a_o\gamma + w)^2$$

$$Q_6 = \frac{+Q_3 + Q_4 + M_5 + M_6}{Q_5} C_6 C_6 + \frac{Q_3 + Q_4 + M_5 + M_6}{Q_5} C_5 C_8$$

$$Q_7 = \frac{-Q_1 + Q_2 + M_3 + M_4}{Q_5} C_5 C_7 - \frac{Q_1 - Q_2 + M_3 + M_4}{Q_5} C_6 C_8$$

$$Q_8 = \frac{-Q_3 - Q_4 + M_5 + M_6}{Q_5} C_5 C_7 - \frac{Q_3 + Q_4 + M_5 + M_6}{Q_5} C_6 C_8$$

$$Q_9 = \frac{Q_1 - Q_2 - M_5 - M_4}{Q_5} C_6 C_7 - \frac{Q_1 - Q_2 + M_5 + M_4}{Q_5} C_5 C_8$$

REFERENCES

1. Abajingin D.D Derivation of the six components of Non - uniform field sources propagated in low Latitude, Journal of Biological and Physical Sciences. Vol. 1, No. 1, 247 - 265, 1999.
2. Oni, E. A mathematical structure for a theory of dectromagnetic induction in the Earth in low Latitude, Pure Appl. Geophys. 101, 162 - 173, 1972.

3. Oni E. and Agunloye, Result of recent studies of electrical conductivity of subsurface structure in parts of the western and mid-western states of Nigeria. Pure Appl. Geophys., 114, 53 - 60, 1976.
4. Oni E. and O. Agunloye, The application of upward continuity concepts to induction studies, Pure Appl. Geophys. 112, 787 - 791, 1974.
5. Wait, J.R., On the relation between telluric current and the Earth's magnetic field, Geophys, 19, 281 - 296 1954.
6. Henry, T. E. On electromagnetic induction in low latitude. Pure and Applied Geophysics. 102, 342-347. 1991
7. Sogade J. A. and Oni, E. Recent Development in the application of Electro-magnetic Field ın sources propagated in low latitudes to the study of 1 and 2 Dimensional Earth Modes.J. Geomag, Geoelectr. 44, 391-402, 1992 Pure and Applied Geophysics. 102, 112-117. 1992