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THE DYNAMIC ANALYSIS OF AN ELASTIC PLATE WITH POINT-MOVING LOAD.

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ABSTRACT

This paper gives a mathematical analysis of the deflection profiles of an elastic plate resting on an viscoelastic foundation subject to the effect of a point load whose position at any time t is defined parametrically as ;

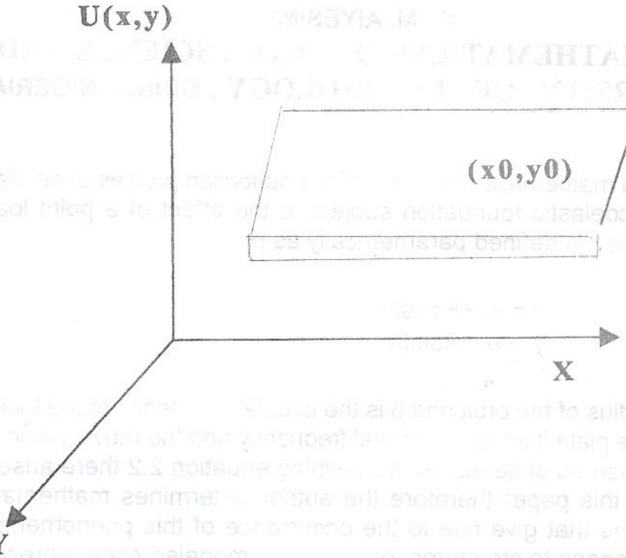
$$x = x_0 + R\cos\beta t$$
$$y = y_0 + R\sin\beta t$$

where R is the radius of the orbit and β is the circular frequency of the load(angular velocity). Since the plate itsef has a natural frequency and the moving load constitute an external vibration as observed in the defining equation 2.2 there arise the case of resonance. In this paper therefore the author determines mathematically the vibrating conditions that give rise to the occurrence of this phenomena which in actual fact is a menace to structures for which the modeled plate represents .

INTRODUCTION

Over the year several papers have been published on the dynamic response of elastic plates to moving loads . In his work on the dynamic response of plate with finite dimensions Holl [1] obtained the solutions for rectangular plates acted upon by uniformly moving loads establishing a critical velocity for each vibrational mode The studies of the response of elastic uniform rectangular plate with moving concentrated masses was carried out by Stanisic et` al [2 ] and Wu et`al [3] . The former presented a method based on Fourier transform technique to solve the multi - masses moving system . The convergence of the solutions was thoroughly examined . It was established that for the same natural frequency resonance is attained prior considering the moving multi-masses problem than the moving multi-force problem .In the latter case a general numerical analysis theory capable of solving the dynamic responses of nonuniform rectangular flat plate with differential boundary conditions subjected to moving mass was presented .

Most of the cases cited above are for structure without viscoelastic effect and moreover they are for loads with uniform velocities . Here we present the case where the foundation is viscoelastic and the load moving with variable velocities which are properties that are generally



exhibited by practical moving load problems . It is also a well known fact that viscoelastic materials are in practical point of view very useful in that they posses energy dissipating properties used in practice for damping vibrations .
Therefore for all practical purposes a more reliable analysis is obtained by considering these properties to be able to cover a more sizeable class of problems . We have therefore considered here the analysis of the dynamic response of a thin elastic plate resting on a viscoelastic foundation to a concentrated moving load traversing a circular orbit about a point (x_0,y_0) with varying magnitude and velocity.

2 MATHEMATICAL FORMULATIONS

The uniform elastic plate of dimension *a* by *b* mass *m* with simple ends resting on a viscoelastic foundation of coefficient ε_0. At time t =0 the load is dropped on the plate which then starts from its equilibrium position from rest . Following Raske[5] the governing differential equation for the plate is therefore given by ;

$$D[(\partial^4 u/\partial x^4) + (\partial^4 u/\partial x^2\partial y^2) + (\partial^4 u/\partial y^4)] - N_x(\partial^2 u/\partial x^2) + N_y(\partial^2 u/\partial y^2) + \rho(\partial^2 u/\partial t^2) + \varepsilon_0(\partial u/\partial t) + \kappa u = F(x,y,t) \quad 0 \le x \le a,\ 0 \le y \le b \qquad \text{...2.1}$$

where

D = Bending stiffness of plate
u = Deformation of plate from resting position
N_x = Prestressed axial force in x- direction
N_y = Prestressed axial force in y- direction
ρ =Density of plate
ε_0 = Viscous Parameter of foundation
κ = coefficient of elasticity of plate

Now for a circularly orbiting load F(x,y,t) has the representation

$$F(x,y,t) = P\cos\omega t \,.\, \delta\,[x-(x_0+R\cos\beta t)].\ \delta\,[y-(y_0+R\sin\beta t)] \qquad .....2.2$$

where P is the magnitude of the load, R the radius of orbit, ω the transverse frequency of load, β the circular frequency of load and δ the Dirac delta function.
In particular a simply - supported plate satisfies the boundary conditions

$$[\,u\,,\,u''\,]_B = 0 \qquad ...2.3$$

The subscript B above implies values taken at the boundary]

3 METHOD OF SOLUTION

The governing differential equation of motion for the plate – load system is seen to be a constant – coefficient inhomogeneous ordinary differential equation we now consider a solution of the form

$$u(x,y,t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} Y_{mn}(t) \sin(m\pi x/a)\sin(n\pi y/b) \qquad ...3.1$$

Thus eq (2.2) becomes

$$\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} [\,\rho Y_{m,n} + \varepsilon_0 Y_{m,n} + (\,D\{\,(m\pi/a)^4 + 2(mn\pi^2/ab)^2 + (n\pi/b)^4\,\} + M_x(m\pi/a)^2$$
$$+ N_y(n\pi/b)^2 + \kappa\,)Y_{mn}]\sin(m\pi x/a)\sin(n\pi y/b)$$
$$= P\cos\omega t\,.\delta[x - (x_0 + R\cos\beta t)]\ \delta\,[y - (y_0 + R\sin\beta t)\,] \qquad ...3.2$$

On setting the expression in the brace bracket above as $\rho\Omega^2$ and $\varepsilon_0 = 2\rho\gamma$ then we have :

$$\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} [\,Y_{mn} + 2\gamma Y_{mn} + \Omega^2 Y_{mn}\,]\sin(m\pi x/a)\sin(n\pi y/b)$$
$$= P\cos\omega t\ \delta[x - (x_0 + R\cos\beta t)]\ \delta\,[y - (y_0 + R\sin\beta t)] \qquad ....3.3$$

Multiplying through (3.3) by $\sin(m\pi x/a)\sin(n\pi y/b)$ and integrating we obtain

$$\ddot{Y} + 2\gamma\dot{Y} + \Omega^2 Y = Q\sin(A + R_a\cos\beta t)\sin(B + R_b\sin\beta t) \qquad ...3.4$$

where

$$Q = (4P/ab\rho),\ A = m\pi x_0/a,\ B = n\pi y_0/b,\ R_a = m\pi R/a,\ R_b = n\pi R/b \qquad ...3.5$$

The complimentary function of (3.4) Is given as

$$Y_c = e^{\gamma t}(F\sin\lambda t + G\cos\lambda t)$$

where

$$\lambda^2 = \gamma^2 - \Omega^2 \qquad ...3.6$$

On invoking the Bessel function identities [4]. In particular we recall the following identities:

$$\mathrm{Cos}(a\cos bt) = J_0(a) + 2\sum_{m=1}^{\infty}\Sigma(-1)^m J_{2m}(a)\mathrm{Cos}2mbt$$

$$\mathrm{Cos}(a\sin bt) = J_0(a) + 2\sum_{m=1}^{\infty}\Sigma J_{2m}(a)\mathrm{Cos}2mbt$$

$$\mathrm{Sin}(a\cos bt) = 2\sum_{m=1}^{\infty}(-1)^m J_{2m+1}(a)\mathrm{Cos}(2m+1)bt$$

$$\mathrm{Sin}(a\sin bt) = 2\sum_{m=1}^{\infty}J_{2m}(a)\mathrm{Cos}(2m+1)bt \qquad 3.7$$

The particular integral Y_p of the problem is given as:

$$\begin{aligned} Y_p = (\lambda/2)[[\, & J_0(R_a)J_0(R_b)((\gamma\sin(a_0-\lambda)t - a_0\cos(a_0-\lambda)t)/(a_0^2+\gamma^2) \\ & +(\gamma\sin(a_1-\lambda)t - a_1\cos(a_1-\lambda)t)/(a_1^2+\gamma^2)) \\ & + \sum_{n=1}^{\infty} J_0(R_a)J_{2n}(R_b)((\gamma\cos(a_7\lambda)t + \sin(a_7-\lambda)t)/(a_7^2+\gamma^2) \\ & - (\gamma\cos(a_6-\lambda)t + a_6\sin(a_6-\lambda)t)/(a_6^2+\gamma^2) \\ & - (\gamma\cos(a_8-\lambda)t + a_8\sin(a_8-\lambda)t)/(a_8^2+\gamma^2) \\ & + (\gamma\cos(a_9-\lambda)t + a_9\sin(a_9-\lambda)t)/(a_9^2+\gamma^2)) \\ & + \sum_{m=1}^{\infty}(-1)^m J_0(R_b)J_{2m}(R_a)((\gamma\sin(a_2-\lambda)t) - a_2\cos(a_2-\lambda)t)/(a_2^2+\gamma^2) \end{aligned}$$

$$+ (\gamma\sin(a_3-\lambda)t - a_3\cos(a_3-\lambda)t)/(a_3^2+\gamma^2)$$
$$+ (\gamma\sin(a_4-\lambda) - a_4\cos(a_4-\lambda)t)/(a_4^2+\gamma^2)$$
$$+ (\gamma\sin(a_5-\lambda)t - a_5\cos(a_5-\lambda)t)/(a_5^2+\gamma^2)\ )$$

$$+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} J_{2n}(R_b)J_{2m}(R_a)(\ (\gamma\cos(a_{19}-\lambda)t + a_{19}\sin(a_{19}-\lambda)t)/(a_{19}^2+\gamma^2)$$
$$+(\gamma\cos(a_{21}-\lambda)t - a_{21}\sin(a_{21}-\lambda)t)/(a_{21}^2+\gamma^2)$$
$$-\ (\gamma\cos(a_{18}-\lambda)t - a_{18}\sin(a_{18}-\lambda)t)/(a_{18}^2+\gamma^2) - (\gamma\cos(a_{20}-\lambda)t + a_{20}\sin(a_{20}-\lambda)t)/(a_{20}^2+\gamma^2)$$
$$-\ (\gamma\cos(a_{22}-\lambda)t + a_{22}\sin(a_{22}-\lambda)t)/(a_{22}^2+\gamma^2) - (\gamma\cos(a_{25}-\lambda)t + a_{25}\sin(a_{25}-\lambda)t)/(a_{25}^2+\gamma^2)$$
$$+(\gamma\cos(a_{23}-\lambda)t + a_{23}\sin(a_{23}-\lambda)t)/(a_{23}^2+\gamma^2) + (\gamma\cos(a_{24}-\lambda)t + a_{24}\sin(a_{24}-\gamma)t)/(a_{24}^2+$$
$$\gamma^2))]\sin A\sin B$$

$$+[\ \sum_{n=1}^{\infty} J_0\ (R_a)\ J_{2n+1}(R_b)(\ (\gamma\cos(a_{15}-\lambda)t + a_{15}\sin(a_{15}-\lambda)t)/(a_{15}^2+\gamma^2)$$
$$-\ (\gamma\cos(a_{14}-\lambda)t + a_{14}\sin(a_{14}-\lambda)t)/(a_{14}^2+\gamma^2)$$
$$-\ (\gamma\cos(a_{16}-\lambda)t + a_{16}\sin(a_{16}-\lambda)t)/(a_{16}^2+\gamma^2) + (\gamma\cos(a_{17}-\lambda)t + a_{17}\sin(a_{17}-\lambda)t)/(a_{17}^2+\gamma^2)\ )$$

$$+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} (-1)^m J_{2n+1}(R_b)J_{2m}(R_a)(\ (\gamma\cos(a_{27}-\lambda)t + a_{27}\sin(a_{27}-\lambda)t)/(a_{27}^2+\gamma^2)$$
$$+(\gamma\cos(a_{28}-\lambda)t + a_{28}\sin(a_{28}-\lambda)t)/(a_{28}^2+\gamma^2)$$
$$-\ (\gamma\cos(a_{26}-\lambda)t + a_{26}\sin(a_{26}-\lambda)t)/(a_{26}^2+\gamma^2) - \gamma\cos(a_{29}-\lambda)t + a_{29}\sin(a_{29}-\lambda)t)/(a_{29}^2+\gamma^2)$$
$$-\ (\gamma\cos(a_{32}-\lambda) + a_{32}\sin(a_{32}-\lambda)t)/(a_{32}^2+\gamma^2) - (\gamma\cos(a_{35}-\lambda)t + a_{35}\sin(a_{35}-\lambda)t)/(a_{35}^2+\gamma^2)$$
$$+\ (\gamma\cos(a_{33}-\lambda)t + a_{33}\sin(a_{33}-\lambda)t)/(a_{33}^2+\gamma^2) + (\gamma\cos(a_{34}-\lambda)t + a_{34}\sin(a_{34}-\lambda)t)/(a_{34}^2+$$
$$\gamma^2))]\sin A\cos B$$

$$-[\ \sum_{m=1}^{\infty} (-1)^m J_0(R_b)J_{2m+1}(R_a)(\ (\gamma\cos(a_{10}-\lambda)t - a_{10}\sin(a_{10}-\lambda)t)/(a_{10}^2+\gamma^2)$$
$$+\ (\gamma\cos(a_{11}-\lambda)t - a_{11}\sin(a_{11}-\lambda)t)/(a_{11}^2+\gamma^2) + (\gamma\cos(a_{12}-\lambda)t - a_{12}\sin(a_{12}-\lambda)t)/(a_{12}^2+\gamma^2)$$
$$+\ (\gamma\cos(a_{13}-\lambda)t - a_{13}\sin(a_{13}-\lambda)t)/(a_{13}^2+\gamma^2)\ )$$

$$+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} (-1)^m J_{2n}(R_b)J_{2n+1}(R_a)(\ (\gamma\cos(a_{27}-\lambda)t + a_{27}\sin(a_{27}-\lambda)t)/(a_{27}^2+\gamma^2)$$
$$+\ (\gamma\cos(a_{28}-\lambda)t + a_{28}\sin(a_{28}-\lambda)t)/(a_{28}^2+\gamma^2)$$
$$-\ (\gamma\cos(a_{26}-\lambda)t + a_{26}\sin(a_{26}-\lambda)t)/(a_{26}^2+\gamma^2) - (\gamma\cos(a_{29}-\lambda)t + a_{29}\sin(a_{29}-\lambda)t)/(a_{29}^2+\gamma^2)$$
$$-\ (\gamma\cos(a_{32}-\lambda)t + a_{32}\sin(a_{32}-\lambda)t)/(a_{32}^2+\gamma^2) - (\gamma\cos(a_{35}-\lambda)t + a_{35}\sin(a_{35}-\lambda)t)/(a_{35}^2+\gamma^2)$$
$$+\ (\gamma\cos(a_{33}-\lambda)t + a_{33}\sin(a_{33}-\lambda)t)/(a_{33}^2+\gamma^2) + (\gamma\cos(a_{34}-\lambda)t + a_{34}\sin(a_{34}-\gamma)t)/(a_{34}^2+\gamma^2)\ )$$

$$]\cos A\sin B + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} (-1)^m J_{2n+1}(R_b)J_{2m+1}(R_a)(\ (\gamma\cos(a_{20}-\lambda)t + a_{20}\sin(a_{20}-\lambda)t)/(a_{20}^2+\gamma^2)$$
$$+\ (\gamma\cos(a_{25}-\lambda)t + a_{25}\sin(a_{25}-\lambda)t)/(a_{25}^2+\gamma^2)$$
$$-\ (\gamma\cos(a_{21}-\lambda)t + a_{21}\sin(a_{21}-\lambda)t)/(a_{21}^2+\gamma^2)$$
$$-\ (\gamma\cos(a_{24}-\lambda)t + a_{24}\sin(a_{24}-\lambda)t)/(a_{24}^2+\gamma^2)$$
$$-\ (\gamma\cos(a_{38}-\lambda)t + a_{38}\sin(a_{38}-\lambda)t)/(a_{38}^2+\gamma^2) - (\gamma\cos(a_{40}-\lambda)t + a_{40}\sin(a_{40}-\lambda)t)/(a_{40}^2+\gamma^2)$$
$$+\ (\gamma\cos(a_{39}-\lambda)t + a_{39}\sin(a_{39}-\lambda)t)/(a_{39}^2+\gamma_2) + (\gamma\cos(a_{41}-\lambda)t$$
$$+a_{41}\sin(a_{41}-\lambda)t)/(a_{41}^2+\gamma_2)\ ]\cos A\cos B\ ]\ e^{\gamma t} \qquad \text{...3.8}$$

where the $a_k\{k=0(1)40$ are defined as follows:

$a_0 = \lambda+2m\beta$, $a_1 = \lambda-2m\beta$, $a_2 = \lambda+2n\beta$, $a_3 = \lambda-2n\beta$, $a_4 = \lambda+2(m+n)\beta$, $a_5 = \lambda-2(m+n)\beta$, $a_6 = \lambda+2(m-n)\beta$, $a_7 = \lambda-2(m-n)\beta$, $a_8 = \lambda+(2m+1)\beta$, $a_9 = \lambda-(2m+1)\beta$, $a_{10} = \lambda+(2m+1)\beta$, $a_{11} = \lambda-(2m+1)\beta$, $a_{12} = \lambda+2\ (m+n+1)\beta$, $a_{13} = \lambda-2\ (m+n+1)\beta$, $a_{14} = \lambda+2\ (m-n-1)\beta$, $a_{15}=\lambda-2\ (m-n-1)\beta$, $a_{16} = \lambda+\ (2m+2n+1)\beta$, $a_{17}=\lambda-\ (2m+2n+1)\beta$, $a_{18} = \lambda+(2m-2n+1)\beta$, ...**3.9**

$a_{19} = \lambda-(2m-2n+1)\beta$, $a_{20} = \lambda+\omega+2m\beta$, $a_{21} = \lambda-\omega+2m\beta$, $a_{22} = \lambda+\omega-2m\beta$, $a_{23} = \lambda-\omega-2m\beta$,
$a_{24} = \lambda+\omega+2n\beta$, $a_{25} = \lambda-\omega+2n\beta$, $a_{26} = \lambda-\omega-2n\beta$, $a_{27} = \lambda+\omega+(2m+1)\beta$,
$a_{28} = \lambda-\omega+(2m+1)\beta$, $a_{29} = \lambda-\omega-(2m+1)\beta$,
$a_{30} = \lambda+\omega+2(m+n+1)\beta$, $a_{31} = \lambda-\omega+2(m+n+1)\beta$,
$a_{32} = \lambda-\omega+2(m+n+1)\beta$, $a_{33} = \lambda-\omega-2(m+n+1)\beta$,
$a_{34} = \lambda+\omega+(2m-2n-1)\beta$, $a_{35} = \lambda-\omega+(2m-2n-1)\beta$,
$a_{36} = \lambda+\omega-(2m-2n-1)\beta$, $a_{37} = \lambda-\omega-(2m-2n-1)\beta$,
$a_{38} = \lambda+\omega+(2m+2n+1)\beta$, $a_{39} = \lambda-\omega+(2m+2n+1)\beta$,
$a_{40} = \lambda+\omega-(2m+2n+1)\beta$, $a_{41} = \lambda-\omega-(2m+2n+1)\beta$.

and $\mathbf{A = m\pi\ x_0/a}$, $\mathbf{B = n\pi\ y_0/b}$

The parameters above are obtained as a results of the double summations emanating from the Bessel's identity above making use of compound angle trigonometric formula.
In particular for drop load problem we have the corresponding initial conditions:

$$u(x,y,0) = 0 = \dot{u}(x,y,0) \Rightarrow Y(0) = \dot{Y}(0) = 0 \qquad ...\mathbf{3.10}$$

Hence we have the following expression for the deflection profile for the moving load problem:

$u(x,y,t) =$

$$\lambda^{-1}[[\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}[J_0(R_b)J_0(R_a)](\gamma\{\sin(\lambda-a_0)\ t\ -e^{-\gamma t}\sin\lambda t\} + a_0\{\cos(\lambda-a_0)t-a_0{}^*e^{-\gamma t}$$
$$\cos\lambda t\})/(a_0{}^2+\gamma^2) + \gamma\{\sin(\lambda-a_1)t-e^{-\gamma t}\sin\lambda t\} + a_1\{\cos(\lambda-a_1t - a_1{}^*e^{-\gamma t}\cos\lambda t\})/(a_1{}^2+\gamma^2;))]$$
$$+\sum_{r=1}^{\infty}J_{2r}(R_b)J_0(R_a)(\gamma\{\cos(\lambda-a_6)t-e^{-\gamma t}\cos\lambda t\}-a_6\{\sin\lambda-a_6{}^*e^{-\gamma t}\sin\lambda t\})/(a_6^2+\gamma^2)$$
$$-(\gamma\{\cos(\lambda-a_7)t-e^{-\gamma t}\cos\lambda t\}-a_7\{\sin(\lambda-a_7)t-a_7{}^*e^{-\gamma t}\sin\lambda t\})/(a_7{}^2+\gamma^2)$$
$$-(\gamma\{\cos(\lambda-a_9)t-e^{-\gamma t}\cos\lambda t\}-a_9\{\sin(\lambda-a_9)t-a_9{}^*e^{-\gamma t}\sin\lambda t\})/(a_9{}^2+\gamma^2)$$
$$+(\gamma\{\cos(\lambda-a_8)t-e^{-\gamma t\ t}\cos\lambda t\}-a_8\{\sin(\lambda-a_8)t-a_8{}^*e^{-\gamma t}\sin\lambda t\})/(a_8{}^2+\gamma^2)]$$
$$+\sum_{p=1}^{\infty}(-1)^p J_{2p}(R_a)J_0(R_b)[(\gamma\{\sin\lambda-a_2)t-e^{-\gamma t}\sin\lambda t\}-a_2\{\cos(\lambda-a_2)t-a_2{}^*\ e^{-\gamma t}\cos\lambda t\})/(a_2{}^2$$

$$+\gamma^2)+ (\gamma\{\sin(\lambda-a_3)t - e^{-\gamma t} \sin\lambda t\} - a_3\{\cos(\lambda-a_3)t - a_3{}^*e^{-\gamma t} \cos\lambda t\})/(a_3{}^2 + \gamma^2)$$
$$+ (\gamma\{\sin(\lambda-a_4)t - e^{-\gamma t} \sin\lambda t\} - a_4\{\cos(\lambda-a_4)t - a_4{}^* e^{-\gamma t} \cos\lambda t\})/(a_4{}^2 +\gamma^2)$$
$$+ (\gamma\{\sin\lambda-a_5)t - e^{-\gamma t} \sin\lambda t\} - a_5\{\cos(\lambda-a_5)t - a_5{}^* e^{-\gamma t} \cos\lambda t\})/(a_5{}^2 + \gamma^2)]$$

$$+\sum_{r=1}^{\infty}\sum_{p=1}^{\infty} J_{2r}(R_b)J_{2p}(R_a)[\ (\gamma\{\cos\lambda-a_{18})t - e^{-\gamma t} \cos\lambda t\} - a_{18}\{\sin(\lambda-a_{18})t - a_{18}{}^* e^{-\gamma t}$$
$$\sin\lambda t\})/(a_{18}{}^2 + \gamma^2)+ (\gamma\{\cos(\lambda-a_{20})t - e^{-\gamma t} \cos\lambda t\} - a_{20}\{\sin(\lambda-a_{20})t - a_{20}{}^* e^{-\gamma t} \sin\lambda t\})/(a_{20}{}^2 +\gamma^2)$$
$$- (\gamma\{\cos(\lambda-a_{21})t - e^{-\gamma t} \cos\lambda t\} - a_{21}\{\sin(\lambda-a_{21})t - a_{21}{}^* e^{-\gamma t} \sin\lambda t\})/(a_{21}{}^2 +\gamma^2)$$
$$- (\gamma\{\cos(\lambda-a_{19})t - e^{-\gamma t} \cos\lambda t\} - a_{19}\{\sin(\lambda-a_{19})t - a_{19}{}^* e^{-\gamma t} \sin\lambda t\})/(a_{19}{}^2 +\gamma^2)$$
$$- (\gamma\{\cos(\lambda-a_{23})t - e^{-\gamma t} \cos\lambda t\} - a_{23}\{\sin(\lambda-a_{23})t - a_{23}{}^* e^{-\gamma t} \sin\lambda t\})/(a_{23}{}^2 +\gamma^2)$$
$$- (\gamma\{\cos(\lambda-a_{24})t - e^{-\gamma t} \cos\lambda t\} - a_{24}\{\sin(\lambda-a_{24})t - a_{24}{}^* e^{-\gamma t} \sin\lambda t\})/(a_{24}{}^2 +\gamma^2)$$
$$+ (\gamma\{\cos(\lambda-a_{22})t - e^{-\gamma t} \cos\lambda t\} - a_{22}\{\sin(\lambda-a_{22})t - a_{22}{}^* e^{-\gamma t} \sin\lambda t\})/(a_{22}{}^2 +\gamma^2)$$
$$+ (\gamma\{\cos(\lambda-a_{25})t - e^{-\gamma t} \cos\lambda t\} - a_{25}\{\sin(\lambda-a_{25})t - a_{25}{}^* e^{-\gamma t} \sin\lambda t\})/(a_{25}{}^2 +\gamma^2)]\sin A\sin B$$

$$+[\sum_{r=1}^{\infty} J_{2r}(R_b)J_0(R_a) \ (\gamma\{\cos(\lambda-a_{14})t - e^{-\gamma t} \cos\lambda t\} - a_{14}\{\sin(\lambda-a_{14})t - a_{14}{}^* e^{-\gamma t} \sin\lambda t\})/(a_{14}{}^2$$
$$+ \gamma^2)$$
$$+ (\gamma\{\cos(\lambda-a_{29})t - e^{-\gamma t} \cos\lambda t\} - a_{29}\{\sin(\lambda-a_{29})t - a_{29}{}^* e^{-\gamma t} \sin\lambda t\})/(a_{29}{}^2 +\gamma^2)$$
$$- (\gamma\{\cos(\lambda-a_{27})t - e^{-\gamma t} \cos\lambda t\} - a_{27}\{\sin(\lambda-a_{27})t - a_{27}{}^* e^{-\gamma t} \sin\lambda t\})/(a_{27}{}^2 +\gamma^2)$$
$$- (\gamma\{\cos(\lambda-a_{28})t - e^{-\gamma t} \cos\lambda t\} - a_{28}\{\sin(\lambda-a_{28})t - a_{28}{}^* e^{-\gamma t} \sin\lambda t\})/(a_{28}{}^2 +\gamma^2)$$
$$- (\gamma\{\cos(\lambda-a_{33})t - e^{-\gamma t} \cos\lambda\} - a_{33}\{\sin(\lambda-a_{33})t - a_{33}{}^* e^{-\gamma t} \sin\lambda t\})/(a_{33}{}^2 +\gamma^2)$$
$$- (\gamma\{\cos(\lambda-a_{34})t - e^{-\gamma t} \cos\lambda t\} - a_{34}\{\sin(\lambda-a_{34})t - a_{34}{}^* e^{-\gamma t} \sin\lambda t\})/(a_{34}{}^2 +\gamma^2)$$
$$+ (\gamma\{\cos(\lambda-a_{32})t - e^{-\gamma t} \cos\lambda t\} - a_{32}\{\sin(\lambda-a_{32})t - a_{32}{}^*e^{-\gamma t} \sin\lambda t\})/(a_{32}{}^2 + \gamma^2)$$
$$+ (\gamma\{\cos(\lambda-a_{35})t - e^{-\gamma t} \cos\lambda t\} - a_{35}\{\sin(\lambda-a_{35})t - a_{35}{}^* e^{-\gamma t} \sin\lambda t\})/(a_{35}{}^2 +\gamma^2)] \ ]\sin A\cos B$$

$$+[\sum_{p=1}^{\infty}(-1)^p J_{2p+1}(R_a)J_0 \ [\ (\gamma\{\sin(\lambda-a_{10})t - e^{-\gamma t} \sin\lambda t\} - a_{10}\{\cos(\lambda-a_{10})t -a_{10}{}^* e^{-\gamma t} \cos\lambda t\})/(a_{10}{}^2$$
$$+\gamma^2) \ )+ (\gamma\{\sin(\lambda-a_{11})t - e^{-\gamma t} \sin\lambda t\} - a_{11}\{\cos(\lambda-a_{11})t - a_{11}{}^* e^{-\gamma t} \cos\lambda t\})/(a_{11}{}^2 +\gamma^2) \ )$$
$$+ (\gamma\{\sin(\lambda-a_{12})t - e^{-\gamma t} \sin\lambda t\} - a_{12}\{\cos(\lambda-a_{12})t - a_{12}{}^* e^{-\gamma t} \cos\lambda t\})/(a_{12}{}^2 +\gamma^2) \ )$$
$$+ (\gamma\{\sin(\lambda-a_{13})t - e^{-\gamma t} \sin\lambda t\} - a_{13}\{\cos(\lambda-a_{13})t - a_{13}{}^* e^{-\gamma t} \cos\lambda t\})/(a_{13}{}^2 + \gamma^2) \ ) \ ]$$

$$+\sum_{r=1}^{\infty}\sum_{p=1}^{\infty}(-1)^p J_{2r}(R_b)J_{2p+1}(R_a) \ (\gamma \ \{\cos \ (\lambda-a_{26})t - e^{-\gamma t} \cos\lambda t\} - a_{26}\{\sin(\lambda-a_{26})t - a_{26}{}^* e^{-\gamma t}$$
$$\sin\lambda t\})/(a_{26}{}^2 +\gamma^2) \ )+(\gamma\{\cos(\lambda-a_{29})t - e^{-\gamma t} \cos\lambda t\} - a_{29}\{\sin(\lambda-a_{29})t- a_{29}{}^*e^{-\gamma t}\sin\lambda t\})/(a_{29}{}^2 +\gamma^2))$$
$$- (\gamma\{\cos(\lambda-a_{27})t - e^{-\gamma t} \cos\lambda t\} - a_{27}\{\sin(\lambda-a_{27})t - a_{27}{}^* e^{-\gamma t} \sin\lambda t\})/(a_{27}{}^2 +\gamma^2) \ ))$$
$$- (\gamma\{\cos(\lambda-a_{28})t - e^{-\gamma t} \cos\lambda t\} - a_{28}\{\sin(\lambda-a_{28})t - a_{28}{}^* e^{-\gamma t} \sin\lambda t\})/(a_{28}{}^2 +\gamma^2) \ )$$
$$- (\gamma\{\cos(\lambda-a_{33})t - e^{-\gamma t} \cos\lambda t\} - a_{33}\{\sin(\lambda-a_{33})t - a_{33}{}^* e^{-\gamma t} \sin\lambda t\})/(a_{33}{}^2 + \gamma^2) \ )$$
$$- (\gamma\{\cos(\lambda-a_{34})t - e^{-\gamma t} \cos\lambda t\} - a_{34}\{\sin(\lambda-a_{34})t - a_{34}{}^* e^{-\gamma t} \sin\lambda t\})/(a_{34}{}^2 +\gamma^2) \ )$$
$$+ (\gamma\{\cos(\lambda-a_{33})t - e^{-\gamma t} \cos\lambda t\} - a_{33}\{\sin(\lambda-a_{33})t - a_{33}{}^* e^{-\gamma t}\sin\lambda t\})/(a_{33}{}^2 +\gamma^2))$$
$$+ (\gamma\{\cos(\lambda-a_{35})t - e^{-\gamma t} \cos\lambda t\} - a_{35}\{\sin(\lambda-a_{35})t - a_{35}{}^* e^{-\gamma t} \sin\lambda t\})/(a_{35}{}^2 +\gamma^2) \ ) \ ]]\cos A\sin B$$

$$+\sum_{r=1}^{\infty}\sum_{p=1}^{\infty}(-1)^p J_{2r+1}(R_b)J_{2p+1}(R_a) \ \{\gamma\{\cos(\lambda-a_{21})t - e^{-\gamma t} \cos\lambda t\}- a_{21}\{\sin(\lambda-a_{21})t - a_{21}{}^* e^{-\gamma t}$$
$$\sin\lambda t\})/(a_{21}{}^2 + \gamma^2))$$

$+ (\gamma\{\cos(\lambda-a_{24})t - e^{-\gamma t}\cos\lambda t\} - a_{24}\{\sin(\lambda-a_{24})t- -a_{24}^* e^{-\gamma t}\sin\lambda t\})/(a_{24}^2 + \gamma^2))$

$- (\gamma\{\cos(\lambda-a_{20})t - e^{-\gamma t}\cos\lambda t\} - a_{20}\{\sin(\lambda-a_{20})t - a_{20}^* e^{-\gamma t}\sin\lambda t\})/ (a_{20}^2 +\gamma^2))$
$- (\gamma\{\cos(\lambda-a_{25})t - e^{-\gamma t}\cos\lambda t\} - a_{25}\{\sin(\lambda-a_{25})t - a_{25}^* e^{-\gamma t}\sin\lambda t\})/(a_{25}^2 +\gamma^2))$
$- (\gamma\{\cos(\lambda-a_{39})t - e^{-\gamma t}\cos\lambda t\} - a_{39}\{\sin(\lambda-a_{39})t - a_{39}^* e^{-\gamma t}\sin\lambda t\})/(a_{39}^2 + \gamma^2))$
$- (\gamma\{\cos(\lambda-a_{41})t - e^{-\gamma t}\cos\lambda t\} - a_{41}\{\sin(\lambda-a_{41})t - a_{41}^* e^{-\gamma t}\sin\lambda t\})/(a_{41}^2 +\gamma^2))$
$+(\gamma\{\cos(\lambda-a_{38})t - e^{-\gamma t}\cos\lambda t\} - a_{38}\{\sin(\lambda-a_{38})t - a_{38}^* e^{-\gamma t}\sin\lambda t\})/(a_{38}^2 + \gamma^2))$
$+ (\gamma\{\cos(\lambda-a_{40})t - e^{-\gamma t}\cos\lambda t\} - a_{40}\{\sin(\lambda-a_{40})t - a_{40}^* e^{-\gamma t}\sin\lambda t\})/(a_{40}^2 +\gamma^2))]\times$
$\cos A\cos B]\sin(m\pi x/a)\sin n\pi y/b)$ 3.12

[where $a_k^* = (\gamma^2- (a_k - \lambda))/\lambda a_k$]
For a particular case of very light damping ($\gamma \ll 1$) we have the following conditions corresponding to the resonance of the plate-moving load system:

$$\lambda =2m\beta,\ \lambda + \omega = 2m\beta,\ \lambda + \omega = (2m+1)\beta$$
$$|\lambda - \omega| = 2m\beta$$
$$|\lambda - \omega| = 2(m+n+1)\beta$$
$$|\lambda - \omega| = 2(m-n+1)\beta,$$
$$|\lambda - \omega| = 2(2m-2n+1)\beta$$
$$|\lambda - \omega| = 2(2m-2n-1)\beta \quad (\text{where } m \in I) \qquad ...3.13$$

3 CONCLUSION

Resonance is a frequently occurring phenomenon in the fields of electronic, acoustic, atomic physics, highway and structural engineering, to name a few which has an interesting ambivalent characteristic. It is desirable in certain fields like electronics. For instance radio communication system is entirely dependent on the principle of resonance. In fact the receiving transistor is set into resonance with the incoming radio wave before a radio broadcast is achieved. On the other hand in structural and highway engineering resonance is a nuisance which causes destruction. Usually plates are used as a mathematical model to represent such engineering structures like bridges. The analysis in this paper thus establish the conditions under which resonance of the structure would occur and hence the designer of such structures can predetermine the allowable frequencies ω and β knowing fully in advance the natural frequency λ of the structure to forestall the occurrence of resonance.
The above conditions in (2.13) are the required conditions for the resonance of the forced plate- moving load problem . It is therefore pertinent that the designer of such structure as buildings , bridges , flyovers etc be aware and make adequate allowance to forestall the occurrence of same .

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