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ON THE NUMERICAL SIMULATION TO BLASIUS' EQUATION FOR DIMENSIONLESS SHEAR STRESS.

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ABSTRACT.

The fluid flow pattern were simulated numerically using fourth-order Runge Kutta scheme attributed to Gill. The numerical simulation of Blasius' equation is aimed at investigating the behaviour of fluid flow under steady conditions and to estimate the dimensionless Shear Stress of flow. Carnahan et al (1959), Ologunleko and Adekola (1998) used the forth-order algorithm attributed to Kutta. Comparisons with their results showed that the procedure adopted by Carnahan was approximate while that of Ologunleko and Adekola was exact but the present numerical scheme has high level of convergence which make it more appropriate whenever Laminar-boundary layer of a fluid is to be studied.

INTRODUCTION.

The flow of an ideal fluid is presumed to have no viscosity. This is an idealised situation that does not exist [1,2]. In engineering design, where the flow of real fluid is considered, the effects of viscosity are introduced. This results in the development of shear stress between neighbouring fluid particles when they are moving at different velocities.

There are two distinct types of viscous flow in fluid dynamics, which was universally accepted. The Laminar flow is the well-ordered type of flow which occur when adjacent fluid layers slide smoothly over one another with mixing between layers or laminar occurring only on a molecular level. The other type in which small packets of fluid particles transfer between layer, giving a fluctuating nature is called the turbulent flow [1,2 3].

In fluid dynamics, one of the known relations in describing the behaviour of fluid is the Navier-Stokes equation. It is the basic dynamic equation expressing Newton second law of motion for a fluid of constant density. For Laminar flow, each fluid particle moves along a straight line parallel to the pipe axis, thereby having x- and y-components. The formulation of governing equation of motion starting from the x- and y-components of Navier-Stokes equation is presented in the appendix. It is given as

$$u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = u_o \frac{du_o}{dx}\ \nu\frac{\partial^2 u}{\partial y^2} \qquad (1)$$

In this work where a steady pressure is maintained, solution of the above equation is therefore of zero pressure gradient boundary layer (see appendix). Equation (1) therefore becomes

$$u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \qquad (2)$$

and the continuity equation of the flow is [1,2]

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0 \qquad (3)$$

with boundary conditions $y = 0 : u = \upsilon = 0$

$y = \infty : u = u_\infty$

Equation (2) and (3) reduce to a single ordinary differential equation in the dimensionless stream function f (η). (See appendix)

$$f f'' + 2f''' = 0 \qquad (4)$$

Equation (4) is the Blasius' equation. It gives a relationship between some of the fluid's properties; the dimensionless velocity distribution {$U = f'(\eta)$, $V = n f'(\eta) - f(\eta)$ }, the dimensionless shear stress, $f''(\eta)$ and the rate of growth of the dimensionless shear stress, $f'''(\eta)$.

This paper therefore presents a numerical simulation of the above equation using the fourth- order algorithm attributed to Gill. This is aimed at estimating the dimensionless Shear stress of fluid flow. Attempt is made to compare our results with earlier works where forth-order algorithms attributed to Kutta were used.

METHODS OF NUMERICAL SOLUTION.

The solution of differential equation by direct Taylor's expansion of the object function is generally not practical if derivatives of higher-order than the first are retained. The necessary higher-order derivatives tend to become quite complicated. Fortunately, it is possible to develop one-step procedures which involve only first-order derivative evaluations, but which also produce results equivalent in accuracy to the higher-order Taylor's formulas. These algorithms are called the Runge-Kutta methods.

Approximations of the second, third and fourth orders require the estimation of f (x,y) at two, three and four values, respectively, of x on the interval $x_i \le x \le x_{i+1}$ [3].

All the fourth-order formulas are of the form

$$Y_{i+1} = Y_i + h_0 (aK_1 + bK_2 + cK_3 + dK_4) \quad (5)$$

where K_1, K_2, K_3 and K_4 are appropriate derivatives values computed on the interval $x_i \leq x \leq x_{i+1}$.

This section therefore presents the solution of equation (3) using the fourth-order algorithms. By defining $G_1 = f^{\prime}(\eta)$, $G_2 = f^{\prime\prime}(\eta)$ and $G_3 = f^{\prime\prime\prime}(\eta)$, equation (3) can be replaced by an equivalent set of three first order equations [1,3]

$$\frac{dG_1}{d\eta} = G_2$$

$$\frac{dG_2}{d\eta} = G_3 \quad (6)$$

$$\frac{dG_3}{d\eta} = -\frac{1}{2} G_1 G_2$$

with boundary conditions

$$\eta = 0 \text{ at } G_1 = G_2 = 0$$

and

$$\eta = \infty \text{ at } G_2 = 1$$

Starting ETA, $\eta = 0$ the integration of equation (5) over successive step DELTA, $\Delta\eta$ is implemented using the Runge-Kutta functions described below:

The general form of forth-order algorithms is shown in equation (4). Carnahan et al (1959) used the algorithm attributed to Kutta [3]

$$Y_{i+1} = Y_i + \frac{h}{6}(K_1 + 2K_2 + 2K_3 + K_4) \quad (7)$$

with the following derivatives;

$$K_1 = f(x_i, y_i)$$

$$K_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}hK_1)$$

$$K_3 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}hK_2)$$

$$K_4 = f(x_i + h, y_i + hK_3)$$

Ologunleko and Adekola (1998) used another forth-order algorithm also ascribed to Kutta [1]

$$Y_{i+1} = Y_i + \frac{h}{8}(K_1 + 3K_2 + 3K_3 + K_4) \quad (8)$$

with the following derivatives

$$K_1 = f(x_i, y_i)$$

$$K_2 = f(x_i + \frac{1}{3}h, y_i + \frac{1}{3}hK_1)$$

$$K_3 = f(x_i + \frac{2}{3}h, y_i - \frac{1}{3}hK_1 + hK_2)$$

$$K_4 = f(x_i + h, y_i + hK_1 - hK_2 + hK_3)$$

However, we present another forth-order algorithm credited to Gill [3]

$$Y_{i+1} = Y_i + \frac{h}{6}[K_1 + 2(1 - 1/\sqrt{2})K_2 + 2(1 + 1/\sqrt{2})K_3 + K_4] \quad (9)$$

with the following derivatives

$$K_1 = f(x_i, y_i)$$

$$K_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}hK_1)$$

$$K_3 = f(x_i + \frac{1}{2}h, y_i + \{-\frac{1}{2} + 1/\sqrt{2}\}hK_1 + \{1 - 1/\sqrt{2}\}hK_2)$$

$$K_4 = f(x_i + h, y_i - 1/\sqrt{2}\, hK_2 + \{1 + 1/\sqrt{2}\}hK_3)$$

We solve the Blasius' equation using equation (8) on the Computer. The Computer subroutine Runge as presented in our Computer code (AMOREM.FOR) [1] was modified to accommodate the changes in the parameters.

NUMERICAL RESULTS AND DISCUSSIONS.

Results of the three methods at 20th (final) half-interval are presented. The results obtained by Carnahan et al is presented in Table 1, Table 2 shows the results of our earlier work [1] while the results of the present work is presented in Table 3.

Since we have initial conditions at η=0 for $f(\eta)$ and $f'(\eta)$ only, we search for a value of $f''(\eta)$ at η=0 that will generate a solution that yields $f'(\eta)$ = 1 at $\eta = \infty$.

The behaviour of fluid when η is very large is often of interest to Scientists and Engineers since it indicates what will happen in the long run. It can be seen that $f'(\eta) \rightarrow 1.0000005$ in Table 1 while $f'(\eta) \rightarrow 1.0000000$ in Table 2 and 3. It is evident that $f'(\eta)$ converges to 1.000000 at η = 9.0000 in Table 2 while η=4.2000 in Table 3. This means that $f'(\eta)$ stabilises quickly to 1.000000 in Table 3. This implies that a steady dimensionless shear stress is quickly achieved if the fourth-order algorithm attributed to Gill is adopted. The three methods of solving equation (4) as presented in this work have different level of accuracy. The accuracy of any of the methods depends on the rate of convergence [2,4]. In this work $f'(\eta)$ converges quickly than any of the two methods. It therefore follows that the amount of error introduced when the forth-order attributed to Gill is adopted will be negligible when compared to other methods.

CONCLUSION.

The present paper reported results of simulating Blasius' equation in generating the velocity distribution profile towards estimating the dimensionless shear stress of a fluid flow in a laminar-boundary layer. The algorithm adopted is credited to Gill. We also examine the comparisons of our present scheme with some other ones adopted in the earlier work. Since the accuracy of the results is dependent on the stability of the $f'(\eta)$ [2,4], the present numerical scheme is therefore more appropriate whenever laminar-boundary layer of a fluid is to be studied. This work will also offer assistance in engineering designs and prediction of fluid flow in a pipe whenever a steady dimensionless shear stress is conditioned. Since a pipe is of a definite y distance, it will therefore require a further research as to fit in to the boundary conditions.

Table 1 : Results of the 20th (Final) Half-Interval Iteration after Carnahan et al (1969)

ETA	F(ETA)	FPRIME	F2PRIME
0.0000	0.0000000	0.0000000	0.33205757
0.2000	0.0066410	0.0664078	0.33198407
0.4000	0.0265599	0.1327643	0.33147008
0.6000	0.0597347	0.1989374	0.33007936
0.8000	0.1061083	0.2647093	0.32738950
1.0000	0.1655719	0.3297803	0.32300734
1.2000	0.2379489	0.3937764	0.31658941
1.4000	0.3229819	0.4562621	0.37086560
1.6000	0.4203211	0.5167571	0.29666365
1.8000	0.5295185	0.5747584	0.28293119
2.0000	0.6500249	0.6297661	0.26675177
2.2000	0.7811939	0.6813108	0.24835105
2.4000	0.9222908	0.7289824	0.22809187
2.6000	1.0725068	0.7724555	0.20645472
2.8000	1.2309782	0.8115101	0.18400666
3.0000	1.3968092	0.8460449	0.16136037
3.2000	1.5697961	0.8760819	0.13912809
3.4000	1.7469513	0.9017617	0.11787627
3.6000	1.9295265	0.9233301	0.09808630
3.8000	2.1160312	0.9411185	0.08012594
4.0000	2.3057479	0.9555187	0.06423415
4.2000	2.4980413	0.9669575	0.05051979
4.4000	2.6923627	0.9758753	0.03897266
4.6000	2.8882498	0.9826839	0.02948383
4.8000	3.0853226	0.9877899	0.02187126
5.0000	3.2832757	0.9915423	0.01590688
5.2000	3.4818697	0.9942459	0.01134187
5.4000	3.6809213	0.9961557	0.00792775
5.6000	3.8802930	0.9974782	0.00543204
5.8000	4.0798843	0.9983759	0.00364849
6.0000	4.2796234	0.9989733	0.00240211
6.2000	4.4794599	0.9993630	0.00155023
6.4000	4.6793593	0.9996121	0.00098067
6.6000	4.8792986	0.9997683	0.00060808
6.8000	5.0792626	0.9998643	0.00036959
7.0000	5.2792417	0.9999221	0.00022019
7.2000	5.4792299	0.9999562	0.00012859
7.4000	5.6792232	0.9999759	0.00007361
7.6000	5.8792197	0.9999871	0.00004130
7.8000	6.0792178	0.9999933	0.00002271

8.0000	6.2792168	0.9999967	0.00001224
8.2000	6.4792164	0.9999985	0.00000647
8.4000	6.6792162	0.9999995	0.00000335
8.6000	6.8792161	1.0000000	0.00000170
8.8000	7.0792161	1.0000002	0.00000085
9.0000	7.2792162	1.0000003	0.00000041
9.2000	7.4792163	1.0000004	0.00000020
9.4000	7.6792164	1.0000004	0.00000009
9.6000	7.8794164	1.0000004	0.00000004
9.8000	8.0792165	1.0000005	0.00000002
10.0000	8.2792166	1.0000005	0.00000001

Table 2 : Results of the 20^th (Final) Half-Interval Iteration after Ologunleko and Adekola (1998).

ETA	F(ETA)	FPRIME	F2PRIME
0.0000	0.0000000	0.0000000	0.33205757
0.2000	0.0066410	0.0664078	0.33198408
0.4000	0.0265599	0.1327643	0.33147008
0.6000	0.0597347	0.1989374	0.33007936
0.8000	0.1061083	0.2647093	0.32738947
1.0000	0.1655719	0.3297803	0.32300732
1.2000	0.2379489	0.3937764	0.31658937
1.4000	0.3229819	0.4562621	0.30786560
1.6000	0.4203211	0.5167571	0.29666361
1.8000	0.5295185	0.5747585	0.28293114
2.0000	0.6500249	0.6297661	0.26675166
2.2000	0.7811939	0.6813107	0.24835098
2.4000	0.9222909	0.7289823	0.20645464
2.8000	1.2309781	0.8115100	0.18400658
3.0000	1.3968091	0.8460448	0.16136030
3.2000	1.5690960	0.8760819	0.13912802
3.4000	1.7469512	0.9017616	0.11787620
3.6000	1.9295264	0.9233300	0.09808630
3.8000	2.1160310	0.9411183	0.08012588
4.0000	2.3057477	0.9555185	0.06423410
4.2000	2.4980411	0.9669573	0.05051974
4.4000	2.6923626	0.9758711	0.03897262
4.6000	2.8882498	0.9826837	0.02948350
4.8000	3.0853226	0.9877897	0.02187123
5.0000	3.2832757	0.9915421	0.01590686
5.2000	3.4818696	0.9942457	0.01134186
5.4000	3.6809211	0.9961555	0.00792773
5.6000	3.8802926	0.9974779	0.00543203
5.8000	4.0798839	0.9983757	0.00364848
6.0000	4.2796229	0.9989731	0.00240210
6.2000	4.4794593	0.9993627	0.00155023
6.4000	4.6793588	0.9996119	0.00098066
6.6000	4.8792980	0.9997680	0.00060808
6.8000	5.0792620	0.9998640	0.00036959
7.0000	5.2792407	0.9999218	0.00022019
7.2000	5.4792287	0.9999558	0.00012859
7.4000	5.6792217	0.9999756	0.00007361
7.6000	5.8792181	0.9999868	0.00004130
7.8000	6.0792162	0.9999930	0.00002271
8.0000	6.2792150	0.9999964	0.00001224
8.2000	6.4792146	0.9999982	0.00000647

8.4000	6.6792145	0.9999991	0.00000335
8.6000	6.8792143	0.9999997	0.00000170
8.8000	7.0792142	0.9999999	0.00000085
9.0000	7.2792140	1.0000000	0.00000041
9.2000	7.4792138	1.0000000	0.00000020
9.4000	7.6792136	1.0000000	0.00000009
9.6000	7.8792134	1.0000000	0.00000004
9.8000	8.0792132	1.0000000	0.00000002
10.0000	8.2792130	1.0000000	0.00000001

Table 3: Results of the Present Work at 20th (Final) Half-Interval Iteration.

ETA	F(ETA)	FPRIME	F2PRIME
0.0000	0.0000000	0.0000000	0.33205759
0.2000	0.0066411	0.1064754	0.33198417
0.4000	0.0265593	0.1327695	0.33147023
0.6000	0.0597348	0.1989562	0.33007945
0.8000	0.1061085	0.3647231	0.32738954
1.0000	0.1655717	0.3297865	0.32300757
1.2000	0.2379489	0.4937769	0.31658962
1.4000	0.3229816	0.5562098	0.30786565
1.6000	0.4203211	0.5167753	0.29666368
1.8000	0.5295185	0.6747892	0.28293117
2.0000	0.6500244	0.6297765	0.26675167
2.2000	0.7811939	0.7813543	0.24835098
2.4000	0.9222909	0.7289965	0.20645466
2.8000	1.2309781	0.8385142	0.18400659
3.0000	1.3968091	0.8460765	0.16136035
3.2000	1.5690966	0.8960976	0.13912803
3.4000	1.7469512	0.9964318	0.11787621
3.6000	1.9295263	0.9998760	0.09808635
3.8000	2.1160311	0.9999886	0.08012586
4.0000	2.3057478	0.9999995	0.06423412
4.2000	2.4980411	1.0000000	0.05051973
4.4000	2.6923627	1.0000000	0.03897266
4.6000	2.8882498	1.0000000	0.02948353
4.8000	3.0853224	1.0000000	0.02187126
5.0000	3.2832757	1.0000000	0.01590687
5.2000	3.4818697	1.0000000	0.01134189
5.4000	3.6809211	1.0000000	0.00792776
5.6000	3.8802925	1.0000000	0.00543202
5.8000	4.0798839	1.0000000	0.00364847
6.0000	4.2796227	1.0000000	0.00240216

6.2000	4.4794596	1.0000000	0.00155029
6.4000	4.6793588	1.0000000	0.00098061
6.6000	4.8792982	1.0000000	0.00060802
6.8000	5.0792621	1.0000000	0.00036955
7.0000	5.2792408	1.0000000	0.00022016
7.2000	5.4792286	1.0000000	0.00012856
7.4000	5.6792216	1.0000000	0.00007368
7.6000	5.8792184	1.0000000	0.00004138
7.8000	6.0792163	1.0000000	0.00002279
8.0000	6.2792152	1.0000000	0.00001225
8.2000	6.4792145	1.0000000	0.00000642
8.4000	6.6792147	1.0000000	0.00000331
8.6000	6.8792145	1.0000000	0.00000176
8.8000	7.0792143	1.0000000	0.00000088
9.0000	7.2792142	1.0000000	0.00000049
9.2000	7.4792139	1.0000000	0.00000025
9.4000	7.6792137	1.0000000	0.00000008
9.6000	7.8792138	1.0000000	0.00000004
9.8000	8.0792134	1.0000000	0.00000002
10.0000	8.2792132	1.0000000	0.00000001

APPENDIX: Derivation of Blasius' equation from the Navier-Stokes equation.

This appendix presents the main points of the mathematical formulation of the Blasius' equation. The fundamental assumptions on Navier-Stokes equation are also highlighted.

Consider the boundary layer to have length scales L and δ in the x- and y-directions. We may expect that the velocity scales will also be different in different directions and we denote the scales of u and υ by U and V.

Similarly the order of magnitude of the pressure differences across the boundary layer in the y-direction may not be the same as the order of magnitude of the imposed pressure differences outside the boundary layer; we denote the scale of the former by ϒ and the scale of the latter by Π. We now consider each of the equations in turn, labelling the terms with their orders of magnitude.

The continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0 \qquad (1)$$

$$\frac{U}{L} \qquad \frac{V}{\partial}$$

Fluid entering or leaving the boundary layer must be association with variations in the amount of fluid travelling downstream within the boundary layer. Hence

$$V \approx U\partial/L$$

The x-component of the Navier-Stokes equation is [5];

$$u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{v\partial^2 u}{\partial x^2} + v\frac{\partial^2 u}{\partial y^2} \qquad (3)$$

$$\frac{U^2}{L} \qquad \frac{VU}{\partial} \approx \frac{U^2}{L} \quad \frac{\Pi}{\rho L} \quad \frac{vU}{L^2} \qquad \frac{vU}{\partial^2}$$

The second expression for the order of $\upsilon\partial u/\partial y$ has been written using the relationship in equation (2). The two parts of the inertial terms are comparable with one another, the smallness of V/U compensating for the more rapid variation of u with y than with x. The two parts of the viscous terms are however of different sizes when ∂/L is small, $v\partial^2 u/\partial x^2$ may be neglected.

The y-component of the Navier-Stokes equation is

$$u\frac{\partial \upsilon}{\partial x} + v\frac{\partial \upsilon}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \frac{v\partial^2 \upsilon}{\partial x^2} + v\frac{\partial^2 \upsilon}{\partial y^2} \qquad (4)$$

$$\frac{UV}{L} \approx \frac{U^2\partial}{L^2} \quad \frac{V^2}{\partial} \approx \frac{U^2\partial}{L^2} \quad \frac{\Upsilon}{\partial\delta} \quad \frac{vV}{L^2} \quad \approx \quad \frac{vU\delta}{L^3} \quad \frac{vV}{\partial^2} \quad \approx \frac{vU}{L\partial}$$

In both equation (3) and (4), the pressure term will be of the same order of magnitude as the largest of the order terms, Hence

$$\Pi/ \rho L \approx U^2/L \approx vU/\partial^2 \tag{5}$$
$$\Upsilon/\rho\delta \approx U^2\partial/L \approx vU/L\partial \tag{6}$$

and so

$$\Upsilon/\Pi \approx \partial^2/ L^2 \tag{7}$$

The pressure differences across the boundary layer are smaller than those in the x-direction. Hence at any of the difference between $(1/\rho)\partial P/\partial x$ and $(1/\rho)dP_o/dx$ is much than the significant terms in equation (3) and it can be replaced by

$$u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dP_o}{\partial x} + v\frac{\partial^2 u}{\partial y^2} \tag{8}$$

Outside the boundary layer there is no variation with y and

$$u_o\frac{du_o}{\partial x} + \upsilon\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dP_o}{\partial x} \tag{9}$$

Hence, using Bernoulli's equation we obtain

$$u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = u_o\frac{du_o}{dx} \quad v\frac{\partial^2 u}{\partial y^2} \tag{10}$$

Equation (1) and (10) constitute the governing equations of fluid motion in two variables u and υ.

In this work where a steady pressure is maintained, solution of the above equation is of zero pressure gradient boundary layer;

$$dP_o/dx = 0$$

It therefore follows from equation (9)

$$u_o = \text{constant.}$$

The equations under this condition are;

$$u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} \quad (13)$$

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0 \quad (14)$$

with boundary conditions

$$u = \upsilon = 0 \text{ at } y=0$$

$$u \rightarrow u_o \text{ as } y \rightarrow \infty \quad (15)$$

We look for a solution of the form

$$u = u_o g(y/\Delta) \quad (16)$$

Where Δ is a function of x and is directly proportional to the boundary layer thickness, δ but it is convenient to define it slightly differently from δ. Equation (14) can be satisfied by introducing a stream function Ψ such that

$$u = \partial\Psi/\partial y \text{ and } \upsilon = \partial\Psi/\partial x \quad (17)$$

Equation (16) then becomes;

$$\Psi = u_o\Delta f(y/\Delta) \quad \text{where } g = f' \quad (18)$$

substituting this in equation (13) gives

$$\frac{u_o^2}{\Delta}\frac{d\Delta}{dx} ff'' + \frac{vu_o}{\Delta^2} f''' = 0 \quad (19)$$

where the prime indicates differentiation with respect to

$$\eta = y/\Delta \quad (20)$$

Reducing equation (19) to a total differential equation in f as a function of η, as it must if the solution is of the assumed form (eqn 16), then

$$\frac{u_o^2}{\Delta}\frac{d\Delta}{dx} \propto \frac{vu_o}{\Delta^2} \quad (21)$$

and so

$$\Delta^2 \propto vx/u_o + \text{constant.}$$

Also it is convenient to choose the constant of proportionality and the origin of x so that

$$\Delta = (vx/u_o)^{1/2}$$

It is found experimentally [5] that this choice of the origin of x corresponds to the leading edge of flat plate set up in an otherwise unobstructed flow. Equation (23) now becomes

$$ff^{//} + 2f^{///} = 0 \qquad (24)$$

The boundary conditions transform to

$$f = f^{/} = 0 \text{ at } \eta = 0$$
$$f^{/} \rightarrow 1 \text{ as } \eta \rightarrow \infty \qquad (25)$$

Equation (24) is the Blasius' equation. It is an ordinary differential equation that may be solved numerically for the function of f (η).

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