In this paper, we propose an algorithm, based on the conjugate gradient method for solving discrete optimal control problems with constraints on the states and controls of the dynamical system. It is important to note that, apart from the first class of solution techniques mentioned above, all the others employ at least a bit of mathematical programming. This is basically due to the relationship between Nonlinear programming problems and optimal control problems. We shall start by showing that, ordinarily, the Discrete Optimal Control problem can be viewed as a Nonlinear programming problem with some special structures.

The Nonlinear programming problem can generally be stated as follows:

Problem (P1)

Given continuously differentiable functions $f: R^n \to R^n$, $g: R^n \to R^m$ and $r: R^n \to R^q$, find a $\hat{z}$ in the set $\Omega = \{ z: g(z) \leq 0, r(z) = 0\}$, such that for all $z \,\varepsilon\, \Omega$, $f(z) \leq f(\hat{z})$.

We note here that the functions g and r have components $g_1, g_2, \ldots, g_m$ and $r_1, r_2, \ldots, r_q$ respectively. Thus, statements about g and r hold componentwise.

Problem (P1) can, however, be stated in shorthand form as:

$$\text{Minimize } \{f(z): g(z) \leq 0, r(z) = 0\}. \tag{1.1}$$

The need for problem (P1) in this paper will become obvious when our problem of interest (the Discrete Optimal Control Problem) is shown to be a form of Problem (P1) with special structure. This is the concern of the next section.

2. PROBLEM STATEMENT

Consider a dynamical system, described by the system of difference equations

$$x_{i+1} = g_i(x_i, u_i),\ i = 0, 1, \ldots, k,\ x(0) = x_0 \tag{2.1}$$

where $x = (x_1, x_2, \ldots, x_n)^T$ and $u = (u_1, u_2, \ldots, u_m)^T$ are respectively the state and control vectors; $g_i: R^n \times R^m \to R^n$ is a continuously differentiable function and $x_0 = (x_{1,0}, x_{2,0}, \ldots, x_{n,0})^T \,\varepsilon R^n$ is a given initial vector and $x_{i,0}$ stands for the i-th component of x_0.

Problem (P2)

Given the system (2.1), find a control $\hat{U} \varepsilon R^m$ and a corresponding trajectory $\hat{x} \,\varepsilon\, R^n$, such that the cost functional

$$J(x, u) = \sum_{i=0}^{k} f_i(x_i, u_i), \tag{2.2}$$