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ON ESTIMATION OF ENTROPY VALUE FOR AN ORGANISATION

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ABSTRACT

The paper considers the use of entropy to measure the stability of workforce of an organisation by making use of mainly wastage data of Edo State Teaching service to estimate the wastage probability of each length of service interval. It is found necessary to study the degree of experience present in Edo State Teaching Service due to the present economic situation and the State government embargo on employment in recent years. The study also modifies the log-normal model of Chu and Lin (1994) to calculate the wastage probabilities of tenure classes. Entropy is also calculated based on this modified log-normal wastage rates (probabilities) and a comparison is made between the two entropy values.

Bowey's stability curves are drawn for the actual wastage proportions (probabilities) of PPEB and the modified log-normal wastage proportions to authenticate the entropy results.

INTRODUCTION

According to McClean and Abodunde (1978), entropy "is defined as the thermodynamics variable of the system under consideration". They also see entropy as a measure of "the number of ways in which the elementary particles of the system may be arranged in the given circumstances".

Entropy has been used as a measure of the experience distribution in a manpower system, Vassiliou (1984), McClean (1986). It has been used in the study of combat degradation, Rodrigues (1989), queueing systems, Guiasu (1986), Kouvatsos (1989).

In this paper we estimate the entropy of the distribution of staff in Edo State Teaching Service Board. We do this by examining data on the organisation.

The rest of the paper is arranged as follows. In the next section, we examine the concept of Entropy as used in the study of manpower system. We then estimate the entropy of the Edo State Teaching Service Board based on the data available to us. Finally, we discuss the results.

CONCEPT OF ENTROPY

McClean and Abodunde (1978) defines entropy as

$$H = \frac{\sum_{i=1}^{k} P_i \log P_i}{\log K} \qquad (1)$$

where K is the maximum number of tenure classes; P_i is the probability that a member of staff is in tenure class i. It is evident that $0 \leq H \leq 1$. $H = 0$ if and only if $P_1 = 1$, $P_2 = P_3 = \ldots = P_k = 0$ and $H = 1$ if and only if $P_i = 1/K$ for all i. When $0 < H < 1$, the entropy is a good measure of the distribution of experience in the organisation.

In an organisation with data on its workforce, the estimation of the entropy is normally based on the use of relative frequencies as probabilities. The distribution of experience can also be examined for a set of workers.

ENTROPY MEASURE FOR PPEB

In this section, we calculate the entropy for a set of workers who left the Post Primary Education Board (PPEB) Edo State in the same year. All the workers in the set left the service in 1998 as a result of several reasons. Some retired on health ground, compulsorily or voluntarily. Some transferred to other States, absconded, died or were dismissed. The data was classified using length of service interval of 5 years. Table I shows the distribution of the length of service for the set of workers. There were 434 workers in the set.

We estimate P_i's the proportion of staff in tenure classes using completed length of service of the data collected from Teaching Service of Edo State as at May 1998. The total wastage is collated, thereafter the proportion of wastage from each length of service interval of 5 years.

This is a good estimate of wastage rates from each length of service interval as it deals directly with number of staff already out of the services of PPEB. One can now estimates the probability that any leaver belongs to any length of service interval i using the relative frequency P_i which is defined as the frequency of a class i over the total frequency of the distribution. The total number of leavers from the teaching service are classified into their various length of service intervals of 5 years .

The data collected from Edo State PPEB is presented in Table I below.

TABLE I

DISTRIBUTION OF LENGTH OF SERVICE

Length of Service Interval	No. of Staff	Proportion of Staff in the Interval (P_i)
$0^+ - 5$	16	16/434
$5^+ - 10$	61	61/434
$10^+ - 15$	75	75/434
$15^+ - 20$	84	84/434
$20^+ - 25$	34	34/434
$25^+ - 30$	37	37/434
$30^+ - 35$	127	127/434
	434	

Table I makes use of actual length of service and the number of staff falling into the various service length intervals. The entropy calculated for the data in Table I is 0.92. This shows a high degree of experience for the set of workers.

In line with the suggestion of McClean and Abodunde (1978); and Chu and Lin (1994), we also use the log-normal model to estimate the probability P_i of a staff belonging to a tenure class i and then the entropy for the workforce.

The log-normal model as suggested by Chu and Lin (1994) is given by

$$W(t) = \frac{1}{\sigma t\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{\ln(t)-w}{\sigma}\right]^2}, \quad t \geq 0 \qquad (2)$$

where w is the mean of ln(t), t is the length of service and σ is the standard deviation of ln(t).

This model operates as $t \to \infty$. But in our case, t is finite. Hence a modification of the system in (2) above is attempted to cater for t = T + 1, T + 2, ... that are not included in this study. T is the maximum length of service. This modified system of (2) is given by

$$W_i^*(t) = W_i(t) + L_i \qquad (3)$$

where W(t) is as defined in (2) above and

$$L_i = \frac{W_i(t)}{\sum_{i=1}^{n} W_i(t)}\left[1 - \sum_{i=1}^{n} W_i(t)\right]$$

$$= \frac{W_i(t)}{\sum_{i=1}^{n} W_i(t)} - W_i(t) \qquad (4)$$

$$\Rightarrow W_i^*(t) = W_i(t) + \frac{W_i(t)}{\sum_{i=1}^{n} W_i(t)} - W_i(t)$$

$$= \frac{W_i(t)}{\sum_{i=1}^{n} W_i(t)} \qquad (5)$$

(5) is termed in this study modified wastage rate of tenure class i = 1, 2, 3, ..., n. where n is the total number of tenure classes (length of service intervals). The implication of this is that (2) is used to generate the wastage probabilities (proportion) for t = 1, 2, ..., T; where T is the maximum number of years a staff can spend in the organisation. In this case, T = 35 years. After this, the proportions are grouped into length of service intervals of 5 years by adding W(1), W(2), W(3), W(4) and W(5) together as the first service length interval, then W(6), W(7) and so on for the second and subsequent i intervals. Thereafter, (5) is introduced to augument for the deficiency of (2). Equation (5) is simply taking the proportions of the total proportion of wastage from the log-normal model results.

The results on Table II is obtained from the modified log-normal model in (5).

TABLE II
DISTRIBUTION OF LENGTH OF SERVICE USING LOG-NORMAL MODEL

Length of Service Interval	Proportion of Staff (P_i)	No. of Staff
$0^+ - 5$	0.224	97
$5^+ - 10$	0.236	103
$10^+ - 15$	0.176	76
$15^+ - 20$	0.130	56
$20^+ - 25$	0.099	43
$25^+ - 30$	0.076	33
$30^+ - 35$	0.059	26
		434

On Table II, the number of staff column is obtained by multiplying the proportion of staff by 434, the total number of staff in the Cohort. The entropy computed for the distribution on table II using the system in (1) is equal to 0.94 indicating a very high stable workforce for Edo State Teaching Service.

The proportion of staff as proposed by Bowey as cited by McClean (1986) is used to plot stability curve. The curve is found by plotting the proportion of staff with t or more years against the length of service t. Table III shows the actual proportion of staff with t or more years service P(t) of Edo State Teaching Service.

TABLE III
THE DISTRIBUTION OF PROPORTION OF STAFF WITH t OR MORE YEAR'S SERVICE

Length of Service Interval	Proportion of Staff	Proportion of Staff P(t) with t or More Years' Service
$0^+ - 5$	0.037	1.00
$5^+ - 10$	0.140	0.96
$10^+ - 15$	0.173	0.82
$15^+ - 20$	0.194	0.65
$20^+ - 25$	0.078	0.46
$25^+ - 30$	0.085	0.38
$30^+ - 35$	0.293	0.29

Table III is the data collected from Edo State PPEB (Teaching Service). The resultant curve (fig. I) is of Bowey's type 3 stability curve which shows that there is a high proportion of long-term staff implying labour stability. The implication of this to Edo State PPEB is that a large number of senior staff present will be retiring soon so that Bowey's type 3 reverts to Bowey's type 1 stability curve indicating inexperienced workforce.

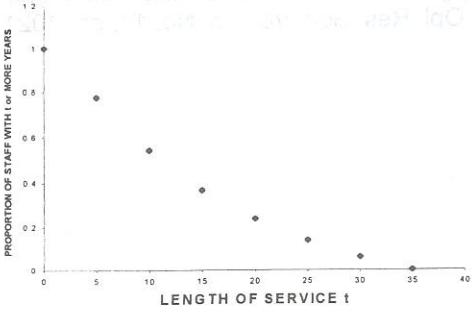
FIG 1: BOWEY STABILITY CURVE FOR PPEB DATA

Also, the wastage proportion obtained from the log-normal model is presented on table IV.

TABLE IV
THE DISTRIBUTION OF PROPORTION OF STAFF WITH t OR MORE YEAR'S SERVICE USING LOG-NORMAL MODEL

Length of Service Interval	Proportion of Staff	Proportion of Staff P(t) with t or More Years' Service
$0^+ - 5$	0.224	1.000
$5^+ - 10$	0.136	0.776
$10^+ - 15$	0.176	0.540
$15^+ - 20$	0.130	0.364
$20^+ - 25$	0.099	0.234
$25^+ - 30$	0.076	0.135
$30^+ - 35$	0.059	0.059

FIG 2: ESTIMATED BOWEY STABILITY CURVE FOR PPEB



The stability curve in fig. 2 is drawn from table IV. It represents Bowey's type 2 stability curve where all lengths of service are almost equally represented. The curve shows a higher degree of stability than that of fig. 1 authenticating the validity of the entropy value obtained for Table II.

CONCLUSION

The entropy measure and the stability curve show that there exists a stable workforce in Edo State Teaching Service. The reasons are that there is

less job opportunity elsewhere as well as the embargo on employment of new employees over the years in Edo State. Also, the entropy value calculated based on the wastage rate obtained from the log-normal model estimates accurately the entropy measure of the organisation authenticating the validity of the log-normal model for estimating manpower wastage in Edo State Teaching Service..

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