J. Nig. Ass. Math. Phys.
Vol. 2 (1998)

COMPUTATION OF IMPEDANCE TENSOR ELEMENTS FROM MAGNETO-TELLURIC DATA.

ASOKHIA, M.B.
DEPARTMENT OF PHYSICS, EDO STATE UNIVERSITY, EKPOMA

ABSTRACT

Magneto-telluric (MT) technique of geophysics exploration is relatively new, being less than a half a century old. A vital parameter in MT survey is impedance. Virtually all other parameters are functions of impedance. Cagniard, in the first paper ever published on MT survey, in 1953, assumed that the earth is one-dimensional so as to simplify interpretation of MT data. However, most structure are either two or three-dimensional in practice.

In this work, mathematical methods of rotating impedance for the purpose of interpreting a nearly two-dimensional earth by inverting the earth by a quasi-one-dimensional technique is presented. This method can also be used for interpreting a two-dimensional structure by a quasi-two-dimensional method. The work was done along the "Blue Road Traverse" in Sweden. Seven stations were covered. MT parameters were computed as functions of the rotated tensor impedance. Mean coherence values, skew factors and strike angles were calculated. Depths to highly conducting layers were derived. Detailed results were tabulated and it was shown that MT survey is a very effective method of mapping geological structures.

INTRODUCTION

This work was done in Sweden along an area known in Scandinavia as the **Blue Road Traverse.** Fig. 1 is a map of the area and the stations covered. The objective was to derive an optimum technique of computing impedance tensor elements from magneto-telluric data.

There are two main geomagnetic induction methods. The two utilise the natural magnetic field variation as the energy source. One is the Geomagnetic Deep Sounding (GDS) technique in which an observer on the surface of the earth measures three components of the varying magnetic field at several stations. The other is the Magneto-Telluric (MT) technique in which observations need only be made at one station. Time variations of two orthogonal components of both electrical and magnetic fields are recorded. The method of deriving conductivity distribution with depth proposed by Cagniard, (1953) assumes that the earth is one-dimensional, that is, that conductivity varies with depth only. However many structures are either two or three-dimensional in practice. Wright, (1970), proposed a method of interpreting a nearly two-dimensional earth by inverting the earth by a quasi-one-dimensional technique. A full two dimensional interpretational method proposed by Wieldelt, (1975a) can also be used. A three-dimensional structure can be analysed by a quasi-two-dimensional method, Haak, (1972).

Conductivity is the reciprocal of resistivity. The term **apparent resistivity** _ is usually used in geophysics because the earth is not homogeneous. Apparent resisitivty is a weighted average of the resistivities of the various formations.

THEORETICAL ANALYSIS

A fundamental parameter in magneto-telluric investigation is **the skew factor** defined by

$$\text{Skew} = (Z_{xx} + Z_{yy}) / (Z_{xy} - Z_{yx}) \quad (1)$$

where Z is impedance. A site is regarded to be two-dimensional if the skew is not greater than 0.4, otherwise, it is three-dimensional.

The **phase angle** between electrical and magnetic field is derived from the equation

$$\tan\phi = \frac{\coth x\,(\coth^2 y + 1) + \cot y(1 - \coth^2 x)}{\coth x(\coth^2 y + 1) - \cot y(1 - \coth^2 x)} \quad (2)$$

where $x = h_1 \text{Real}(k_1) - \text{arc coth}\sqrt{\rho_2/\rho_1}$ and $y = h_1 \text{Imag}(k_1)$, ρ_1 and ρ_2 are resistivities of the first and second layers respectively, k_1 is the wave number of the magneto-telluric field in the first layer.

Let X(t), the imput, and Y(t), the output be stationary time series with power spectra $P_{xx}(f)$ and $P_{yy}(f)$ respectively, and cross-spectrum $P_{xy}(f)$. The **coherence** between the two time series is defined by

$$\gamma^2_{xy}(f) = |P_{xx}|^2 (P_{xx} . P_{yy}) \quad (3)$$

A very vital parameter in the estimation of apparent resisivity in magneto-telluric method is **tensor impedance Z** which is related to apparent resistivity, $\rho_a(w)$ by the equation

$$\rho_a(w) = \frac{1}{\mu_o w} |Z(0)|^2 \quad (4)$$

Were Z(0) is the impendance at the surface of the earth, w is the frequency of magneto-telluric field and μ_0 is the permeability of free space. Impedance at the top of a layer is related to impedance at the bottom of the layer by the equation.

$$Z_{m-1}(Z_{m-1}) = -\frac{\mu_o iw}{K_m}\coth\left[K_m h_m - \text{arc coth}\;\frac{K_m Z_m(Z_m)}{\mu_o iw}\right] \quad (5)$$

where m signifies the m^{th} layer, K is wave number and h is depth, Asokhia 1979).

Any electrical or magnetic field component, F_n can be resolved into the direction x, and y with the equation

$$F_n = a_n H_x + b_n H_y \quad (6)$$

where a_n and b_n are complex coefficients, H is magnetic field and x and y are the coordinate systems used. In tensor notation we have

$$F_i = f_{ij}F_j + f_{ik}F_k \quad (7)$$

Where f_{ij} and f_{ik} are complex coefficients. These complex coefficients depend on the orientation of the coordinate system, the frequency and the geological conditions at the point of investigation. Using the symbols of the MT literature for f_{ij}, f_{ik} and F_i we have at any point of investigation.

$$E_x = Z_{xx}H_x + Z_{xy}H_y \quad (8)$$

$$E_y = Z_{yx}H_x + Z_{yy}H_y \quad (9)$$

In tensor form equation (8) and (9) may be written as

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} Z_{xx} Z_{xy} \\ Z_{xy} Z_{yy} \end{bmatrix} \begin{bmatrix} H_x \\ H_y \end{bmatrix} \quad (10)$$

Equation (10) may be simplified to

$$[E] = [Z]\,[H] \quad (11)$$

The tensor invariant here is given by

$$Z^2_{eff} = Z_{xx}Z_{yy} - Z_{yx}Z_{xy} \quad (12)$$

which is independent of the coordinate system and contains only parameters of the subsurface.

Equations (8), (9) and (12) are the basic MT equations.

The idea of rotating the tensor elements was introduced by Bostick and Smith (1962). This reduces the tensor, as far as possible, to Cagniard represntation, that is, it reduces the complexities in the dimensionality of the structure. Asokhia, (1979) showed how impedance changes for a clockwise rotation of the measuring axis though angle θ from a coordinate system (x,y) with x along the strike, to a new coordinate system (x', y'). The new impedance values obtained.

$$2Z_{xx'}(\theta) = (Z_{xx} + Z_{yy}) + (Z_{xx} - Z_{yy})\cos 2\theta + (Z_{xy} + Z_{yx})\sin 2\theta \quad (13)$$

$$2Z_{xy'}(\theta) = (Z_{xy} - Z_{yx}) + (Z_{xy} + Z_{yx})\cos 2\theta - (Z_{xx} - Z_{yx})\sin 2\theta \quad (14)$$

$$2Z_{xy'}(\theta) = -(Z_{xy} - Z_{yx}) + (Z_{xy} + Z_{yx})\cos 2\theta - (Z_{xx} - Z_{yy})\sin 2\theta \quad (15)$$

$$2Z_{yy'}(\theta) = (Z_{xx} - Z_{yy}) - (Z_{xx} - Z_{yy})\cos 2\theta - (Z_{xy} - Z_{yx})\sin 2\theta \quad (16)$$

The maximum and minimum values of $Z_{xy'}(\theta)$ are called the principal impedances. These principal impedances can be determined by maximising or minimising the right hand side of equation (14) with respect to θ

Swift (1967) demonstrated that the angle at which

$$|Z_{xy'}(\theta)|^2 + |Z_{yx'}|^2 = \text{maximum}$$

also satisfies

$$|Z_{xx'}(\theta)|^2 + |Z_{yy'}|^2 = \text{minimum}$$

and is given by

$$4\theta = \tan^{-1}[(Z_{xx}-Z_{yy})(Z_{xx} - Z_{yy})^* + (Z_{xx} - Z_{yy})^* (Z_{xy} - Z_{yx})]/ [|Z_{xx} - Z_{yy}|^2 - |Z_{xy} + Z_{yx}|^2 - |Z_{xy} + Z_{yx}|^2] \quad (17)$$

(where the symbol * represents the operation of convolution

It is desirable to make more than two independent measurements and employ some forms of averaging of spectral estimates so as to reduce the effect of noise. Sims et al. (1971) suggested a mean square approach to minimise the effect of noise on telluric records. In order to estimate Z_{xx} and Z_{xy} in the mean square sense and in accordance with equations (8) and (9) we may defined

$$\psi = \sum_{i=1}^{n} (E_{xi} - Z_{xx}H_{xi} - Z_{xy}H_{yi})(E^*_{xi} - Z^*_{xx}H^*_{xi} - Z^*_{xy}H_{yi}^*) \quad (18)$$

and then find the vallue of Z_{xx} and Z_{xy} that minimises ψ. Setting the derivative of ψ with respect to the real and imaginary parts of Z_{xx} to zero yields

$$\sum_{i=1}^{n} E_{xi} H^*_{xi} = Z_{xx} \sum_{i=1}^{n} H_{xi} H^*_{xi} + Z_{xy} \sum_{i=1}^{n} H_{yi} H^*_{xi} \quad (19)$$

Similarly, setting the derivatives of ψ with respect to the real and imaginary parts of Z_{xy} to zero yields

$$\sum_{i=1}^{n} E_{xi} H^*_{yi} = Z_{xx} \sum_{i=1}^{n} H_{xi} H^*_{yi} + Z_{xy} \sum_{i=1}^{n} H_{yi} H^*_{yi} \quad (20)$$

The summations represent auto-power and cross-power density spectra. Equations (19) and (20) may be solved simultaneously for Z_{xx} and Z_{xy}. This solution will minimise the error caused by noise on E_x. Defining

$$SE_xH_y = \frac{1}{N} \sum_{i=1}^{N} E_{xi} H^*_{yi} \quad (21)$$

Equation (19) and (20) can be replaced by a more concise form

$$SE_xH_x = Z_{xx}SH_x.H^*_x + Z_{xy}SH_yH^*_x \quad (22)$$

$$SE_xH_y = Z_{xx}SH_xH^*_y + Z_{xy}SH_y.H^*_y \quad (23)$$

Sims and Bostick (1069) show that the comparable relationship which minimises noise on the magnetic components are

$$SE_xE_x = Z_{xx}SH_xE_x + Z_{xy}SH_yE_x \quad (24)$$

$$SE_xE_y = Z_{xx}SH_xE_y + Z_{xy}SH_yE_y \quad (25)$$

Any two of equations (22) to (25) may be solved to obtain estimates of Z_{xx} and Z_{xy}. Sims et, al. (1971) gave the possible solutions. The four commonly used by MT workers are

$$Z_{xy} = (SH_xH_y\,SE_xH_y - SE_x\,H_x.SH_xH_y)/(SH_xH_x.SH_yH_y - SH_yH_x.SH_xH_y) \quad (26)$$

$$Z_{xx} = (SE_xH_x.SE_yH_y - SH_yH_x..SE_xH_y)/(SH_xH_x..SH_yH_y - SH_yH_x.SH_xH_y) \quad (27)$$

$$Z_{yx} = (SE_yH_x.SH_yH_y - SH_yH_x.SE_yH_y)/(SH_xH_x.SH_yH_y - SH_yH_x.SH_xH_y) \quad (28)$$

and

$$Z_{yy} = (SH_xH_x.\,SE_yH_y - SE_yH_x.SH_xH_y)/(SH_xH_x.SH_yH_y - SH_yH_x.SH_xH_y) \quad (29)$$

The denominators of equations (26) and (29) may be zero in practice. Kunetz, (1972) suggested the following stabilizing procedure for example, equation (26) may be written in the form

$$Z_{xy} = \frac{N(w)}{D(w)} \quad (30)$$

Where N stands for numerator, D for denominator and w is frequency. Then a stabilized estimate of Z_{zy} would be

$$Z_{xy} = \frac{N(w)\mathbf{D}(w)}{D(w)\mathbf{D}(w)+\lambda^2} \quad (31)$$

where λ^2 is a conveniently chosen positive constant and D is the complex conjugate of D(w). In carrying out the computations we should note that equations (26) to (29) contain complex magnitudes that should be separated into real (Re) and imaginary (Im) parts. Thus, for equation (26) we have

$$Z_{xy} = \mathrm{Re}\,Z_{xy} + \mathrm{Im}\,Z_{xy} \quad (32)$$

$$|Z_{xy}| = \sqrt{(\mathrm{Re}\,Z_{xy})^2 + (\mathrm{Im}\,Z_{xy})^2} \quad (33)$$

$$\mathrm{Arg}\,Z_{xy} = \mathrm{Im}\,Z_{xy}/\mathrm{Re}\,Z_{xy} \quad (34)$$

Strike angles were computed by maximising the equation

$$|Z_{xy}(\theta)|^2 + |Z_{yx}(\theta)|^2$$

Reddy and Rankin, (1971), and by minimising the equation

$$|Z_{xx}(\theta)|^2 + |Z_{yy}(\theta)|^2$$

Swift (1967)

Considering a resistive top layer of resistivity ρ_2 , overlain by a resistive top layer of thickness h_1, the following equations hold for h_1 and ρ_2 respectively

$$h_1 = \frac{1}{\mu w}(\mathrm{Im}\,Z(0) - \mathrm{Re}Z(0)) \qquad (35)$$

$$\rho_1 = \frac{2}{\mu w}(\mathrm{Re}\,Z(0))^2 \qquad (36)$$

provided that the phase lies between $\pi/4$ and $\pi/2$. If $\delta_2 >> \delta_1$ and $k_2 >> k_1$ where δ = conductivity then the thickness of the first layer is

$$h_1 = 800\ (\rho_a/w)^{1/2} \text{ meters.} \qquad (37)$$

The subscript 1 and 2 in equation (35) to (37) indicate the order of the layer. All the MT data thus obtained have to be assessed to determine whether or not further processing would be meaningful. The first **acceptance criterion** is that the phase of the major impedance tensor element, $Z_{xy'}(\theta)$, had to be between 0° and 90^{0}, for the value of conductivity is greater than zero but less than infinity. This ensured that the data were physically realisable.

The second acceptance criterion is that there should be more coherent signal in the anti-diagonal elements than in the diagonal elements.

EXPERIMENTAL WORK

The seven stations shown in fig 1, were covered along the Blue Road Traverse of Scandinavia.

The MT instrument used for the measurements was constructed in Kiruna, north Sweden. The total weight of the instruments was 113kg, the heaviest components being the magnetometer, 36kg; cables, 13kg, and paper chart recorder, 10kg. The other components were loop with preamplifier, 6kg; and two electrodes, each weighing 3kg. Thus, weight was a little constraint in the field. As such, measurements were taken at accessible venues to the instrument bus. Fig. 2 shows the layout of sensors at the measuring site. The raw data in form of voltage histories were recorded on magnetic tape using ASCII (American Standard Code for Information Interchange). The analog records were digitized at sampling intervals shown in table 1. This table summarises the field work in all the seven stations covered. Table 2 shows 25 digital values of an acceptable data (Station K_3S_{63}). These data were acceptable because they satisfied the acceptance criteria above.

Linear trends were removed before the values in table 2 could meaningfully represent magnetic units. For details see Asokhia (1979)

The response data were then transformed from time domain into complex frequency domain by using Fourier transform given by

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-iwt}\,dt \qquad (38)$$

The aim of Fourier intransient data processing is the estimation of frequency dependent characteristics such as impedance, conductivity, etc.

Skew factors, phase angles between electric and magnetic fields as well as coherence were computed using equations (1), (2), and (3) respectively. Tensor impedances were calculated and rotated to reduce the tensor, as far as possible, to Cagniard representation. Knowing the tensor impedance, equation (4) was used for estimating apparent resistivity. Depth estimates were made using equations (35) and (37).

RESULTS AND DISCUSSION

A summary of the result is presented in table 3. Coherence is a type of measure of the quality of the data. Station K_3S_{63} with a coherence value of 0.85 had the highest quality data while station G_1S_4 had the noisiest data. The skew factors for stations K_3S_{63}, $K_3\,S_{10}$ and G_1S_1 were equal to or less than 0.4 and were therefore two-dimensional in structure. The other stations were three dimensional. The inversion techniques presented in the theory were used for the various stations, depending on the dimensionality. Nearness to the Baltic sea appeared to be a main factor in the depth of the conducting layer. Station G_1S_1 which was closest to the Baltic sea had a highly conducting depth of only 3km while the farthest station inland station, K_3S_{63} had the deepest highly conducting layer of almost 40km. All the other parameters derived are shown in table 3. Fig. 3 is a typical graph of log auto-power versus log period (NS) while fig. 4 is a typical graph for tensor impedance versus log period. Both graphs were for station K_3S_{63}.

CONCLUSION

The MT method is in its developing stage, as such, there is need for a lot of research work so as to refine it in order to increase the accuracy of its interpretation. It is undoubtedly a very effective method of mapping geological structures. It has a lot of advantages over the conventional techniques of geophysical exploration. For instance, Lund, (1973) gave reasons why seismic methods failed in several areas of the Baltic shield. The main obstacle being the "hidden layer" problem as a result of the geosyncline nature of the region. When a low velocity layer is situated between two high velocity layers, refractions from the low velocity layer are usually completely masked by the arrivals from the high velocity layers. No layer is hidden from MT survey. Also, in vertical electrical sounding, readings are taken at several points for a single station investigation whereas readings are taken at a single points for a single station investigation in MT work. MT survey is relatively cheap in terms of cost and time. Energy source for MT work is the natural variations of geomagnetic field whereas energy sources in several other geophysical investigation techniques are artificial and subject to failure. MT investigation is effective for tens of kilometer whereas data from vertical electrical sounding are not quite reliable beyond a depth of 1 km.

Table 1. Summary of data acquisition

Station	Play-back speed (cm/s)	Recording speed (cm/s)	Sampling rate (Samplings/s)	Sampling intervals/s
K_3S_{63}	76	4.75	1.9954	8.0186
K_3S_{62}	76	4.75	1.9954	8.0186
K_3S_{61}	76	4.75	1.9954	8.0186
K_3S_{10}	19	4.75	2.5224	1.5958
G_1S_4	4.75	4.75	22.2816	0.0449
G_1S_2	38	4.75	2.3873	3.3510
G_1S_1	38	4.75	1.5916	5.0265

Table 2. A typical acceptable data (Station K_3S_{63})

(1)	100.00	(2)	38.00	(3)	94.00	(4)	144.00	(5)	63.00
(6)	119.00	(7)	24.00	(8)	94.00	(9)	136.00	(10)	215.00
(11)	165.00	(12)	99.00	(13)	140.00	(14)	134.00	(15)	115.00
(16)	93.00	(17)	130.00	(18)	95.00	(19)	23.00	(20)	118.00
(21)	42.00	(22)	115.00	(23)	94.00	(24)	0.00	(25)	-15.00

Table 3. Summary of results

PARAMETERS	STATIONS						
	K_3S_{63}	K_3S_{62}	K_3S_{61}	K_3S_{10}	G_1S_4	G_1S_2	G_1S_1
Mean coherence	0.85	0.60	0.80	0.75	0.57	0.65	0.80
Mean skew factor	0.38	0.71	0.57	0.40	0.90	0.68	0.39
Mean strike angle	58°	38°	52°	59°	41°	48°	45°
Mean electrical auto-power $(mV/km)^2/Hz$	1.5	1.2	1.0	1.1	-2.0	0.8	1.0
Mean magnetic auto-power $(gamma)^2$ /Hz	0.1	-1.8	0.8	-1.0	-2.5	0.9	-1.5
Mean Z_{11} ((mV/km) /gamma	5.2	55	1.0	12.0	4.0	2.0	10.0
Mean Z_{12}((mV/km) gammma	5.4	11.0	0.9	40.0	5.5	5.0	30.0
Mean Z_{21}((mV/km) Gamma	11.5	90	2.2	20.0	6.0	10.0	40.0
Mean Z_{22}((mV/km) Gamma	3.6	32	0.7	10.0	3.8	2.0	8.0
Highly conducting depth (km)	41.0	26.0	24.5	14.0	-	12.0	3.0
Corresponding $\rho_a(\Omega\text{-mm})$	200	200	60	100	-	15	10

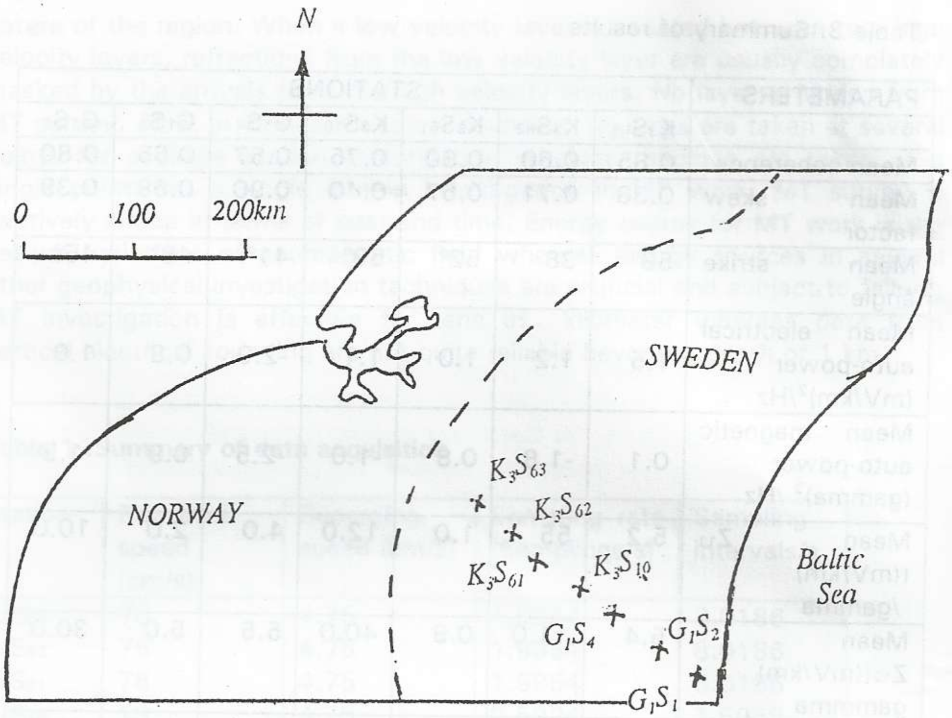

Fig. 1. MT sites along the "Blew Road Traverse, "Sweden
Note: MT sites indicated by crosses.
The frontier between Sweden and Norway is shown in broken lines.

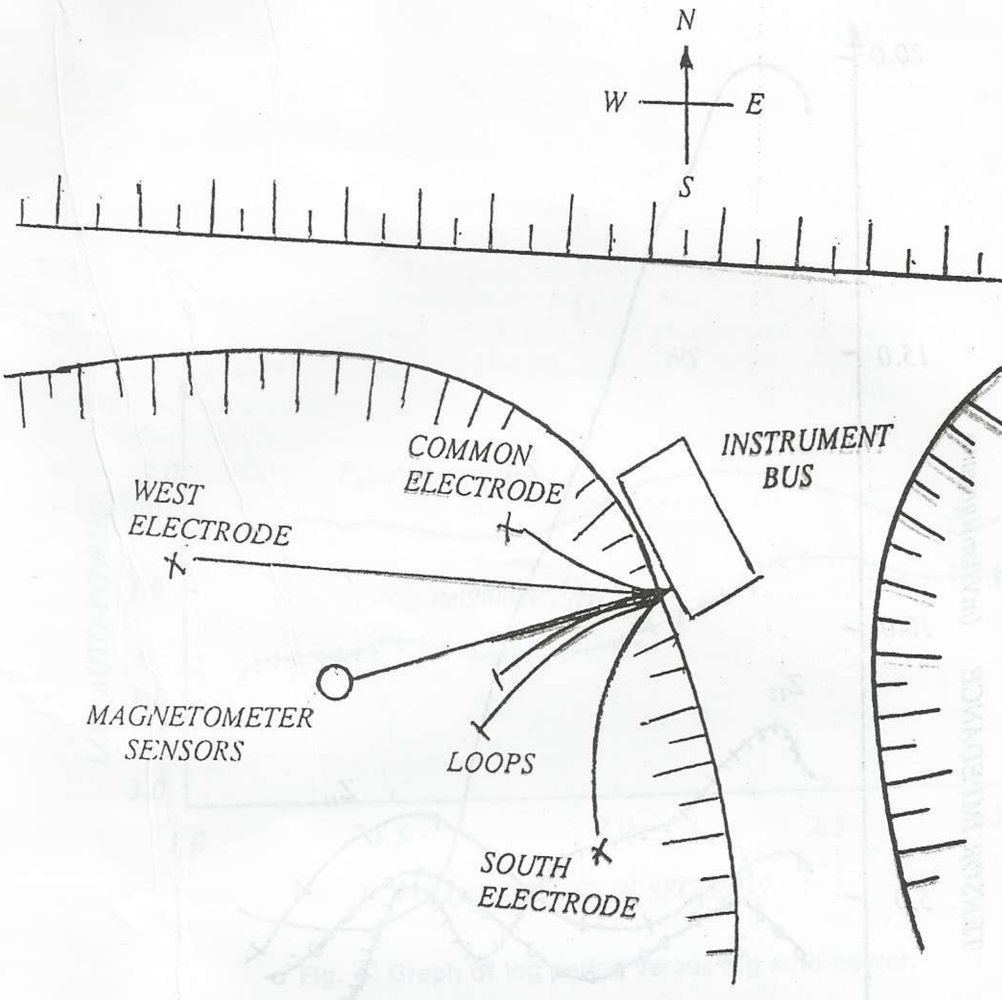

Fig. 2. Layout of sensors at the measuring site

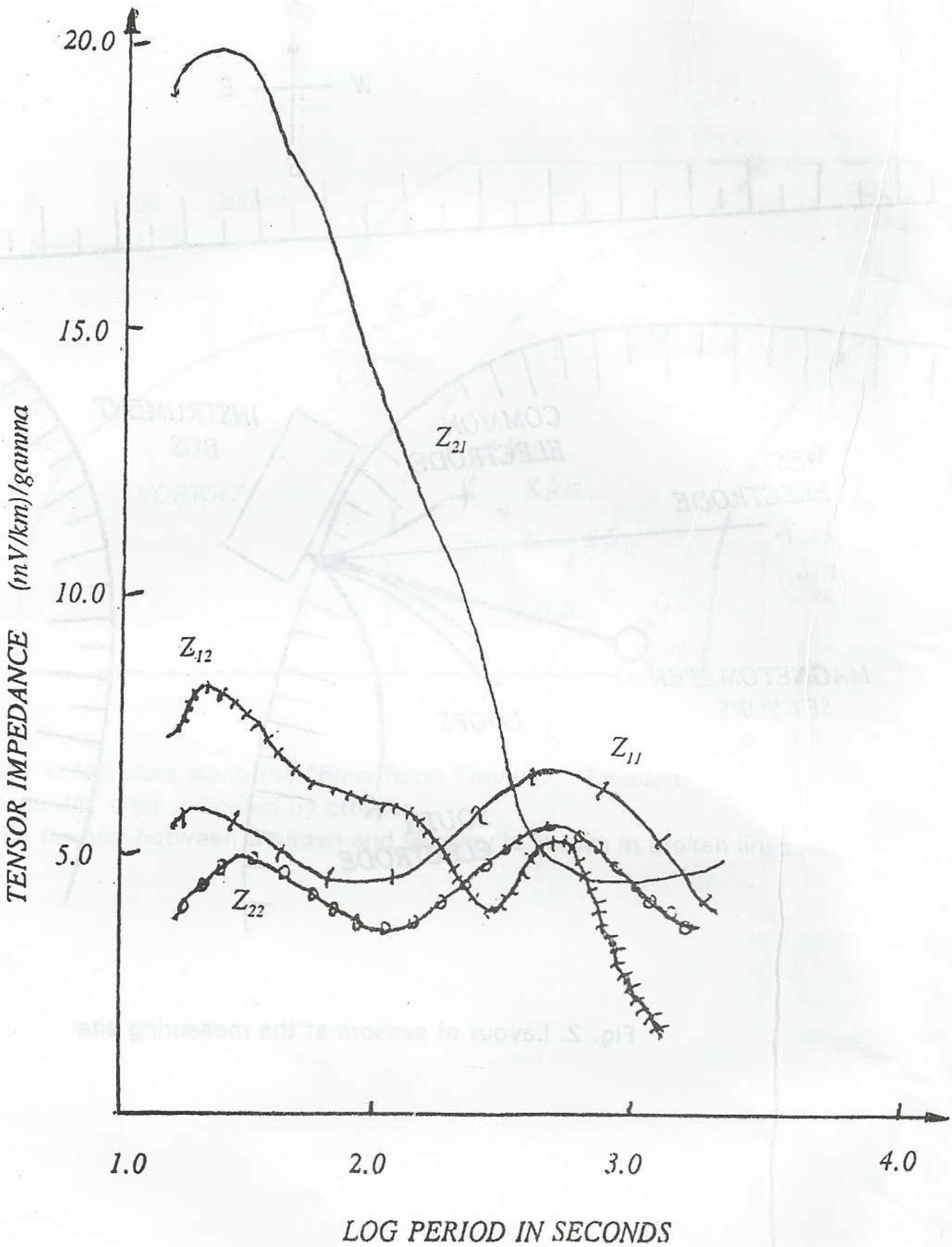

Fig. 3. Graph of log period versus impedance tensor.

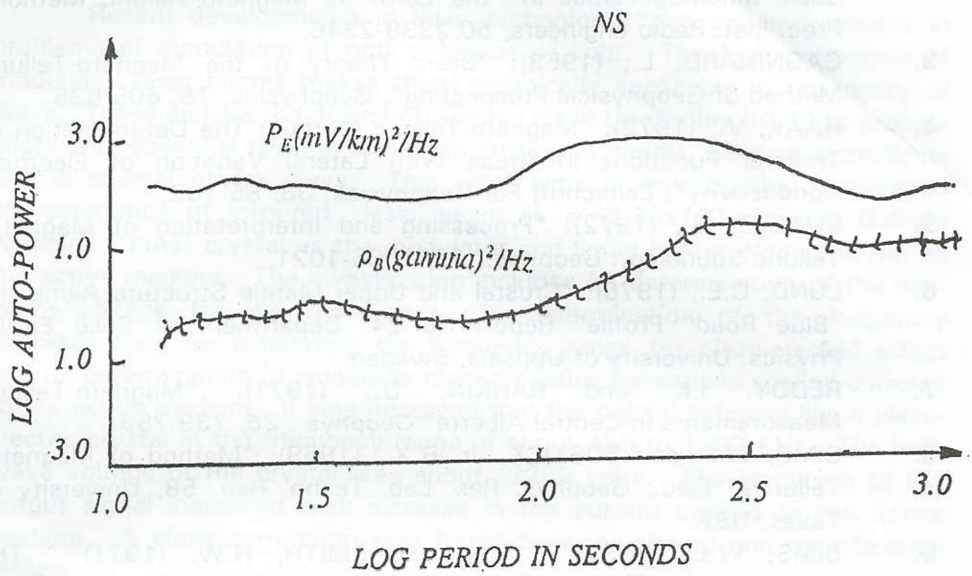

Fig. 4. Graph of log period versus log auto-power.

ACKNOWLEDGEMENT

The author is very grateful to the Swedish authorities for the assistance in making this field work a success.

REFERENCES

1. ASOKHIA, M.B., (1979): "One Dimensional Analysis of Magento-Telluric Data for a Multi- Dimensional Structure" Ph.D Thesis, University of Lagos Nigeria.
2. BOSTICK, F.X. and SMITH, H.W.(1962): "Investigation of Large Scale Inhomogeneities in the Earth by Magneto-Telluric Method" Proc. Inst. Radio Engineers, *50,*2339-2346.
3. CAGNINARD, L., (1953): "Basic Theory of the Magneto-Telluric Method of Geophysical Prospecting", Geophysics, *18*, 605-635.
4. HAAK, V., (1972): "Magneto-Telluric Method: The Determination of Transfer Functions in Areas With Lateral Variation of Electrical Conductivity", Zeitschrift Fur Geophysick, *38,* 85-102.
5. KUNETZ, G., (1972): "Processing and Interpretation of Magneto-Telluric Sounding,: Geophys., *37*, 1005-1021.
6. LUND, C.E., (1976): "Crustal and Upper Mantle Structure Along the "Blue Road" Profile" Report No 24. Department of Solid Earths Physics. University of Uppsala, Sweden.
7. REDDY, I.K. and RANKIN, D., (1971): "Magneto-Telluric Measurements in Central Alberta" Geophys.,_*36,* 739-753.
8. SIMS, W.E. and BOSTICK, Jr. R.X., (1969): "Method of Magneto-Tellurics" Elec., Geophys. Res. Lab. Techn. Rep. 58, University of Texas, USA.
9. SIMS, W.E., BOSTICK, F.N. and SMITH, H.W. (1971) " The Estimation of Magneto-Telluric Impedance Tensor Elements from Measurement Data" Geophys. *36,*938-942.
10. SWIFT, C.M., (1967): "A Magneto-Telluric Investigation of an Electrical Conductivity Anomaly in the South-Western United States". Ph.d. Thesis Department of Geology and Geophyics, M.I.T., Cambridge, Massachusetts.
11. WIELDELT, P., (1975a): "Inversion of Two-Dimensional Conductivity Structure" Phys. Earth. Planet Int., *10,* 282-291.
12. WRIGHT, J.A., (1970) : "Anisotropic Apparent Resistivity Arising from non-homogenous Two-Dimensional Structures. "Can. J. Earth Sc., I, 527-531.