J. Nig. Ass. Math. Phys.
Vol. 2 (1998)

THE COMMENTS ON THE SPECTRUM OF MICRO-SEISMIC FREQUENCIES

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SUMMARY:

The spectrum of frequencies associated with the motion of the seabed forced by gravity sea wave is examined. The dependence of the peak frequency and energy spectral height on the induced shear modulus is derived. Deduced from the inherent variation of the shear modulus with the depth below the seabed, this model suggests that the seismometer buried at a depth of 10m below the earth's surface is likely to record intense ground movements. Below this depth, further increase will be modified by the exponential decay with the depth usually associated with the wave amplitudes.

1. INTRODUCTION

Microseisms are defined as microscale vibrations of the solid earth with amplitude of about 10 microns (1 micron = 10^{-6} metre). They often appear on seismic records and were first observed by Father Bertelle of Florence nearly a century ago. Although regarded as nuisance by seismologists as they tend to distort earthquake records, they are nevertheless of great use to the meteorologists as their appearances on seismograph usually herald the approach of a storm.

A number of mechanisms have been proposed to explain the origin of the background noise usually observed on seismograms. Weichart[15] attributed this to the action of surf on the coastlines. This refers to the wave activities along the shoreline over which the breaking waves are actively dissipating their energy against the extended steep coast. Sholte[13] reasoned, among previous workers investigating the geophysical phenomena, that pressure fluctuations in the atmosphere were capable of generating microseisms.

Recently, it was established that microseisms could be excited by the evolution of the bottom pressure field on the sea floor as free water waves propagate towards the shoreline[3,4,11]. The effective bottom pressure field may be of the first order in the form of a system of quasi-monochromatic wave trains and thus, has the same frequency as that of the associated elastic modes. This was initially observed by Hinde et al[6]. It was followed by elaborate studies carried out by Munk et al[10], Hasselmann[4], Darbyshire et al[3] and Okeke[11,12] using different formulations. However, they arrived at identical conclusions. The microseisms which usually originate from this mechanism are called, primary frequency microseisms.

There are also microseisms, associated with second order pressure fluctuations. They are called, double frequency microseisms for they have the frequency half that of the generating sea waves. Longuet-Higgins[9,4,3] established the existence of this second-order pressure effects. It has been established that the pressure effects are induced by the piston-like vertical motion of water columns. These follow the existence of the randomly distributed groups of standing oscillations that are in phase in the areas of deep water. It was further demonstrated successfully that the motion is maintained by the coupling between the oscillations of the centre of gravity of water column and waves of compression[3]. This, in turn, is capable of communicating energy in the form of the acoustic vibrations to the seabed. This process is independent of the depth of the water layer.

Thus, the present work is an attempt meant to have a closer look at the two established microseisms bands, attempt to determine how close they are and examine the inherent layer effects in this regard.

2. FREQUENCY DISPERSION RELATIONS

The far field characterization of seismic wave form can be explained through the consistency equation[11]. In the present analysis, it is given as follows:

$$(2k^2 - \omega^2/\alpha^2)(2k^2 - \omega^2/\beta^2) - 4k^2 k_\alpha k_\beta = 0 \quad (2.1)$$

Here k, k_α k_β are parameters for the surface wave, compression and shear waves respectively; α and β are respectively the speed of compression wave and that of shear wave, ω is the observed frequency for the waves. 2.1 can be re-arranged to give:

$$\omega^4 - 2k^2\omega^2(\alpha^2 + \beta^2) + 4\alpha^2\beta^2k^2(k^2 - k_\alpha k_\beta) = 0 \quad (2.2)$$

2.2 is a quadratic in ω^2. The two roots are ω_1^2 and ω_2^2, where the representations are as follows:

$$\omega_1^2 = k^2[(\alpha^2 + \beta^2) - R_1] \quad (2.3)$$

$$\omega_2^2 = k^2[(\alpha^2 + \beta^2) + R_1] \quad (2.4)$$

$$R_1(k) = 2\left[(\alpha^2 - \beta^2)^2 + \frac{4\alpha^2\beta^2 k_\beta k_\alpha}{k^2}\right]_0^{\frac{1}{2}} \quad (2.5)$$

Since $k > k_\beta > k_\alpha$, the wave length of compression wave is longer than that of shear wave and that of the latter is longer than that of associated surface wave. Thus $k_\beta k_\alpha < k^2$

The solutions $\omega_1^2(k)$ and $\omega_2^2(k)$ define two completely decoupled solutions in the frequency spectrum of the microseisms. The smaller root ω_1^2 is for the primary and ω_2^2 is for double frequency microseisms respectively. In the two solutions, the term $R_1(k)$ given by 2.5, is strictly positive. This implies that the expression will not vanish nor be negative for any physically realizable value that may be assigned to the parameters of equation 2.5.

On the other hand, in the presence of the sea-wave forcing the seabed, we have

$$-f(\omega) = \omega^4 - 2k^2\omega^2(\beta^2 + \alpha^2) + 4\alpha^2\beta^2k^2(k^2 - k_\beta k_\alpha) \quad (2.6)$$

If we take $\omega^2 = \omega_1$,

$$f(\omega_1) = k^2(\alpha^2 + \beta^2)^2 - 4\alpha^2\beta^2(k^2 - k_\beta k_\alpha) - [\omega_1 - k^2(\alpha^2 + \beta^2)]^2 \quad (2.7)$$

2.7 peaks when $\omega_1 = k^2(\alpha^2 + \beta^2)$ and thus, the peak frequency is given by:

$$\omega_1 = \omega_m^2 = k(\alpha^2 + \beta^2)^2 \quad (2.7a)$$

The height of the spectrum H_m corresponding to the peak frequency is given by

$$H_m = k^2(\alpha^2 + \beta^2) - 4\alpha^2\beta^2(k^2 - k_\alpha k_\beta) \quad (2.8)$$

Further, another physical quantity which needs to be determined is the spectral frequency band width. Consequently, this is calculated from:

$$\overline{\Delta\omega} = C_m \Delta k = 0.93\beta\Delta k = 0.93\left(\frac{\mu}{\rho}\right)^{\frac{1}{2}}\Delta k$$

Here, ρ is the material density, μ is the shear modulus characterising the material rigidity with C_m as the group velocity. Thus, O varies as the square root of the material rigidity.

Usually, $\beta = 2.8\ kms^{-1}$, $\omega = 1.0\ rad\ s^{-1}$; and $\Delta k = 1.0^{-4} kms^{-1}$ from which $\overline{\Delta\omega} = 2.852x10^{-4}\ s^{-1}$. O

Further, using Poissons's relations, $\alpha 5 = 3\beta 5$, 2.7 gives:

$$\omega_m = 2k\beta = 2k(\mu/\rho)^2 \quad (2.9)$$

$$H_m = \left(\frac{\mu}{\rho}\right)^2\left(1 + \frac{k\beta k\alpha}{k^2}\right) \quad (2.10)$$

2.10 is interesting because, since $0 < k_\beta k_\alpha < k^2$, then $H_m > 0$ always.

The dependence of ω_m and H_m on the material rigidity μ is clearly noticed in 2.9 and 2.10. Because, in solid materials, rigidity varies with depth[2] from the earth's surface, ω_m and H_m will accordingly vary.

3. INTERMEDIATE MICROSEISMS RANGE OF FREQUENCIES

Solutions 2.3 and 2.4 on the k-axis vanish separately when $k = 0$, but respectively diverge towards two different directions. There is evidently a distinct zone between them. This zone is called, intermediate frequencies zone.

As clearly shown by the analysed microseisms records, this zone centres at a period of about 8 seconds. In this range, there is no clearly defined relationship between the spectral amplitudes of the seismic modes and that of the associated water waves. It is suggested that the coastal reflection of the swell along the shoreline can give rise to the observed distortion in the frequency spectrum within the intermediate range of frequencies. If that is the case, the dependence of reflection coefficient on frequency must be determined. Consequently, if we take $\lambda(\omega)$ as the reflection coefficient along the shoreline, empirical relation gives the representation as follows:

$$\lambda(\omega)=\frac{\alpha_0}{2}-\beta_0\left(1-\frac{\omega_n}{\omega}\right)^2_0 \tag{3.1}$$

Here, ω is also the frequency of the incident water wave. α_0 is the beach slope and generally ranges from about 0.5 for very steep beach to negligible quantity for nearly horizontal bottom. The parameter β_0 ranges from 1/30 when the coastline is a very steep rock to nearly zero when the adjacent beach is approximately horizontal. Also, within the range of microseisms frequencies, $.5\omega_n$? ω ? $2.5\omega_n$, ω_n is the value of ω for which $\lambda(\omega) = \alpha_0/2$. Series of numerical calculations give $\omega_n = 0.185523$ c.p.s corresponding to a period of approximately 8 seconds and interestingly is the centre of the intermediate frequency range. Further, within the range of microseisms frequencies, 2.5 ? 10^{-2} ? $\lambda(\omega)$? 9.2 ? 10^{-2}, $\lambda(\omega) = 10^{-1}$ is the stationary value. In the case of negligible $\lambda(\omega)$ outside this range, β_0 will be correspondingly negligible. Thus 3.1 is still valid.

4. THE FREQUENCY SPECTRUM OF THE GENERATING SOURCE.

We consider the reflection of the beam of the shallow water waves incident on a shore line over which the adjoining beach has constant gradient α_0. Using Darbyshires' theory on the coastal reflection of sea waves, we take $\Delta\omega$ as the frequency difference between the incident and reflected beams. Divide the frequency band $\Delta\omega$ into n subdivisions each of width $\delta\omega_R$. Let $h_1(\omega, \theta)$, $h_2(\omega, \theta)$... $h_n(\omega, \theta)$ and $g_1(\omega,\theta)$, $g_2(\omega,\theta)$ ... $g_n(\omega,\theta)$ be the power associated with each subdivision with respect to the incident and reflected beams respectively. Further, the total power available for the excitation of the seismic modes in this process is given by:

$$\sum_i \sum_j h_i g_j \delta_{ij}$$

; where $\delta_{ij} = 0$ if $j \neq i$ and $\delta_{ij} = 1$ if $i = j$, $\delta_{ij} = 3$

$j \neq i$ represents the case in which the two components are out of phase, given rise to the destructive interference; on the other hand, $j = i$ represents the case of constructive interference.

Consequently,

$$S_a(\omega)\delta\omega = \begin{cases} \sum_j h_j g_j \delta\omega_R, & \text{if } j = i \\ 0, & \text{if } j \neq i \end{cases} \quad (4.1)$$

But $g_j = \lambda(\omega) h_j$ (4.2)

Thus, $Sa(\omega)\ \delta\omega = \lambda(\omega)\ \Sigma h_j^2\ \delta\omega_R$ and $Sa(\omega)$ is the amplitude spectral density. If $S_A(\omega)$ and $S_\omega(\omega)$ are the energy spectrum of the ground movements and the corresponding sea waves respectively, we have:

$$\frac{S_A(\omega)}{S_\omega(\omega)} = 0.32\left(\frac{C_\omega \rho_\omega S_a(\omega)\delta\omega}{C_m \rho d^{5/2}}\right)^2 H_m^2(\mu,\omega)\delta k \quad (4.3)$$

d = width of the breaker zone measured from the shoreline.

c_ω and ρ_ω are the phase velocity and density for the fluid layer. As before, $C_m(z)$, $\rho(z)$ are respectively, the group velocity and density for the solid seabed.

Computational result from 4.3. gives a ratio of 3.2×10^{-12} using a fixed period of 8 seconds. This is quite close to the previous calculations.[3,11].

5. CONCLUSIONS

In the previous models[3,4,11] of the ground movements forced by the water waves, the effects of layering were neglected. The implication of this seems to be that the elastic half space is assumed to be statistically homogeneous. Interestingly, this work strongly suggests the inherent dependence of the elastic wave quantities (such as the peak frequency, spectral width and height) on the elastic shear modulus. The latter, in turn varies with the depth of the elastic half space.

Consequently, it has been shown that the elastic shear modulus associated with the ground movements generally increases with the depth below the seabed until a depth of 2,885 km is reached[5]. However, in the case of the ground movement induced by an 8 record sea wave[14], we are interested in the maximum value of the material rigidity which occurs at a depth of 10m. It follows that the observed seismic record will depend on the distance below the earth's surface at which the seismogram is buried.

Usually, this dependence will be modified by the inherent exponential decay with depth associated with elastic surface waves. Studies in this direction are currently in progress.

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