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A SWITCHING FROM HANSEN' S METHOD TO SCULTZ - HERZBERGER'S METHOD FOR THE GUARRANTEED INCLUSION OF MATRIX INVERSION

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ABSTRACT

We herein discuss a switching from Hansen's method to Schultz Herberger's method for the numerical inversion of a square matrix in interval arithmetic. This method enhances the approach earlier used by the authors[10]. The resultant method leads to the popular Ostrowski's identity method of order five for the Scultz-Herzberger's method.

The analysis of efficiency index of this method gave rise to optimization problem in the set of integer, $r \in J_7$(J is the set of integers). The interval of the best combined method of this algorithm is also discussed. Numerical example is also given.

Key Words: Matrix inversion, Hansen's method, Scultz- Herzberger's method, efficiency index.

1. INTRODUCTION

This paper is concerned with the problems of bounding inverse of interval matrices. The concept of an interval in mathematics is a natural extension of that of a scalar. A scalar value is a discrete point, where as an interval in a continuum defined by two scalar end values. We may regard an interval type as being derived from a scalar type, in which case we refer to the scalar type as the base type.

Error analysis can be performed by implementing interval arithmetic [3, 8 and 9] where each real number X is replaced by a pair of floating point number (a, b) defining a sufficiently narrow interval such that $X \in [a.b]$.
Let OP denote a binary, real arithmetic operation. Let the set g of real interval numbers be defined by:

$$g = \{[s,t] | s \leq t \text{ are real numbers}]\}$$

If A, B $\in$ g then

$$A \text{ OP } B = \{z | z = s \text{ o p } t \text{ for some } S \in A \text{ and } t \in B\}.$$

If A = [v, w]and B = [x, y], then it can be shown [6] that A OP B yields.

$$A + B = [v + x, w + y] \quad (1.1)$$

$$A - B = [v - y, w - x] \quad (1.2)$$

$A*B=[\min(v*x, v*y, w*x, w*y), \max(v*x, v*y, w*x, w*y)]$ (1.3)

$A/B = [\min(v/x, v/y, w/x, w/y), (\max(v/x, v/y, w/x, w/y)]$ (1.4)

The operations +, -, * are defined for all interval numbers, however, the operation/is undefined if $O \in B$. It is easily verified that g is closed under these operations. The operations * and / can be simplified computationally by examining the signs of the end points. It is hereby remarked that the usual implementation of [6] replaces +, -, * and / by computer interval arithmetic, computers result from - (1.4), and then if necessary rounds the right and left endpoints up and down so that the final result is guaranteed to contain the real arithmetic result. Thus computer interval arithmetic is a finite - precision interval arithmetic defined on a set of such numbers with the additional features of Optimal rounding and magnitude violation indicator. For this reason, the main heart of discussion in this paper is primarily on iterative methods which use computer interval arithmetic for bounding errors in matrix inversion. The advantage of our method lies purely on the fact that any non vanishing square matrix A can always be inverted with guaranteed error bounds.

The remaining sections in this paper are arranged as follows:

In section two, inclusion method of Schulz, popularly known as Hyperpower method is discussed. In section three, we derived the algorithm for the initial inclusion of the inversion of the matrix $A \in R^n \times R^n$ using the technique of Hansen's method [2]. The much celebrated algorithm of Schulz – Herzberger's method and convergence analysis are discussed in section four. The efficiency index of our combined method which endorses Ostrowski's identity of order five is studied. The analysis of this hybrid method leads to optimisation problem in the set of integer numbers, $r \in J_7$. The best combined floating point method with interval Schulz – Herzberger's method is also derived. We also discussed our method in the light of numerical evidence.

2. THE INTERVAL SCHULZ'S METHOD

Following the idea in [1], an iterative method for inverting an arbitrary bounded operator in a Hilbert space is derived. This method is called a hyperpower method. Let $p > 1$ be a fixed natural number, let A be a given non-vanishing point matrix and $x^{(o)}$ be initial matrix whose procedure of obtaining will be discussed in section three, such that the Frobenius matrix norm$\|I - A x^{(0)}\| < 1$, holds where I is a unit $R^n \times R^n$ matrix.

The generalized iterative method of Schulz of order p for inverting square matrix A is given by the recurrence relation:

$$X^{(k+1)} = x^{(k)} \sum_{r=0}^{p-1} (I - AX^{(k)})^r \qquad k = 0,1 \qquad (2.1)$$

Denote $B = I - AX^{(k)}$, k = o, 1,

Then $$\sum_{r=0}^{p-1} (I - AX^k)^r = I + B + B^2 + \ldots + B^{p-1}$$

So that we now rewrite equation (2.1) in the form

$x^{(k+1)} = x^{(k)}(I + B + B^2 + \ldots + B^{p-1})$

For example, for, p = 2 we have the expression given by Altman [1]:

$$X^{(k+1)} = X^{(k)} (2I - Ax^{(k)}) \qquad (2.3)$$

It has been shown in [1] that the highest efficiency index is when p = 3.

In [8], a method for the initial inclusion of $x^{(o)}$ is given in the form

$$a = \frac{1}{1 - \|I - AX\|}$$

Where || .|| denotes any of the Holder's matrix norm. with this the inclusion matrix takes the form

$$x_{i,j}^{(0)} = \begin{cases} [-a, a], & i \neq j \\ [-a, 2 + a], & i = j \end{cases}$$

However, it is our aim in this research paper to provide an alternative approach to the provision of initial values for $x^{(o)}$. This has been treated ahead in section three. In the sequel,

Let $M(aii) = 0 = \left[\frac{s+t}{2}\right]$

Then the recurrence relation holds true for

$$A^{-1} \in M(x) \sum_{r=0}^{p-2} (I - AM(x) + A^{-1}(I - AM(x))^{p-1}$$

$$\in M(x) \sum_{r=0}^{p-2} (I - AM(x)^r \mathrm{x} (I - AM(x))^{p-1}$$

Thus, the iterative interval version of the relation (2.5) is given by

$$x^{(k+1)} = M(x^{(k)}) \sum_{r=0}^{p-2} (I - AM(x^{(k)})^r + x^{(k)} (I - AM(x^{(k)})^{p-1} \qquad (2.6)$$

(k = 0,1,......,$x^{(o)}$ contains A^{-1})

3. INITIAL INCLUSION OF INVERSE MATRIX:

In this section we shall provide our starting value for the approximate inverse of the matrix $A \in R^n \times R^n$. Following closely the idea in [2], we shall first of all consider the case of 3 x 3 matrix as

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

We set our system as

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= 1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= 0 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= 0 \end{aligned} \tag{3.1}$$

With $a_{11} \neq 0$, then

$$\det(A) = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$= a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31}$$

Therefore one step of Crammer's rule applied to (3.1) yields.

$$\begin{aligned} x_1^{(1)} &= (a_{22}a_{33} - a_{23}a_{32}) / \det(A) \\ x_2^{(1)} &= -(a_{21}a_{33} - a_{23}a_{31}) / \det(A) \\ x_3^{(1)} &= (a_{21}a_{32} - a_{22}a_{31}) / \det(A) \end{aligned} \tag{3.2}$$

Hence

$$\begin{bmatrix} x_1^{(1)} \\ x_2^{(1)} \\ x_3^{(1)} \end{bmatrix} = \begin{bmatrix} (A^{-1})_{11} & (A^{-1})_{12} & (A^{-1})_{13} \\ (A^{-1})_{21} & (A^{-1})_{22} & (A^{-1})_{23} \\ (A^{-1})_{31} & (A^{-1})_{32} & (A^{-1})_{33} \end{bmatrix} = \begin{bmatrix} (A^{-1})_{11} \\ (A^{-1})_{21} \\ (A^{-1})_{31} \end{bmatrix}$$

In the same way, we solve

$$\begin{aligned} a_{11}x_1^{(2)} + a_{12}x_2^{(2)} + a_{13}x_3^{(2)} &= 0 \\ a_{21}x_1^{(2)} + a_{22}x_2^{(2)} + a_{23}x_3^{(2)} &= 1 \\ a_{31}x_1^{(2)} + a_{32}x_2^{(2)} + a_{33}x_3^{(2)} &= 0 \end{aligned}$$

Giving

$$x_1^{(2)} = \left(-a_{12}a_{33} + a_{13}a_{32}\right)/\det(A)$$
$$x_2^{(2)} = \left(a_{11}a_{33} - a_{13}a_{31}\right)/\det(A)$$
$$x_3^{(2)} = \left(-a_{11}a_{32} + a_{12}a_{31}\right)/\det(A)$$

Thus,

$$\begin{bmatrix} x_1^{(2)} \\ x_2^{(2)} \\ x_3^{(2)} \end{bmatrix} = \begin{bmatrix} (A^{-1})_{11} & (A^{-1})_{12} & (A^{-1})_{13} \\ (A^{-1})_{21} & (A^{-1})_{22} & (A^{-1})_{23} \\ (A^{-1})_{31} & (A^{-1})_{32} & (A^{-1})_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} (A^{-1})_{12} \\ (A^{-1})_{22} \\ (A^{-1})_{32} \end{bmatrix}$$

Also if we solve

$$a_{11}x_1^{(3)} + a_{12}x_2^{(3)} + a_{13}x_3^{(3)} = 0$$
$$a_{21}x_1^{(3)} + a_{22}x_2^{(3)} + a_{23}x_3^{(3)} = 0$$
$$a_{31}x_1^{(3)} + a_{32}x_2^{(3)} + a_{33}x_3^{(3)} = 1$$

We have

$$\begin{bmatrix} x_1^{(3)} \\ x_2^{(3)} \\ x_3^{(3)} \end{bmatrix} = \begin{bmatrix} (A^{-1})_{11} & (A^{-1})_{12} & (A^{-1})_{13} \\ (A^{-1})_{21} & (A^{-1})_{22} & (A^{-1})_{23} \\ (A^{-1})_{31} & (A^{-1})_{32} & (A^{-1})_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} (A^{-1})_{13} \\ (A^{-1})_{23} \\ (A^{-1})_{33} \end{bmatrix} \tag{3.5}$$

Collectively the set of interval solution of x is

$$\begin{bmatrix} (A^{-1})_{11} & (A^{-1})_{12} & (A^{-1})_{13} \\ (A^{-1})_{21} & (A^{-1})_{22} & (A^{-1})_{23} \\ (A^{-1})_{31} & (A^{-1})_{32} & (A^{-1})_{33} \end{bmatrix} \tag{3.6}$$

Thus for a linear system of order n, we shall have the following resultant interval matrix inversion.

$$\begin{bmatrix} (A^{-1})_{11} & (A^{-1})_{12} & \cdots & (A^{-1})_{1n} \\ (A^{-1})_{21} & (A^{-1})_{22} & \cdots & (A^{-1})_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots \\ (A^{-1})_{n1} & (A^{-1})_{n2} & \cdots & (A^{-1})_{nn} \end{bmatrix} \tag{3.7}$$

With interval solution X as

$$\begin{bmatrix} x_1^{(1)} & x_1^{(2)} & \cdots & x_1^{(n)} \\ x_2^{(1)} & x_2^{(2)} & \cdots & x_2^{(n)} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ x_n^{(1)} & x_n^{(2)} & \cdots & x_n^{(n)} \end{bmatrix} \tag{3.8}$$

4. THE SCHULZ – HERZBERGER'S METHOD

The monotonicity of $x^{(1)} \subset x^{(o)}$ has earlier been proved by J.W. Schmidt in 1967; which is a necessary ingredient for the convergence of Schulz's method.

$$x^{(k+1)} = m(x^{(k)}) + x^{(k)}(I - Am(x^{(k)})) \tag{4.1}$$

In [4], the necessary and sufficient condition for the monotonicity of method (2.6) was proved. We herein generalized the Herzberger's method for the iterative method of (4.1) by considering theorem 1 given below.

THEROEM 1:[8]

Let A be a non – singular n x n matrix and $X^{(o)}$ an n x n interval matrix for which $A^{-1} \in x^{(o)}$. If the sequence of matrices $\{x^{(k)}\}$ is produced by method (2,6), then

(a) each matrix $x^{(k)}$, $k \geq 0$ contain A^{-1}

(b) if the inequality $(I - AX1) < 1$ is satisfied for all $X \in X^{(o)}$, then the sequence $\{X^{(k)}\}$ EA $^{-1}$

(c) the $\{X^{(k)}\}$is bounded as follows $|| d(x^{(k+1)}) || \leq \alpha || d(X^{(k)}) ||^p$,

$\alpha \geq 0$ that is, the order of convergence of the iterative process (2.6) is at least P.

THEOREM 2: [8]

Let $A^{-1} \in X^{(o)}$ and $(I - Am(X^{(O)}) < 1$

Then the generalized interval method of (2.6) converges to A^{-1}, where $A^{-1} \in X^{(k)}$, $k = 0, 1, \ldots$ and if

$$2| m(X^{(0)}) (I - Am(X^{(0)}) | \leq d(X^{(0)}) (I - AM(X^{(0)}) | \tag{4.2}$$

holds, then the method of equation (2.6) is monotone.

The proof of theorems 1 and 2 can be found in [8] and the cited references therein.

5. **The Ostrowski's identity for the Schulz's method of order 5.**
In [9], the Schulz – type method of (2.6) has been rearranged as follows:

$$\Phi_5(X^{(k)},A)=x^{(k)}\left[I+\frac{5^{\frac{1}{2}}+1}{2}(I-Ax^{(k)})+(I-AX^{(k)})^2\right]*$$

$$\left[I-\frac{5^{\frac{1}{2}}-1}{2}(I-Ax^{(k)})+(I-AX^{(k)})^2\right]$$

$$=X^{(k)}\sum_{r=0}^{p-1}(I-AX^{(k)})^r$$

The combined method of (5.1) has the form

$$X^{(v+1)}=x^{(v)}\left[I+\frac{5^{\frac{1}{2}}+1}{2}(I-Ax^{(v)})+(I-AX^{(v)})^2\right]*$$

$$\left[I-\frac{5^{\frac{1}{2}}-1}{2}(I-Ax^{(v)})+(I-AX^{(v)})^2\right] \qquad (5.2)$$

(v = 0,1,...(In floating point arithmetic)

$$(X^{(1,k)}) = X^{(0)}(I - AX^{(k)}) + X^{(k)}(I - AX^{(k)}) + X^{(k)} \qquad (5.3)$$

(interval arithmetic)

In this paper our aim is to discuss the efficiency of the combined floating point arithmetic of (5.2) and the interval arithmetic of (5.3).

In line with Traub [10] we define the efficiency index to be $q^{1/\theta}$ where θ is the amount of work for one iteration step and q is an integer. In (5.2) and (5.3) we measure θ in terms of only matrix multiplication and all other computation costs are negligible compared with these. Counting computational cost this way, the following implication holds that one interval matrix multiplication costs at least about two times as much as a point matrix multiplication.

kp = multiplications for the applications of Schulz – type method, where p is the number of multiplications for the evaluation of θ, r + 1 = interval matrix multiplication for the Horner – Scheme interval evaluation or approximately 2(r + 1) point matrix multiplication for one step of equation (5.2) and (5.3) where all occurring multiplications are treated equally.

The lower bound of the combined method (5.2) and (5.3) is given by

$$\theta = \begin{bmatrix} 4k+2(r+1), & p=5 \\ pk+2(r+1), & p \neq 5 \end{bmatrix}$$

where θ was earlier defined in the sense of Traub's computational efficiency index.

In this case, we write,

$$e(p,r,k) = \begin{pmatrix} (r5^k+1)^{\frac{1}{4k+2r+2}}, & p=5 \\ (rp^k+1)^{\frac{1}{pk+2r+2}}, & p \neq 5 \end{pmatrix}$$

The optimal values of p and r could be obtained in the following setting which leads to the set of integer optimisation problem. In the spirit of this analysis we state the following relevant Lemmas and theorems.

Lemma 1

If $r \in J_7$, J being set of integers

then

$$\ln(6r+1) - 1 > r + 1/3 \quad (5.6)$$

$$\ln r - (r+1)\ln(6/5) > \ln(36/25) \quad (5.7)$$

$$\ln(5r+1)/(2r+6) > \ln(3r+1)/(2r+5) \quad (5.8)$$

Lemma 2

Let $k \geq 1$ and $r \in J_7$.

Then a function p - > f(p) defined by

$$f(p) = (rp^k+1)^{\frac{1}{pk+2r+2}} \quad (5.9)$$

is monotonically decreasing for $p \geq 6$

THEOREM 3

Let $r \in J_7$ and let $k \geq 1$ and $p(p \geq 2$ and $p \neq 5)$ be arbitrary integers. Then $e(5, r, k) > e(p,r,k)$

Proof of lemma 1 (see [8])

Proof of lemma 2

Using logarithmic derivative of f we find,

$$f^1(p)/f(p) = kg(p)/[(rp^k+1)(pk+2r+2)^2] \quad (5.10)$$

where

$$g(p) = rp^{k-1}(pk+2r+2) - (rp^k+1)\ln(rp^k+1) \quad (5.11)$$

We prove the necessity of lemma 2 if we can show that $g(p) < 0$ for $P \geq 6$, that is $rp^{k-1}(pk+2r+2) < (rp^k+1)$, for $p \geq 6$ (5.12)

Instead of using equation (5.12), it is enough to show the inequality,

$$rp^{k-1}(pk+2r+2) < rp^k \ln(rp^k+1) \quad (5.13)$$

which reduces to

$$2r+2 < p[\ln(rp^k+1) - k] \quad (5.14)$$

The function $z(p.k) = p[\ln(rp^k + 1) - k]$ appearing in equation (5.14) is monotonically increasing with respect to p as well as to k ($p \geq 6$, $k \geq 1$). Therefore, $z(p,k) \geq z(6,1)$ and hence, the inequality (5.14) will be valid if,

$2r + 2 < z(6,1) = 6\{\ln(6r + 1) - 1\}$ holds. Thus the desired result conforms with equation (5.6)

Proof of Theorem 3

Considering the definition of efficiency index of equation (5.5), we have to prove that

$$(r5^k + 1)^{\frac{1}{4k+2r+k}} > (rp^k + 1)^{\frac{1}{pk+2r+2}} \quad (5.15)$$

under the conditions given in the above theorem 3. To show this, two cases are to be considered,

(1) $p \geq 6$ and (2) $p = 2, 3, 4$.

Case (1) $P \geq 6$

By Lemma 2, it follows that

$$\underset{p\geq 6}{Max} f(p) = f(6) = (r6^k + 1)^{\frac{1}{6k+2r+2}}$$

so that equation (5.15) holds for all $p \geq 6$ if,

$$(r5^k + 1)^{\frac{1}{4k+2r+2}} > (r6^k + 1)^{\frac{1}{6k+2r+2}}$$

is true.

The last inequality can be written in the form.

$$\frac{\ln(r5^k t_1(r,k))}{2k+r+1} = \frac{\ln(r5^k + 1)}{2k+r+1} > \frac{\ln(r6^k + 1)}{3k+r+1} = \frac{\ln(r6^k t_2(r,k))}{3k+r+1}$$

where $t_1(r, k) = 1 + 1/(r5^k)$ and

$t_2(r, k) = 1 + 1/(r6^k)$

Since $t_1(r, k) > t_2(r, k)$ for fixed r and k, to prove equation (5.16), it suffices to show that,

$$\frac{\ln(r5^k t_1(r,k))}{2k+r+1} > \frac{\ln(r6^k t_1(r,k))}{(3k+r+1)}$$

factorization yields.

$$\ln t_1(r,k) + \ln r - (r+1)\ln\frac{6}{5} > k\ln\frac{36}{25}$$

The inequality (5.17) can be proved taking into cognisance that $\ln t_1(r, k) > 0$ and (5.8) holds the maximum of the right hand side of (5.17) attained for $k = 1$.

Case 2: ($p = 2, 3, 4$.)

Observe that (5.15) is trivial when $p = 4$. For $p = 3$ (5.15) becomes

$$\frac{\ln(r5^k+1)}{4k+2r+2} > \frac{\ln(r3^k+1)}{3k+2r+2} \tag{5.18}$$

By setting k=1, method (5.18) reduces to (5.8). Thus, we assume that $k \geq 2$, since,

$\ln(r5^k + 1) > \ln(r5^k)$ and

$\ln(r3^k + 1) \leq \ln(10/9r3^k)$

Our inequality (5.18) will hold if,

$$\frac{\ln(r5^k)}{4k+2r+2} > \frac{\ln(\frac{10}{9}r3^k)}{3k+2r+2} \tag{5.19}$$

is satisfied. The inequality (5.19) can be written in the form.

$$k^2 \ln\frac{125}{81} + k[(r+1)\ln\frac{25}{9} - \ln r - 4\ln\frac{10}{9}] - (r+1)\ln\frac{100}{81} > 0 \tag{5.20}$$

The left hand side of (5.20) may be regarded as a function of k say s(k). It is enough to prove that s(k)>0 for $k \geq 2$. Differentiation of (5.20) w. r. t. k, we have.

$S^1(k) = 2k \ln 125/81 + (r+1)\ln 25/9 - \ln r - 4\ln 10/9$ and conclude that $s^1(k) > 0$ for $k \geq 2$and $2 \geq 1$. Thus, the function s(k) is monotonically increasing ↑ for $k \geq 2$. We also have that

$S(2) = 4\ln 5/4 - 2\ln r + (r+1)\ln 25/4 > 0$

For $r \geq 1$ so that s(k)>0 for k >2 which implies that inequality (5.20) holds. Thus the inequality (5.15) for p=3 is also valid. The case when p =2 can be proved similarly, as in the case of p=3. This completes the theorem 3.

Remark: It can be shown that the best optimal choice of the value of r in the combined Schultz-Herzberger's method which endorses Ostrowski's identity of order 5 lies in the interval [1,2]. The details of this can be found in current Ph.D thesis of the author which is in progress.

6. Numerical Examples.

We consider a matrix $A \in R^2 \times R^2$ given by

$$A = \begin{bmatrix} 3 & 1 \\ 3 & 2 \end{bmatrix}$$

$X^{(0)}$ to A^{-1}

$$X^{(0)} = \begin{bmatrix} (0.662, & 0.672), & (-0.338, & -0.332) \\ (-1.01, & -0.999), & (0.99, & 1.011) \end{bmatrix}$$

Using equation (2.6) and computing

$$M(X^{(0)}) = \begin{bmatrix} 0.667, & -0.335 \\ -1.0045, & 1.0005 \end{bmatrix}$$

$$AM(X^{(0)}) = \begin{bmatrix} 0.9965, & 0 \\ -0.008, & -005 \end{bmatrix}$$

and

$$X^{(0)}(I - AM(X^{(0)}) = \begin{bmatrix} (0.662, & 0.672), & (-0.338, & -0332) \\ (-1.01, & -0.9997), & (0.99, & 1.011) \end{bmatrix} \begin{bmatrix} 0.0035, & \\ 0.008 & -0. \end{bmatrix}$$

$$= \begin{bmatrix} (-0.000387, & -0.000304), & (0.00169, & 0.00166) \\ (0.004385, & 0.0045915), & (-0.00495, & -0.0050-55) \end{bmatrix}$$

compute

$$x^{(1)} = M(X^{(0)}) + x^{(0)}(I - AM(x^{(0)}) =$$

$$\begin{bmatrix} (0.666613, & 0.666696), & (-0.33331, & -0.33334) \\ -(-1.000115, & 0.9999085) & (0.99555, & 0.995445) \end{bmatrix}$$

Next

$$M(X^{(1)}) = \begin{bmatrix} 0.6666545, & -\ 0.333325 \\ -1.00001175, & 0.9954975 \end{bmatrix}$$

The iteration is stopped at the third iteration having obtained desired value and the completed value of $X^{(3)}$ is

$$0X^{(3)} = \begin{bmatrix} (0.66667029, & 0.666667029), & (-0.333387229, & -033338722) \\ (-1.000000352 & -1.000000352) & (1.00008084, & 1.0008084) \end{bmatrix}$$

7. CONCLUSION

We have been able to experiment Hansen's method with the algorithm of Schultz Herberger's method. This was tested with a simple matrix inversion using computer interval arithmetic. Further experimentation may be extended to Schultz-Herzber's method which endorses the Ostrowski's identity of order 5. The resultant effect leads to the integer optimization problem in the set of integers when taking into consideration the effects of efficiency index in the sense of Traub's computational complexity [10]. The advantage of our method is that it may be easier to provide initial inclusion $X^{(o)}$ with Hansen's method.

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