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N = 1 SUPERSYMMETRY WITH A TENSOR BOSON

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ABSTRACT

The supersymmentry transformation for the members of the triplet $\{A_{\mu\nu\rho}(X), (X), \psi(X)\}$
Consisting of an antisymmentric tensor, $A_{\mu\nu}(X) = \sigma_{\mu\nu} A(x)$, the pseudoscalar particle B(x) and the Majorana spinor $\psi(x)$, are constructed. The generators of these transformations are subsequently found.

The first field theoretical realization of supersymmetry was the Wess-Sumino model [1], [2], in which the triplet consists of a scalar field A(x), a pseudoscalar field B(x), and a spinor field $\psi(x)$. Here the transformations which change bosons into fermions etc are [3] (with ε an x-independent infinitestimal Grassmann parameter and $\bar{\varepsilon} = \varepsilon^{+}\gamma^{o}$

$$\delta A(x) = \varepsilon\,\psi(x) \tag{1.1a}$$

$$\delta B(x) = \bar{\varepsilon}\gamma^{5}\psi(x) \tag{1.1b}$$

$$\delta 4 = -(i\partial + m)(A - i\gamma^{5}B)\varepsilon \tag{1.1c}$$

$$\delta\bar{\psi} = -\bar{\varepsilon}(A - i\gamma^{5}B)\varepsilon\,(A - i\gamma^{5}B)\left(i\overleftarrow{\partial} - m\right) \tag{1.1d}$$

Let ϕ_i be any of the field in the above triplet, then from the definition of the supercurrent

$$j^{\mu} = V^{\mu} - \sum_{i}\frac{\partial}{\partial\partial\mu\phi_{i}}\,\delta\phi_{i} \tag{1.2}$$

where

$$V^{\mu} = \bar{\varepsilon}[\partial^{\mu}(A - i\gamma^{5}B)]\psi$$

The conserved Majorana spinor current K^{μ} is defined by

$$j^{\rho} = \frac{1}{\lambda}\varepsilon k^{\rho}$$

where λ is a constant such that the spinor charge is defined by

$$Q_a = -\int d^{3}x k_{a}^{o}$$

Where

$$K_a^o = = \frac{i}{2}\lambda\left[[(A - i\gamma^5 B)(\overleftarrow{\partial} \leftarrow -m]\gamma^o\psi\right]_a$$

Let u $((\vec{p},s), v(\vec{p},s)$ (s = 1, 2) be the solutions of the Dirac Equations corresponding to $p_o = \pm\,\omega_p = \pm\,(p^2 + m^2)^{1/2}$ then

$$u(\vec{p},s) = N(\vec{p})(\not{p} + m)\,u(O,S) = ½\,\frac{(p+m)u(O,S)}{(2m(m+\omega_p))^{1/2}} \quad (1.3a)$$

and $$v(\vec{p},s) = (-\not{p}+m)N(\vec{p})v(0,s) = \frac{(-\not{p}+m)v(0,s)}{(2m(m+\omega_p))^{1/2}} \quad (1.3b)$$

The requirement that the Dirac spinor be a Majorana spinor i.e that

$$\psi(x) = \psi^c(x) = C[\Psi(x)]^T$$

where $\psi^c(x)$ is the charge-conjugate solution, imposes the following conditions on the spinors

$$u^c(\vec{p},s) = v(\vec{p},s)$$

$$v^c(\vec{p},s) = u(\vec{p},s)$$

since

$$b(\vec{p},s) = d(\vec{p},s)$$

and

$$b^+(\vec{p},s) = d^+(\vec{p},s)$$

the Majorana spinor has the following Fourier decompositions:

$$\psi^M(x) = \frac{1}{(2\pi)^{\frac{3}{2}}}\sum_s\int d^3p\left(\frac{m}{\varpi_p}\right)^{1/2}[d(\vec{p},s)u(\vec{p},s)e^{-ip.x} + d^+(\vec{p},s)v(\vec{p},s)e^{ik.x}] \quad (1.4)$$

By using the Fourier decomposition of A(x) and B(x), the spinor Q_a is given by

$$Q_a = i\lambda\,(m/2)^{1/2}\sum_s\int d^3p[C(\vec{p})d^+(\vec{p},s)v(\vec{p},s) - D(\vec{p},s)u(\vec{p},s)]_a \quad (1.5)$$

where

$$C(\vec{p}) = a(\vec{p})1 - i\gamma^5 b(\vec{p}) \quad (2.\ 1a)$$

$$D(\vec{p}) = a^+(\vec{p})1 - \gamma^5 b^+(\vec{p}) \quad (2.\ 1b)$$

Let us define the projection operators by

$$\Delta_{\pm} = \frac{\pm p + m}{2m}$$

Then the following completeness relations hold

$$\Lambda_{+ab}(\vec{p}) = \sum_j u_a(\vec{p}, s) \otimes u_b(\vec{p}, s)$$

$$\Lambda_{-ab}(\vec{p}) = \sum_j v_a(\vec{p}, s) \otimes v_b(\vec{p}, s)$$

THE SUPERSYMMETRY TRANSFORMATION

We shall now consider the N = 1 supersymmetry model with the multiplet consisting of a spin ½ particle represented by the Majorana spinor ψ_a (a = 1,...,4) satisfying

$\partial\psi$ = 0, an odd parity spin zero particle represented by the pseudoscalar field B(x) statisfying $\Box B$ = o, and a spin zero tensor boson $A_{\mu\nu}$ = $-A_{\nu\mu}$ behaving under the guage transformation as

$$\delta_{A_{\mu\nu}} = \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu$$

The field strength tensor corresponding to $A_{\mu\nu}$

$$F_{\mu\nu\rho} = \partial_{\mu\rho} A_{\nu\rho} + \partial_\rho A_{\mu\nu} + \partial_\nu A_{\rho\mu}$$

Clearly this is antisymmetric in the three (free) indices and satisfies

$$\partial^\mu F_{\mu\nu\rho} = 0$$

We may note that tensor mesons occur as intermediate states in the reactions of the corresponding pseudoscalar particles [4]

Our purpose in this section is to derive the on-shell supersymmetry transformations. After the supersymmetry transformations have been derived, in the next section appropriate spinor charges Q_a will be written down which will be shown to be generators of the supersymmetry transformations.

We now use the requirements of linearity of the transformation, dimension, Lorentz invariance and parity to determine the supersymmetry transformations of the fields. The transformation for B is the same as that for the same field in the Wess-Zumino model comprising the triplet [ψ_a, A, B. In natural units (i.e $\hbar$ = c = 1), the (length) dimension for the boson is −1, while that of the boson is +1 while that of the ferminion is +3/2. The supersymmetry transformation of B(x) given by Eq. (1.1b) remains unchanged; and it is clear that ε must have the (length) dimension + ½. Thus

$$\delta B(x) = -i\bar{\varepsilon}\gamma_5\psi \qquad (2.1)$$

On dimensional grounds and taking into account Lorentz invariance, the supersymmetry transformations of $A_{\mu\nu}(x)$ and $\psi(x)$ are

$$\delta A_{\mu\nu} = \bar{\varepsilon}\,\sigma_{\mu\nu}\psi \qquad (2.2)$$

$$\delta\psi = [\alpha(\gamma^{\mu}\sigma^{\nu\rho}\partial_{\mu}A_{\nu\rho} + \gamma^{\rho}\sigma^{\mu\nu}\partial_{\rho}A_{\mu\nu} + \gamma^{\nu}\sigma^{\rho\nu}\partial_{\nu}A_{\rho\mu}) + \beta\gamma^{\mu}\partial_{\nu}A_{\mu}{}^{\nu} + \alpha'\,\partial\!\!\!/\gamma^{5}B]\varepsilon \qquad (2.3)$$

Here α, α' and β are parameters, the determination of whose values requires that the supersymmetry algebras, Equs. (2.1) to (2.3.), and the operation of gauge transformation should form a closed. Let us now choose $A_{\mu\nu}$ (x) to be of the following simple form

$$A_{\mu\nu}(x) = \sigma_{\mu\nu}A(x)$$

with this choice

$$F_{\mu\nu\rho} = \sigma_{\mu\nu}\partial_{\rho}A(x) + \sigma_{\rho\nu}\partial_{\rho}A(x) + \sigma_{\nu\rho}\partial_{\mu}A(x)$$

Moreover the gauge transformation

$$\partial_{\Lambda}F_{\mu\nu\rho} = 0$$

Hence, bearing in mind the gauge transformations

$$\delta_{\Lambda}\psi(x) = 0 = \partial_{\Lambda}B(x)$$

$$[\delta_{\Lambda}, \delta_{\varepsilon}]\psi = \delta\Lambda\delta_{\varepsilon}\psi(x)$$

$$= \delta_{\Lambda}\left[\begin{array}{l}\alpha(\gamma_{\mu}\sigma^{\nu\rho}\partial_{\mu}A_{\nu\rho} + \gamma^{\rho}\sigma^{\mu\upsilon}\partial_{\rho}A_{\mu\nu} + \gamma^{\nu}\sigma^{\rho\nu}\partial_{\nu}A_{\rho\mu}) \\ \qquad + \beta\gamma^{\mu}\partial_{\nu}A_{\mu}{}^{\nu} + \alpha\partial\!\!\!/\gamma^{5}B\end{array}\right]\varepsilon$$

$$\beta\gamma^{\mu}(\partial_{\nu}\delta_{\nu}\partial^{\upsilon}\Lambda_{\mu}\partial_{\nu}\partial_{\mu}\Lambda^{\nu})$$

β must vanish because the term on the right is not part of the supersymmetry algebra. Hence ;

$$\delta_{\varepsilon}\psi = \alpha(\gamma^{\mu}\sigma^{\nu\rho}\partial_{\rho}A_{\nu\rho} + \gamma^{\rho}\sigma^{\rho\nu}\partial_{\rho}A_{\mu\nu} + \gamma^{\nu}\sigma^{\rho\mu}\partial_{\nu}A_{\rho\mu})\varepsilon - \partial\!\!\!/\gamma^{5}B\varepsilon$$

with the choice $\alpha = -i, \quad \alpha' = -1$

$$\delta\Psi = -i(\gamma^{\mu}\sigma^{\nu\rho}\partial_{\mu}A_{\nu\rho} + \gamma^{\rho}\sigma^{\mu\nu}\partial_{\rho}A_{\mu\nu} + \gamma^{\nu}\sigma^{\rho\mu}\partial_{\nu}A_{\rho\mu})\varepsilon - \partial\gamma^{5}B\varepsilon \qquad (2.3a)$$

THE INFINITESIMAL SUPERSYMMETRY TRANSFORMATIONS AND THEIR GENERATORS

We shall now find the generators of the infinitesimal supersymmetry transformations (2.1) , (2.2) and (2.3a). The generator of B(x) remains unchanged [3], [5], [6].

$$-i[\bar{\varepsilon}Q, B(x)] = -i\bar{\varepsilon}_a[Q_a, B(x)] = -i\bar{\varepsilon}\gamma^5\psi = \delta B(x)] \qquad (2.4)$$

We shall now show by making use of the fourier expansions of the spinor changes, Q_a and of the fields $A_{\mu\nu}$ and $\psi(x)$ that

$$\delta A_{\mu\nu}(x) = -i[\bar{\varepsilon}Q, A_{\mu\nu}(x)]$$

In fact,

$$-i[\varepsilon Q, A_{\mu\nu}(x)] = -i\bar{\varepsilon}_a[Q_a, A_{\mu\nu}(x)] = -i\bar{\varepsilon}_a[Q_a, \sigma_{\mu\nu}A(x)]$$

(with λ = 2 in Eq .1.5)

$$= \bar{\varepsilon}_a\sum_s(2\pi)^{-3/2}\int d^3p\int d^3k\left(\frac{m}{\omega_k}\right)^{1/2} X$$

$$\{ [C_{ab}(\vec{p}), \sigma_{\mu\nu}a^+(\vec{k})]d^+(\vec{p},s)v_b(\vec{p},s)e^{-ikX}$$

$$+[C_{ab}(\vec{p}), \sigma_{\mu\nu}a^+(\vec{k})]\,d^+(\vec{p},s)v_b(\vec{p},s)e^{ik.X}$$

$$-[D_{ab}(\vec{p}), \sigma_{\mu\nu}a(\vec{k})]\,d(\vec{p},s)u_b(\vec{p},s)e^{-ik.X}$$

$$-D_{ab}(\vec{p}), \sigma_{\mu\nu}a^+(\vec{k})]d(\vec{p},s)u_b(\vec{p},s)e^{ik.X}\} \qquad (2.5)$$

Since

$$C_{ab}(\vec{P}), \sigma_{uv}\ a(\vec{k})] = o$$

and

$$[C_{ab}(\vec{p}), \sigma_{\mu\nu}a(\vec{k})] = \delta(\vec{p}-\vec{k})\delta_{ab}\sigma_{\mu\nu}\mu\nu$$

$$[D_{ab}(\vec{p}), \sigma_{\mu\nu}a(\vec{k})\ ] = -\sigma_{\mu\nu}\delta(\vec{p}-\vec{k})\delta_{ab}$$

$$[D_{ab}(\vec{p}), \sigma_{\mu\nu}a^+(\vec{k})\ ] = o$$

We have, writing the third term in Eq (2.5) first

$$-i[\bar{\varepsilon}Q, A_{\mu\nu}\ ] = \bar{\epsilon}_a\sigma_{\mu\nu}\sum_S(2\pi)^{-3/2}\int d^3p\left(\frac{m}{\omega_p}\right)^{1/2}$$

$$\left[d\left(\vec{p},s\right)u_a\left(\vec{p},s\right)e^{-ip.X}+d^+\left(\vec{p},s\right)v_a\left(\vec{p},s\right)e^{ik.X}\right.$$

$$=\bar{\varepsilon}_a\sigma_{\mu\nu}\Psi_a(X)=\bar{\varepsilon}\sigma_{\mu\nu}\Psi(X)=\delta A_{\mu\nu}(\mathrm{x})$$

Which was to shown

To find the generator of infinitestimal supersymmetry transformation (2.3a) let us write

$$\delta\Psi(\mathrm{X})=-i\left(\gamma^{\mu}\sigma^{\upsilon\rho}\partial_{\mu}\sigma_{\nu\rho}A(X)+\gamma^{p}\sigma^{\mu\nu}\partial_{p}\sigma_{\mu\nu}A(X)+\gamma^{\nu}\sigma^{P\mu}\partial_{p\mu}A(X)\right)\varepsilon \qquad (2.3)$$

$$-\partial\!\!\!/\gamma^5 B\in\equiv[\delta\Psi]_1+[\delta\Psi]_2$$

We shall now show that $[\delta\psi]_1$ is generated by

$$Q_b^{(1)}=i(2m)^{1/2}\sum_S\int d^3p\left[C^{(1)}\left(\vec{\mathrm{p}}\right)d^+\left(\vec{p},s\right)v\left(\vec{p},s\right)-D^{(1)}\left(\vec{p}\right)d\left(\vec{p},s\right)u\left(\vec{p},s\right)\right]_b \qquad \mathbf{(2.6)}$$

where

$$C^{(1)}\left(\vec{\mathrm{p}}\right)=\left(\sigma^{\nu\rho}\sigma_{\nu\rho}+\sigma^{\mu\nu}\sigma_{\mu\nu}+\sigma^{\rho\mu}\sigma_{\rho\mu}\right)a\left(\vec{p}\right)\mathrm{I}_{4x4}$$

$$\equiv\left(\sigma^{\nu\rho}\sigma_{\nu\rho}+\ldots\right)\mathrm{a}\left(\vec{\mathrm{p}}\right)\mathrm{I}_{4x4}$$

We shall start with $m \neq o$ in the field equations and at the end take the limit as $m \rightarrow o$.

We shall require the following two identities:

$$\sum_r u_a\left(\vec{p},r\right)v_b\left(\vec{p},r\right)=\left(\frac{p+m}{2m}C\right)_{ab} \qquad (2\cdot 7a)$$

$$\sum_r v_a\left(\vec{p},r\right)u_b\left(\vec{p},r\right)=\left(\frac{p-m}{2m}C\right)_{ab} \qquad (2.7b)$$

Then

$$-i\left[\bar{\varepsilon}Q^{(1)},\Psi_a(X)\right]$$

$$=-i\bar{\varepsilon}_b\left\{Q_b^{(1)},\Psi_a\right.$$

(by Eqs. (2.6) and (1.4)

$$=\frac{(2\mathrm{m})^{1/2}}{(2\pi)^{3/2}}\bar{\varepsilon}_b\sum_{r,s}\int d^3p\int d^3k\left(\frac{m}{\omega_k}\right)^{1/2}$$

$$X\left\{C_{bd}^{(1)}\left(\vec{\mathrm{p}}\right)d^+\left(\vec{p},s\right)v_d\left(\vec{P},S\right)-D_{bd}^{1}\left(\vec{p},s\right)u_d\left(\vec{p},s\right)\right\},$$

$$d\left(\vec{k},r\right)u_a\left(\vec{k}.r\right)e^{-ik.X}+d^+\left(\vec{k},r\right)v_a\left(\vec{k},r\right)e^{ik.X}\}$$

(expanding the anticommutator and making use of Eqs (2.7a) and (2.7b)

$$= \bar{\varepsilon}_a \sum (2\pi)^{-3/2} \int d^3 p (2\omega_p)^{-1/2}$$

$$\left[a(\vec{p})(\sigma^{\nu\rho}\sigma_{\nu\rho} + ...)(p-m) C e^{-i(p.X)} - a^+(\vec{p})(\sigma^{\nu\rho}\sigma_{\upsilon\rho} + ...)(p-m) C e^{ip.X} \right]_{ba} \quad (2.8)$$

nothing in the second half of this equation that

$$(p-m)e^{ip.X} = (i\partial - m)e^{ip.X} \quad (2.9a)$$

implies

$$(p+m)e^{ip.X} = -(i\partial - m)e^{ip.X} \quad (2.9b)$$

$$-1\left[\in Q^{(1)}, \Psi_a(X)\right] = \bar{\varepsilon}_B \left[(-i\partial^T - m)\right] C(\sigma\sigma^{\nu\rho}\sigma_{\nu\rho} + ...)\,]_{ba}$$

(on using $C\,\gamma^{uT}\,C^{-1} = -\gamma^u$)

$$= \bar{\varepsilon}_b C_{bd} \left[A(X)(-i\partial^T + m)(\sigma^{\nu\rho}\sigma_{\nu\rho} + ...)A(X)\right]_{da}$$

$$\left(u\sin g \quad \bar{\varepsilon}^T = \in^+ \gamma^o \text{ and } C = i\gamma^2\gamma^o \right)$$

$$= -\left[\in^T A(X)(i\vec{\partial}^T + m)(\sigma^{\nu\rho}\sigma_{\nu\rho} + ...)\right]_a$$

On taking the limit $m \to 0$

$$-i\left[\bar{\in} Q^1, \Psi_a(X)\right)$$

$$-i\left[\left(\gamma^\mu \sigma \upsilon \partial_\mu A_{\nu\rho} + \gamma^\rho \sigma^{\mu\rho} \partial_\rho A_{\mu\nu} + \gamma^\nu \sigma^{\rho\mu} \partial_\nu A_{\rho\nu}\right)\varepsilon\right]_a$$

$$= [\delta\Psi_a]_1$$

Finally, we shall show that $[\delta\psi_a]_2$ is generated by

$$Q_a^2 = i(2m)^{1/2} \sum_S \int d^3 p [C^{(2)}(\vec{P}) d^+(\vec{p},s) v(\vec{p},s)$$

$$- D^2(\vec{p}) d(\vec{p},s) u(\vec{p},s)\,]_a$$

where

$$C^{(2)}(\vec{p}) = -i\gamma^5 b(\vec{p}$$

and

$$D^2(\vec{P}) = -i\gamma^5 b^+(\vec{p})$$

Starting with $m \neq 0$ in the pseudoscalar equation and at the end taking the limit as $m \to 0$

$$-i\left[\in Q^{(2)}, \Psi_a(X)\right]$$

$$-i\bar{\varepsilon}_b \left[Q_b^{(2)}, \Psi_a\right]$$

$$= \frac{(2m)^{1/2}}{(2\pi)^{3/2}} \bar{\varepsilon}_b \sum_{r,s} \int d^3p \int d^3k \left(\frac{m}{\omega_k}\right)^{1/2} \times$$

$$\left[C^{(2)}_{bd}\left(\vec{p}\right) d^+\left(\vec{p},s\right) v_d\left(\vec{p},s\right) - D^{(2)}_{bd}\left(\vec{p}\right) d\left(\vec{p},s\right) u_d\left(\vec{p},s\right)\right.$$

$$d\left(\vec{k},r\right) u_a\left(\vec{k},r\right) e^{ik.x} + d^+\left(\vec{k},r\right) v_a\left(\vec{k},r\right) e^{ik.x}$$

(expanding the anticommutator and making use of Eqs. (2.7a) and (2.7b)

$$= \bar{\varepsilon}_b (2\pi)^{(-3/2)} \int d^3p (2\omega_p)^{-1/2} x$$

$$\left[-i\gamma^5_{bd}\, b\left(\vec{p}\right)(p-m)C\right)_{da} e^{-ip.x} + i\gamma^5_{bd}\, b^+\left(\vec{p}\right)(p+m)\,C)_{da}\, e^{ip.x}\right]$$

$$= \bar{\varepsilon}_b (2\pi)^{(-3/2)} \int d^3 p (2\omega_p)^{-1/2} x$$

$$\left[-b\left(\vec{p}\right)\gamma^5 (p-m)Ce^{-ip.x} + ib^+\left(\vec{p}\right)\gamma^5 (p+m)Ce^{ip.x}\right]_{ba}$$

using Eqs. (2.9a) and (2.9b) in the second part of this last equation

$$-i\left[\bar{\varepsilon} Q^{(2)}, \Psi_a(X)\right] = \bar{\epsilon}_a \left(-i\gamma^5 (i\partial - m) C (2\pi)^{-3/2} \int d^3p (2\omega_p)^{-1/2}\right.$$

$$\left. b\left(\vec{p}\right) e^{-ip.x} + b^+\left(\vec{p}\right) e^{ip.x} \right)_{ba}$$

$$= \bar{\varepsilon}_b \left[-i\gamma^5 (i\partial - m) CB(X)\right]_{ba}$$

(using successively $C\gamma^{\mu T} C^{-1} = -\gamma^\mu$ and $C\gamma^{5T} C^{-1} = \gamma^5$)

$$= \bar{\varepsilon}_b \left[-i c \gamma^{5T} \left(-i\partial^T - m\right) B(x)\right]_{ba}$$

$$= \bar{\varepsilon}_b C_{bd} \left[-i\gamma^{5T} B(x) \left(-i\overleftarrow{\partial}^T - m\right)\right]_{da}$$

$$\left(\text{Using } \bar{\varepsilon} = \varepsilon^+ \gamma^o \text{ and } C = i\gamma^2\gamma^o\right)$$

$$= -\left[\varepsilon^T (-i\gamma^5 B(X)^T \left(i\overleftarrow{\partial} + m\right)^T\right]_a$$

$$= \left[-(i\partial + m)\left(-i\gamma^5 B(X)\right)\varepsilon\right]_a$$

On taking the limit as $m \to 0$ we obtain

$$-i\left[\bar{\varepsilon} Q^{(2)}, \Psi_a(X)\right] = \left[-\partial\gamma^5 B(X)\varepsilon\right]_a$$

i.e

$$\left[\delta\Psi(x)\right]_2 = -\partial\gamma^5 B(X)\varepsilon$$

Hence

$$-\left[\bar{\varepsilon} Q, \Psi(X)\right] = -i\left(\gamma^\mu \sigma^{\upsilon\rho} \partial_\mu \sigma_{v\rho} A(X)\right.$$

$$+ \gamma^\rho \sigma^{\mu v} \partial_\rho \sigma_{\mu v} A(X) + \gamma^v \sigma_{\rho\mu} A(X)\, \varepsilon$$

$$-\partial\gamma^5 B(X)\varepsilon = \delta\Psi(X)$$

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