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# THE ONSET AND SATURATION OF THE IONOSPHERIC LOWER HYBRID MODE

by

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ABSTRACT
Ionospheric noise shows strong peaks at lower hybrid frequencies. It is proposed that wave-particle energy exchange processes lead to the growth of a beam instability which has mode frequencies centred on the lower hybrid frequency. The onset of the beam instability and its saturation through energy transfer to part of the ion population are analyzed. The theory has implications for ionospheric radio communications.

## 1. INTRODUCTION

Plasma instabilities in the ionosphere play a role in the emission of background noise and, in this way, on the effectiveness of radio communications. Measurements show that ionospheric noise has intensity prominences at the lower hybrid frequency, $\omega \approx \omega_{pi}$. (Gurnett & Frank, 1972; Maggs, 1978). We analyze the onset and evolution of the lower hybrid instability, and show that the range of the values of its wave frequency are consistent with measured intensity peaks. The paper is organized as follows: Section 2 relates magnetic reconnection processes in the earth's magnetic tail to the inflow of energetic electron beams into **the upper ionosphere.** Section 3 discusses the unstable ionospheric waves which are excited by the electron beams, and section 4 concludes the paper with comments on the saturation of the lower hybrid mode through energy transfer to the ion population.

## 2. DYNAMICS OF THE EARTH'S MAGNETOTAIL

The earth's magnetic tail is produced by interaction between the earth's magnetic field and the solar wind, and contains a neutral layer which separates regions of oppositely directed equilibrium magnetic fields. The magnetotail equilibrium magnetic field configuration can be described by

$$\underline{B} = (B_o x/r_B)\underline{e}_y \qquad (1)$$

where $B_o$ (approximately $1.6 \times 10^{-4}$ G) is the typical field strength outside the neutral layer. Applying Maxwell's equations, we substitute for each time-varying quantity $\underline{g}$ as follows:

$$\underline{g} = \underline{g}_o(x) + \underline{g}_1 \exp(-i\omega t + iky),$$

to find the following results:

$$\xi_x = iA_1 r_B/B_o x \qquad (2)$$

where $\xi_x$ is the x-component of the fluid displacement, and $A_1$, the perturbed magnetic vector potential, is related to the magnetic field by

$$\underline{B}_1 = \nabla \times \underline{A}_1.$$

Then, from eq (2), we see that $\xi_x \to \infty$ as $x \to 0$. Thus, in this description, the fluid displacement is singular on the neutral sheet. In order to resolve this singularity, a more detailed description of the region $x \leqslant \delta_i$, centred on the neutral sheet, is needed, where

$$\delta_i = (2 \rho_i r_B)^{\frac{1}{2}}$$

is a scale length which is typical of ion particle orbits, and $\rho_i$ is the ion Larmor radius. The differential equation for $A_1$, which is valid in the region $x > \delta_i$, the so-called "outer region", is obtained as follows: On one hand, the equations $\underline{J}_1 \times \underline{B}_0 + \underline{J}_0 \times \underline{B}_1 = 0$, $\nabla \cdot \underline{J}_1 = 0$, and $\nabla \cdot \underline{B}_1 = 0$ give

$$J_{1z} = (iB_{1x}/kB_o)(dJ_{0z}/dx).$$

On the other hand, using Ampere's law, $\nabla \times \underline{B}_1 = 4\pi \underline{J}_1/c$, and $\nabla \cdot B_1 = 0$,

$$J_{1z} = (c/4\pi)(i/k)(d^2/dx^2 - k^2)B_x.$$

These equations for $J_{1z}$, with $B_{1x} = -ikA_1$, give the equation for $A_1$ in the outer region:

$$((d^2/dx^2) - k^2 - a_o/x)A_1 = 0 \qquad (3)$$

Eq (3) leads to the "cat's eyes" field configuration shown in figure 1. The closed form solution of eq (3), valid near $x = 0$, is (Chike-Obi, 1991):

$$A_{1\pm}^{out} = A_1(0)(1 + d_1 x \pm \tfrac{1}{2}\Delta' x + d_2 x(\ln(x)) \qquad (4)$$

This solution should be matched to the solution for $A_1$ in the inner region $x < \delta_i$. The constants $d_1$ and $d_2$ are given by the boundary conditions at the outer edges of the plasma.

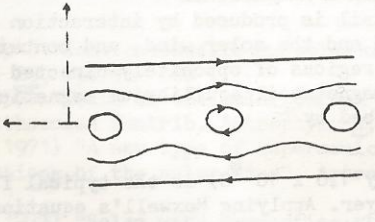

Figure 1: The formation of magnetic islands.

To describe the inner region, we use the appropriate equilibrium particle distribution functions. Then, the linearized Vlasov equation, with Ampere's law for the electrons and ions, gives the inner layer equation for $A_1$:

$$d^2A_1/dx^2 = (\omega/c\lambda_{De}^2 k)(A_1\omega'/k_{\parallel}c)P_e \qquad (5)$$

In this equation, see Chike-Obi (1991), $P_e$ is an anomalous conductivity, $\omega'$ is the Doppler shifted frequency, and $\lambda_{De}$ is the electron Debye length. The boundary conditions are that the inner $A_1$ is constant at $x = 0$, and that it matches with the outer solution given in eq (4), as the inner variable x goes to infinity. The matching gives the growth rate:

$$\gamma = \pi^{-\frac{1}{2}}(2r_B/\rho_e)^{3/2}(r_B/v_e)(T_e/(T_e + T_i)) \qquad (6)$$

In a time interval of the order of $\gamma$, particles are accelerated to kinetic energies

$$(KE)_j = q_j^2B^2r_B^2\gamma^4/2m_j \qquad (7)$$

A variety of satellite measurements (see Chike-Obi (1991) and the references cited therein), give values for $r_B$, $B_o$, $T_e$, and $T_i$, from which we find $\gamma \sim 1$s and $(KE)_e \sim 2.02$MeV. The value for kinetic energy agrees well with measurements of the energies of incoming electrons of geomagnetic origin, and suggests that magnetic reconnection is the mechanism for the acceleration of electron beams which are thought to excite the lower hybrid instability in the upper ionosphere.

## 3. THE IONOSPHERIC LOWER HYBRID MODE

In the beam instability, an incoming beam of electrons, with speeds centred on $v_b$, transfers some of its energy to any waves in the plasma having phase velocities $\omega_r/k \lesssim v_b$, thereby causing the amplification of such waves. In the ionosphere, following magnetic reconnection in the magnetotail, an electron beam is accelerated into a background plasma which is initially Maxwellian, to give the following double-peaked distribution

$$f_e = (n_{eo}/v_{eo})\pi^{-\frac{1}{2}}\exp\{-(v/v_{eo})^2\}$$
$$+ (n_b/v_{eb})\pi^{-\frac{1}{2}}\exp\{-[(v - v_b)/v_{eb}]^2\},$$

where $v_{eo} = (2T_{eo}/m)^{\frac{1}{2}}$ and $v_{eb} = (2T_{eb}/m)^{\frac{1}{2}}$. The ions are also Maxwellian, and described by the equilibrium distribution $f_i$, in which we include a perpendicular hot ion component, as an extension to the analysis in Chike-Obi (1991):

$$f_i = (n_{io}/v_{io})\pi^{-\frac{1}{2}}\exp(-(v/v_{io})^2$$
$$+ (n_{ih}/v_{ih})\pi^{-\frac{1}{2}}\exp -(v/v_{ih})^2$$

where $n_{io} + n_{ih} = n_{eo} + n_b$, $v_{io} = (2T_i/M)^{\frac{1}{2}}$, and M is the ion mass. We postulate $\theta T_i = T_{eb}$ with $\theta > 1$, and $T_{eo} = 2\epsilon T_{eb}$, where $\epsilon <$ 1. Then, the collisionless Vlasov equation gives the first order perturbed electron and ion distributions:

$$f_{e1} = (iq/m)E_{\parallel}\nabla_{v_{\parallel}} f_e/(\omega - k_{\parallel}v_{\parallel})$$

and

$$f_{i1} = -(iq/M)\underline{E} \cdot \nabla_v f_i/(\omega - \underline{k} \cdot \underline{v})$$

From these, the perturbed densities are

$$n_{e1} = \int f_{e1} dv = -(iE_{\parallel}q/k_{\parallel})\,(n_{eo}/T_{eo})W(x_e) + (n_{eb}/T_{eb})W(x_b)$$

and

$$n_{i1} = \int f_{i1} dv = (iEq/kT_{io})(n_{io} + n_{ih})W(x_i),$$

where $x_e = \omega/k\,v_{eo}$, $x_b = (\omega - k_{\parallel}v_b)/k\,v_{eb}$, and $x_i = \omega/kv_{io}$. $W(x)$ is the Landau function,

$$W(x) = -\pi^{-\frac{1}{2}}\int se^{-s^2} ds/(s - x),$$

and is related to the plasma dispersion function $Z(x)$ (Fried & Conte, 1961) by

$$W(x) = -(1 + xZ(x)).$$

Substituting $n_{i1}$ and $n_{e1}$ into Poisson's equation, $\nabla \cdot \underline{E} = 4\pi q(n_{i1} - n_{e1})$, the dispersion relation is,

$$1 = (4\pi q^2/k^2)\Big[(n_b/T_{eb})W(x_b) + (n_{ih}/T_{ih})W(x_i) + n_{eo}k^2/m\omega^2 + n_{io}k^2/M\omega^2\Big] \qquad (8)$$

This mode is of lower hybrid type because, evaluating eq (8) in the realistic limit $x_i \gg 1$ and $x_e \gg 1$, using the asymptotic expansion of the Landau function, the real part $\omega_r$ of the complex frequency $\omega$ is

$$\omega_r^2 = \omega_{pi}^2\left[1 + (k^2/k^2)(M/m)\right] \qquad (9)$$

Eq (9) shows that $\omega_r$ is about the ion plasma frequency, and thus corresponds to the lower hybrid wave (Shohet, 1971). Does $f_{pi} = \omega_{pi}/2\pi$ give values in the range 7.35kHz to 105kHz, measured for ionospheric noise? (Gurnett & Frank (1972); Maggs (1976). Substituting values for $n_{oi}$ for the ionosphere at heights of about 5000km (Ratcliffe, 1972), we find that $f_{pi}$ is in the range 6.6kHz to 105kHz, suggesting that the intensity peaks reported by Perkins (1968) and Gurnett & Frank (1972) are due to lower hybrid oscillations.

## 4. ION EVOLUTION AND CONCLUDING REMARKS

Now, it is seen that for $v_{ph} = \omega_r/k_{\parallel} \lesssim v_b$, the wave resonates with the magnetotail beam electrons, gaining energy from them. This suggests the interesting observation that there will be a part of the ion population for which $v \lesssim \omega_r/k$. These ions will be heated by the unstable lower hybrid wave. In this case the dispersion relation in eq (8) for the lower hybrid wave will be modified to the form

$$\omega = \omega_{pi} + i\gamma_e - i\gamma_i,$$

since the growth which is fed by the beam electrons is damped by the hot ion component. The theory may therefore be suggested as the first step in a sequence of plasma processes which produces upwardly accelerated and field-aligned energetic ions of ionospheric origin (Chang & Coppi, 1981). Broadband noise of lower hybrid frequencies have been observed, and this is a problem in radio communications because of its contribution to undesirable background noise. Energetic electron beams of magnetotail origin have been detected which, in this paper, we propose as the driving mechanism for the ionospheric lower hybrid beam instability. We also suggest that subsequent energy transfer from the wave to the ions is part of the mechanism for the evolution of ionospheric ions.

## 5. REFERENCES

T. Chang & B. Coppi, Geophysical Research Letters, 8, 12(1981)
B. Chike-Obi, accepted (1991), Nigerian Journal of Physics
B. D. Fried & S. D. Conte, The Plasma Dispersion Function (Academic Press, New York, 1961)
D. A. Gurnett & L. A. Frank, Journal of Geophysical Research, 77, 1(1972)
J. E. Maggs, Journal of Geophysical Research, 81, 10(1976)
J. E. Maggs, Journal of Geophysical Research, 83, 7(1978)
J. A. Ratcliffe, An Introduction to the Ionosphere and Magnetosphere (Cambridge at the University Press, 1972)
J. L. Shohet, The Plasma State (Academic Press, New York, 1971)