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ENTROPY RATE IN SOLAR WIND FLOW

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ABSTRACT

In this paper, the entropy rate in solar wind flow is analyzed by considering fluid dynamical models of a spherically symmetric, expanding, and heat conducting gas under gravity. Based on certain simplifying postulates, the fluid equations **of continuity, momentum,** energy, and a thermodynamic relation that involves entropy, are obtained and solved analytically. The entropy rate is shown to **give** two critical solutions, one increasing and the other decreasing.

1. INTRODUCTION

Existence of the solar wind was not widely accepted **until 1962, when** results from the spacecraft Mariner 2 were published (Neugebaver & Snyder, 1962). Bierman (1951) used the evidence that comet tails always seem to have a pronounced anti-solar orientation to support the claim that solar corpuscular radiation is continuous and that emission takes place from the whole surface of the sun, not just from solar flares or other areas of intense activity. Chapman (1957) put forward the idea that the solar corona is essentially a static, ionized gas, dominated by thermal conduction, and **extending** beyond the orbit of the earth. However, a consequence of this is an associated gas pressure at infinity which is several orders of magnitude in excess of what was accepted as interstellar background pressure. Parker (1958) argued that the solar corona could not be in static equilibrium and, inspired by the continuous-emission hypothesis of Bierman, proposed that the solar corona must necessarily be steadily expanding. Parker named such a continuous **expansion** of the corona the "solar wind", set up a simple hydrodynamical model incorporating an isothermal corona out to several solar radii, and predicted that the solar wind would reach supersonic speeds, of the order of several km/s, at the earth.

In this paper, an attempt is made to study the disorderliness in the solar wind particles, mostly ions, in the course of its continuous outflow into interplanetary space.

2. GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

In order to construct a simple fluid model of the quiet **solar wind,** we assume that (i) the solar wind is a fully ionized, electrically neutral hydrogen plasma, that is, an electron-proton gas in which electron number density is equal o the proton number density; (ii) the electron streaming speed c s equal to the proton streaming speed (this assumption preserve charge neutrality of the sun);

(iii) electron and proton temperatures are equal, say T (this assumption characterizes the "one-fluid" model); (iv) viscous dissipation is negligible; (v) the magnetic field is carried along by the flow, without exerting any appreciable force on the plasma, and so can be omitted; (vi) there are no heat sources or sinks in the corona; (vii) the effects of solar rotation can be neglected; and (viii) the solar wind behaves as a conventional fluid, since near the sun, the particle mean free path is small and at greater heliocentric distances, the interplanetary magnetic field causes particles to interact indirectly so that the medium still behaves as a fluid even in the absence of collisions. The fluid equations, based on these assumptions, are the equations of continuity, momentum, energy, and thermodynamic relations:

$$\partial\rho/\partial t + \nabla\cdot(\rho\underline{q}) = 0 \tag{1}$$

$$\partial\underline{q}/\partial t + (\underline{q}\cdot\nabla)\underline{q} = -(1/m\rho)\nabla(R\rho T) + g_S \tag{2}$$

$$\partial(\tfrac{1}{2}\rho mV^2 + R\rho T/(\gamma - 1))/\partial t + \nabla\cdot(\tfrac{1}{2}\rho mV^2\underline{q} + \gamma\rho RT\underline{q}/(\gamma - 1)) = \nabla\cdot(K\nabla T) + \rho m g_S \tag{3}$$

$$T\nabla S = C_p\nabla T - (1/m\rho)\nabla(R\rho T) \tag{4}$$

where m is the mass of the solar particle, γ is the ratio of principal specific heats, R is the ideal gas constant, g_S is the solar gravitational acceleration, K is the coefficient of thermal conduction, and S is the entropy rate for the solar wind. We have assumed a scalar gas pressure p, which can be eliminated from the equations by assuming the ideal gas equation of state:

$$p = \rho RT \tag{5}$$

The solar wind may be considered steady and spherically symmetric, in which case the variables ρ, V, T, and S are functions only of the heliocentric radius r. Then, equations (1) to (4) become:

$$r^{-2}d(r^2\rho V)/dr = 0 \tag{6}$$

$$VdV/dr = -(1/m\rho)d(R\rho T)/dr - GMr^{-2} \tag{7}$$

$$r^{-2}(d/dr)\left[r^2(\tfrac{1}{2}\rho mV^3 + \gamma R\rho TV/(\gamma - 1)\right] = r^{-2}(d/dr)(r^2KdT/dr) - \rho mGMVr^{-2} \tag{8}$$

$$TdS/dr = C_p dT/dr - (1/m\rho)d(\rho RT)/dr \tag{9}$$

where M is the mass of the sun, G the universal constant of gravitation, and C_p is the molar heat capacity at constant pressure. Equations (6) and (8) are readily integrated, and ρ may be eliminated from (7) and (9) by use of the following integral which relates the wind flux per unit solid angle F (= constant):

$$r^2\rho V = F \tag{10}$$

and the relation

$$K = K_o T^{5/2}$$

for the thermal conductivity. Then,

$$mVdV/dr = -Rr^2Vd(T/r^2V)/dr - mGMr^{-2} \tag{11}$$

$$F(\tfrac{1}{2}mV^2 + \gamma RT/(\gamma - 1) - mGM/r) - r^2K_o T^{5/2}dT/dr = E \tag{12}$$

$$dS/dr = (C_p/T)dT/dr - (Rr^2V/mT)d(T/r^2V)/dr \tag{13}$$

where E (= constant) is the energy flux per unit solid angle. The terms on the left hand side of eq (12) represent the convection of kinetic energy, enthalpy, gravitational energy, and thermal conduction respectively. Rewriting equations (10) to (13) in dimensionless form, we solve the resulting equations subject to the boundary conditions that as r goes to infinity, T, p, S, and V go to T_∞, p_∞, S_∞, and V_∞ respectively; and V goes to zero as r goes to zero. Introducing the variables

$$x = \varepsilon FmGM/Er \tag{14a}$$

$$y = \varepsilon FRT/E \tag{14b}$$

$$z = \varepsilon FmV^2/2E \tag{14c}$$

and

$$w = \varepsilon FmS/E \tag{14d}$$

equations (11) to (13) become

$$(1 - y/2z)dz/dx = 1 - x^{-2}d(x^2y)/dx \tag{15}$$

$$z + 5y/2 - x + y^{5/2}dy/dx = \varepsilon \tag{16}$$

$$dw/dx = (1/y)dy/dx + x^{-2}d(x^2y)/dx - (y/2z)dz/dx \tag{17}$$

where $\varepsilon = (K_o mGM)^2E/K^7F^5$, $C_p = E/\varepsilon Fm$, and γ is assumed to have the value 5/3. Thus, the solar wind problem reduces to solving the differential equations (15) to (17) with dependent variables y, z, and w; the independent variable x; and the parameter ε. In terms of these, the boundary conditions become that as x goes to x_o, y, w, and z go to y_o, w_o, and z_o respectively; and z goes to zero as x goes to infinity.

3. METHOD OF ANALYSIS

In broad terms, the topology of solutions of equations (15) and (16) is such that there are two critical solutions (one increasing and the other decreasing) passing through their singular point, which is the point at which

$$y = 2z \tag{18a}$$

$$x^2d^2y/dx^2 + 2xdy/dx - 2y = 0 \tag{18b}$$

$$x + 5y/2 - x + y^{5/2}dy/dx = \varepsilon \tag{18c}$$

These monotonic critical solutions separate families of wholly subsonic, wholly supersonic, and unphysical solutions. It can be shown that all critical supersonic solutions to the solar wind equations (15) and (16) are characterized by one of the asymptotic solutions:

$$y \sim Ax^{2/7} \tag{19a}$$

$$z \sim \varepsilon - 2A^{7/2}/7 - 8Ax^{2/7} \tag{19b}$$

$$y \sim (35x/4)^{2/5} \tag{20a}$$

$$z \sim \varepsilon - 6(35x/4)^{2/5} \tag{20b}$$

and, as x goes to zero,

$$y \sim Bx^{4/4} \tag{21a}$$

$$z \sim \varepsilon + x - 5Bx^{4/3}/2 \tag{21b}$$

where A and B are undetermined constants. The solution in eq (19) corresponds to an energy flux at infinity which is partly thermal and partly kinetic; whilst for the solutions in equations (20) and (21), the energy at infinity is entirely kinetic (Durney, 1971 and 1973). Combining equations (15) and (17), we integrate the outcome to find:

$$w = c + x - z + \log(y) \tag{22}$$

where c is an undetermined constant. Consequently, all the critical supersonic solutions for the entropy rate in solar wind flow are characterized by one of the asymptotic solutions:

$$w \sim x + 8Ax^{2/7} + \log(Ax^{2/7}) + 2A^{2/7}/7 + C - \varepsilon \tag{23}$$

$$w \sim x + 6(35x/4)^{2/5} + (2/5)\log(35x/4) + C - \varepsilon \tag{24}$$

and, as x goes to zero,

$$w \sim 5Bx^{4/3}/2 + \log(Bx^{4/3}) + C - \varepsilon \tag{25}$$

4. DISCUSSION

Despite the fact that solar wind models based on the assumptions in section 2 will be idealized, such models can be remarkably effective in describing some of the gross features of the solar wind. A solar beam of particles of one sign must necessarily disperse before reaching the earth, as a result of mutual electrostatic repulsion. But it has been observed that certain geomagnetic disturbances tend to recur with the 27-day solar rotation cycle. This disturbance is related to the existence of the solar wind and is better explained by taking into consideration the entropy rate of the solar particles as they travel away from the sun. From our calculations in this work, we observe that in regions which are at small distances from the sun, the entropy rate is very high (these regions are referred to as the collision-dominated regions). Also, as the wind moves away, up to and beyond the earth's orbit from the sun, the entropy rate drops sharply, that is, as r goes to infinity, S goes to zero (these regions are referred to as the collision-free regions of the solar wind). One usefulness of models of the collision-free region, compared with the collision-dominated regions, is that they may help to predict phenomena described by the more complex Boltzman equation.

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