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HEAT TRANSFER TO A LIQUID FILM ON AN UNSTEADY STRETCHING SURFACE

by

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ABSTRACT
An invèstigation into heat transfer into a liquid film on an unsteady stretching surface is carried out. Using similarity transformations, both momentum and energy equations are reduced to ordinary differential equations. To obtain heat transfer, we use both asymptotic and numerical techniques.

1. INTRODUCTION

Motion and heat transfer caused by a stretching surface are currently topics of research because of their relevance in many engineering processes; and of typical interest is motion caused in extrusion processes and, generally, motion involving materials between a feed roll and wind-up roll, or on conveyor belts.

One needs to study both velocity and temperature histories because in the extrusion of a polymer sheet from a die, the properties of the final product depend to a great extent on the rate of cooling. Thus, by stretching such a sheet in a viscous fluid whose properties we know, we can control the rate of cooling. Hence, the desired characteristics of the final product can be achieved (see ref 3). In refs 2 and 3, the authors were interested in steady flows, whereas in refs 1 and 4, the principal interest was in the transient case. For ref 1, the temperature was unsteady while the flow was steady; but ref 4 completely focussed on the unsteady flow field. In this paper, we investigate the effect of the unsteady parameter on both the flow field and the heat transfer.

2. THE PHYSICS OF THE PROBLEM

We consider a flow with the velocity vector

$$\underline{q} = t^{-\frac{1}{2}}F(\underline{x}t^{-\frac{1}{2}}) \qquad (1)$$

where $\underline{q}$ is the velocity vector, t is time, and $\underline{x}$ the space variable vector. The surface at y = 0 is being stretched with velocity

$$u = bx/(1 - \alpha t) \qquad (2)$$

where b and α are constants with dimensions of inverse time. Unlike in ref 4, we are not ruling out the possibility of suction or blowing, so that still on y = 0, the y component of velocity is

$$v = -d(\nu b)^{\frac{1}{2}}/(1 - \alpha t)^{\frac{1}{2}} \qquad (3)$$

where ν is the kinematic viscosity and d is a dimensionless constant. For a clear picture, we present figure 1. Moreover, for easy reference, we state the continuity equation, the momentum equation, and the energy equation:

$$\partial u/\partial x + \partial v/\partial y = 0 \qquad (4)$$

$$\partial u/\partial t + u\,\partial u/\partial x + v\,\partial u/\partial y = (\mu/\rho)\,\partial^2 u/\partial y^2 \qquad (5)$$

$$\partial T/\partial t + u\,\partial T/\partial x + v\,\partial T/\partial y = (\lambda/\rho c)\,\partial^2 T/\partial y^2 \qquad (6)$$

where the velocity vector $\underline{q} = (u,v)$, T is temperature, ρ is density, μ is the dynamic viscosity, and λ is the thermal conductivity. The kinematic viscosity ν which was defined earlier is given by $\nu = \mu/\rho$ and c is the specific heat.

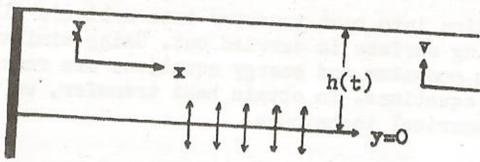

Figure 1

It is important to say that to arrive at (5), we have neglected gravity and end effects. In the physics of the problem, the constant α is important. The stretching is decelerated if α is negative; and accelerated if α is positive, that is, if t is less than $1/\alpha$. As we are not ruling out the possibility that the liquid film is thick or thin, we simply represent its thickness by h(t). We shall assume that the thickness is uniform and stable, so that t is less than $1/\alpha$. We are now ready to specify the boundary **conditions** for temperature:

$$\left.\begin{aligned} T(x, 0, t) &= T_w \\ T(x, h(t), t) &= T_o \end{aligned}\right\} \qquad (7)$$

3. SIMILARITY VARIABLE AND NONDIMENSIONALIZATION

Using the similarity variable described in ref 4,

$$\left.\begin{aligned} u &= bxf'(\eta)/(1 - \alpha t) \\ v &= -(\nu b)^{\frac{1}{2}} f(\eta)/(1 - \alpha t)^{\frac{1}{2}} \end{aligned}\right\} \qquad (8)$$

where

$$\eta = (b/\nu)^{\frac{1}{2}}(1 - \alpha t)^{\frac{1}{2}} y.$$

With these, equation (5) becomes

$$S(f' + \tfrac{1}{2}\eta f'') + (f')^2 - ff'' = f''' \qquad (9)$$

where $S \equiv \alpha/b$ signifies the relative importance of unsteadiness to the stretching rate. Of course, (8) automatically satisfies (4). We let

$$\theta = (T - T_o)/(T_w - T_o) \qquad (10)$$

with T_w greater than T_o, and (6) becomes

$$(\tfrac{1}{2}S\eta - f(\eta))\,\theta' = \sigma\theta'' \qquad (11)$$

where $\sigma = \lambda/\rho\nu c$ and $1/\sigma$ is the Prandtl number. The boundary conditions on the stretching boundary are

$$
\begin{aligned}
f(0) &= d \\
f'(0) &= 1 \\
\theta(0) &= 1
\end{aligned} \tag{12}
$$

On the free surface at $y = h(t)$ on $\eta = \beta$, we require $v = dh/dt$, or

$$
\begin{aligned}
\tfrac{1}{2}S\beta &= f(\beta) \\
\theta(\beta) &= 0
\end{aligned} \tag{13}
$$

Furthermore, zero tangential stress condition gives

$$f''(\beta) = 0 \tag{14}$$

4. MATHEMATICAL TECHNIQUES AND ASYMPTOTIC SOLUTION FOR THIN FILM

We wish to consider three cases: the thin film, the thick film, and an average film. Hence both asymptotic and numerical **treatments** are appropriate here. The case $d = 0$, for the thin film, has been carried out by Wang (see ref 4) with $f \sim \eta$. By putting

$$\eta = \beta\varepsilon \tag{15}$$

$$f = \beta\varepsilon + \beta^3 F_1(\varepsilon) + \beta^5 F_2(\varepsilon) + O(\beta^7) \tag{16}$$

$$S = S_0 + \beta^2 S_1 + \beta^4 S_2 + O(\beta^6) \tag{17}$$

we obtain

$$
\left.
\begin{aligned}
S_0 &= 2 \\
S_1 &= 2F_1(1) \\
S_2 &= F_2(1)
\end{aligned}
\right\} \tag{18}
$$

where (see ref 4),

$$F_1 = \tfrac{1}{2}\varepsilon^3 - 3\varepsilon^2/2 \tag{19}$$

$$F_2 = \varepsilon^5/10 - \tfrac{1}{2}\varepsilon^4 - \tfrac{1}{3}\varepsilon^3 + 3\varepsilon^2 \tag{20}$$

$$S = 2 - 2\beta^2 + 68\beta^4/15 + O(\beta^6) \tag{21}$$

Inversion gives

$$\beta = (1 - \tfrac{1}{2}S)^{\frac{1}{2}}\ 1 + 17(1 - \tfrac{1}{2}S)/15 + O(1 - \tfrac{1}{2}S)^2 \tag{22}$$

Now writing

$$\theta = \phi_0(\varepsilon) + \beta^4\phi_1(\varepsilon) + O(\beta^6), \tag{23}$$

equations (11) to (13) give

$$\phi_0'' = 0 \tag{24}$$

$$
\left.
\begin{aligned}
\phi_0(0) &= 1 \\
\phi_0(1) &= 0
\end{aligned}
\right\} \tag{25}
$$

$$\sigma\phi_1'' = (\tfrac{1}{2}S_1\varepsilon - F_1(\varepsilon))\phi_0' \tag{26}$$

$$\phi_1(0) = \phi_1(1) = 0 \tag{27}$$

Thus

$$\phi_0 = 1 - \varepsilon \tag{28}$$

$$\sigma\phi_1 = \varepsilon^5/40 - \varepsilon^4/8 + \varepsilon^3/12 - \varepsilon/60 \tag{29}$$

$$\partial\theta(0,t)/\partial y = ((b/\nu)(1 - \alpha t))^{\frac{1}{2}}(-\beta^3/60\sigma - 1/\beta) + O(\beta^6) \tag{30}$$

5. THIN FILM WITH SUCTION OR BLOWING

Equations (13) and (9) imply

$$S = 2d/\beta + \beta S_1 + \ldots\ldots \qquad (31)$$

$$f = d + \beta\varepsilon + \beta^2 G_1(\varepsilon) + \ldots\ldots \qquad (32)$$

Hence,

$$G_1''' = 2\delta \qquad (33)$$

$$G_1(0) = G_1'(0) = G''(1) = 0 \qquad (34)$$

and

$$G_1(\varepsilon) = d(\tfrac{1}{2}\varepsilon^3 - \varepsilon^2) \qquad (35)$$

Writing

$$\theta = H_0(\varepsilon) + \beta H_1(\varepsilon) + O(\beta^2) \qquad (36)$$

we obtain from equations (11) to (13)

$$H_0'' = 0 \qquad (37)$$

$$\left.\begin{array}{l} H_0(0) = 1 \\ H_0(1) = 0 \end{array}\right\} \qquad (38)$$

$$\sigma H_1'' = d(\varepsilon - 1)H_0' \qquad (39)$$

$$H_1(0) = H_1(1) = 0 \qquad (40)$$

Thus,

$$H_0 = 1 - \varepsilon \qquad (41)$$

and

$$\sigma H_1 = -d(\varepsilon - \tfrac{1}{2}\varepsilon^2 + \varepsilon^3/6) \qquad (42)$$

$$\partial\theta(0,t)/\partial y = ((b/\nu)(1 - \alpha t))^{\frac{1}{2}}(-1/\beta - d/\sigma) + O(\beta^2) \qquad (43)$$

$$\beta = \tfrac{1}{2}(S \pm (S^2 - 8d^2)^{\frac{1}{2}}) \qquad (44)$$

6. ASYMPTOTIC SOLUTION FOR THICK FILM

For f to be bounded, when η approaches infinity S must approach zero. We therefore seek asymptotic solutions of the form

$$f = m - e^{-m\eta}/m + Sf_1(\eta) + O(S^2) \qquad (45)$$

$$\beta = \beta_0/S + \beta_1 + O(S) \qquad (46)$$

$$\theta = \theta_0 + S\theta_1 + O(S^2) \qquad (47)$$

where

$$d = m - 1/m \qquad (48)$$

Equations (13), (45), and (46) give

$$\left.\begin{array}{l} \beta_0 = 2m \\ \text{and} \\ \beta_1 = 2f_1(\infty) \end{array}\right\} \qquad (49)$$

Equation (11) gives

$$\sigma\theta_0'' = -(m - e^{-m\eta}/m)\theta_0' \qquad (50)$$

Hence,

$$\sigma\theta_0' = A\exp(-(m\eta + m^{-2}e^{-m\eta})) \qquad (51)$$

where

$$A = -\sigma / \left\{ \int_0^\infty \exp\left[-(mz + m^{-2}e^{-mz})\right] dz \right\} \quad (52)$$

Hence

$$\partial\theta(0,t)/\partial y = ((b/\nu)(1 - \alpha t))^{\frac{1}{2}} A e^{-m^{-2}} \quad (53)$$

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