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# DISCRETE DELAY ON THE IMPACT OF MEDIA COVERAGE IN THE TRANSMISSION DYNAMICS OF FOWL POX INFECTION

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**ABSTRACT**

***In this work, a vector-host deterministic mathematical model has been formulated to include the susceptible, infected and vector (mosquito) population at time $> 0$ . The model was then transformed to a vector-host model that incorporate discrete delay parameter, to investigate the impact of media coverage on the transmission dynamics of fowl pox with discrete delay. The model was analysed using methods which provides a general algorithm for determining stability of model with delay differential equations.At both disease free equilibrium and endemic equilibrium, it was observed that the system is stable in the absence of delay and unstable with increasing delay under established conditions***

***Also in the work, the construction of aLyapunov function is done for the Vector-Host model (2.1,2.3) using lyapunov direct methods. The results indicated that the DFE is globally asymptotically stable when $R_b \leq 1$ (i.e. every solution trajectory of the Vector-Host model (2.1) - (2-3) converges to the largest compact invariant set $M = \{(S_0, I, V)\}$and unstable when $R_b \geq 1$.***

## 1.0 INTRODUCTION

Fowl pox, pox, or avian pox is a relatively slow spreading viral disease characterized by skin lesions or plagues in the pharynx. It is prevalent among chickens, turkey, pigeons, canaries, worldwide. Morbidity is 10-95% and mortality usually 0-50%. Infection occurs through the skin abrasions and bites, or by the respiratory route [9.10]. The virus persists in the environment for months, the e duration of the disease is about 14days on individual bird bases. The infected birds display some of the following symptoms: warty spreading eruption, scabs on comb and wattles gaseous deposits in mouth throat and sometimes trachea, depression, poor growth and poor egg production. Because of its slow-spreading nature, it is possible to vaccinate to stop an outbreak. Flocks and individuals still unaffected maybe vaccinated usually with chicken strain by wing web vaccinating method. If there is evidence of secondary bacterial infection, broad-spectrum, antibiotics may be of some benefit. Fowl pox or avian pox is transmitted by direct contact between infected and susceptible birds or by mosquitoes. Virus-containing scabs also can be slough from infected birds and serve as a source of infection [11.12].

The virus can enter the blood stream through the eye, skin wound, or respiratory tracts. Mosquitoes become infected by feeding on birds with fowl pox in their blood stream. There is some evidence that the mosquitoes remain infected for life. Mosquitoes are the primary reservoir and spreaders of fowl pox on poultry ranges.

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Several species of mosquito can transmit fowl pox. Often mosquitoes winter-over in poultry houses, so outbreak can occur during winter and early spring [http://msucares.com/poultry/disease/disviral.htm] [9,10].
In modelling biological and physical sciences, it is sometimes necessary to take into consideration the time delays inherent in the phenomena [5,3]. Delay is used to mean "time lag" in the transmission process of an epidemic. This refers to the period taken for the infectious agent in the infected to develop. It is only after this period that the infected birds become infectious. Fowl pox can be transmitted by direct or indirect contact. The virus is highly resistant in dried scabs and under certain conditions may survive for months on contaminated premises. The disease may be transmitted by a number of species of mosquitoes. Mosquitoes can harbour infective virus for a month or more after feeding on affected birds. After the infection is introduced, it spreads within the flock by mosquitoes as well as direct and indirect contact. Recovered birds do not remain carriers.
Media report plays a key role in the perception, management and even creation of crises [7]. A good percentage of loss recorded in poultry farming is traceable to infectious disease. When it appears and spreads in an environment, farmers will do everything possible to control and prevent the disease spreading. One of the immediate measures that should be taken, is to educate the populace on the preventive methods of the disease through media coverage. It is a common sense that the more preventive knowledge the population has the better to prevent the spreading of the disease. Media coverage has great influence not only the farmer's behaviours but also on the formation and implementation of public intervention and control policies [6,8].
[8] presented the mathematical model of the impact of media coverage on the transmission dynamics of fowl pox in poultry, this model is a deterministic model without taking into consideration the delay inherent in the biological processes of fowl pox transmission dynamics. The model was analysed using the dynamical system theory to determine the behaviour of the system near equilibrium points. That is, Methods from dynamical system theory were employed in analysing the equilibrium stability of the model at both disease free equilibrium and endemic equilibrium. Appropriate conditions for the local asymptotic stability of both equilibrium points have were established. A threshold parameter $R_0$ (the basic reproductive ratio) was also derived analytically to discuss the local stability of the disease free equilibrium.
Mathematical models have been used in providing solutions to real life problems [24], and has over the years been a veritable tool in the hand of mathematicians and epidemiologist in studying, understanding, describing and analysing the dynamics of infectious diseases with the aim of providing solutions to public health challenges [ 14, 15, 16, 17, 18, 19,23].
In this work, we overcome the limitation of [8] by extending the work to incorporate discrete delay inherent in the biological processes of fowl pox transmission dynamics. The model of media coverage in [8] is transformed to mathematical model of media coverage that incorporate discrete delay. The transformed model is analysed using methods from [ 2, 3, 4, 5] which provides a general algorithm for determining stability of models with delay differential equations. We shall also investigate the global asymptotic stability (GAS) of [8] which was lacking

## 2.0The Model Formulation

## 2.1 Model Assumptions

The infection is transmitted by both the vector and the infected birds.
2. Birds that recover from one can become susceptible to the other strain.
3. Media report has positive impact on the transmission rate of the infection.
4. Mosquitoes are the primary reservoir and spreaders of the infection in poultry range [13].
5. The vector has logistic growth.

**2.2 Parameters/Symbols**

$S =$ Susceptible birds

$I = infectted\ birds$

$V =$ Vector (Mosquito)

$\Lambda =$ recruitment term of the susceptible birds

$\alpha_1 =$ infection rate of fowl pox in poultry

μ= the rate at which the infected birds recover and become susceptible

$\varphi =$ natural death rate (it is the same for each sub − population)

$\gamma =$ death due to infection

$\delta =$ death rate of the vector due to effort as a response tomedia coverage

$\frac{\alpha_2 I}{m_1 + I} = the\ measure\ of\ effect\ of\ reduction\ in\ transmission\ rate\ as\ a\ result\ of\ media\ coverage$

$\frac{\alpha_3 V}{m_1+V}$ = the measure of the effect of reduction in transmission rate due to extra effort on the vector as a response of media coverage

$m_1$
$= half\ saturation\ constant\ which\ reflects\ the\ impact\ of\ media\ coverage\ on\ the\ contact\ transmission\ rate$

$\rho =$ growth rate of the vector

$K =$ the carrying capacity for the vector

Our model describes the transmission dynamics of fowl pox infection based on two strains. Here we assume that birds which recover from one strain can become susceptible to the other strain. The susceptible population is increased by recruitment of birds either by birth or immigration and the recovered bird that become susceptible.population is reduced by infection and by natural death or emigration. The infected population is increased by infection of susceptible birds either by infected bird or the vector (mosquito). This population is diminished by natural death, death due to infection, and those that recover and become susceptible. The vector population assumes a logistic growth with K as the carrying capacity and as the growth rate of the vector. This population is decreased by emigration or natural death and death due to extra effort as a response from media coverage. Media coverage is introduced into the system via saturated incidence functions $H(I) = \frac{\alpha_2 I}{m_1+I}$ and $g(V) = \frac{\alpha_3 V}{m_1+V}$

The transmission model with media coverage is given by the following deterministic system of non -linear ordinary differential equations:

$$\frac{dS}{dt} = \Lambda - \left(\alpha_1 - \frac{\alpha_2 I}{m_1 + I}\right)SI - \left(\alpha_1 - \frac{\alpha_3 V}{m_1 + V}\right)SV + \mu I - \psi S \ldots\ldots\ldots\ldots (2.1)$$

$$\frac{dI}{dt} = \left(\alpha_1 - \frac{\alpha_2 I}{m_1 + I}\right)SI + \left(\alpha_1 - \frac{\alpha_3 V}{m_1 + V}\right)SV - \mu I - \psi I - \gamma I \ldots\ldots\ldots\ldots (2.2)$$

$$\frac{dV}{dt} = \rho V\left(1 - \frac{V}{K}\right)SI - \delta V - \psi V \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.3)$$

Let $\tau\ be\ the\ descrete\ delay\ parameter$.When delay is introduced, then (1) - (3) becomes

$$\frac{dS}{dt} = \Lambda - \left(\alpha_1 - \frac{\alpha_2 I}{m_1 + I}\right)S(t)I(t-\tau) - \left(\alpha_1 - \frac{\alpha_3 V}{m_1 + V}\right)S(t)V(t-\tau) + \mu I(t-\tau) - \psi S \ldots\ldots\ldots\ldots (2.4)$$

$$\frac{dI}{dt} = \left(\alpha_1 - \frac{\alpha_2 I}{m_1 + I}\right) S(t)I(t-\tau) + \left(\alpha_1 - \frac{\alpha_3 V}{m_1 + V}\right) S(t)V(t-\tau) - \mu I(t-\tau) - \psi I - \gamma I(t - \tau) \ldots\ldots\ldots\ldots (2.5)$$

$$\frac{dV}{dt} = \rho V\left(1 - \frac{V}{K}\right) - \delta V(t - \tau) - \psi V(t - \tau) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.6)$$

## 3.0 Analysis of the Model

Using the theory of computable criteria for stability of discrete delay system [1, 2, 3, 4]

$$\frac{dx}{dt} = \sum_{i=0}^{n} A_i(t)x(t-h_i) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.1)$$

Where $A_i$ is the matrix of the discrete delay system, $h_i$ is the discrete delay. Then the characteristic equation is

$$F(\lambda) = \left|\lambda I - \sum_{i=0}^{n} A_i(t)e^{-\lambda h_i}\right| \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.2)$$

From the system (2.4) - (2.6), (3.1), (3.2), we have

$$A_0 = \begin{pmatrix} -\xi_1 I^0 - \xi_2 V^0 - \psi & 0 & 0 \\ \xi_1 I^0 + \xi_2 V^0 & \xi_1 S^0 & 0 \\ 0 & 0 & \rho\left(1 - \frac{V}{K}\right) \end{pmatrix}$$

$$A_1 = \begin{pmatrix} 0 & -\xi_1 S^0 + \mu & -\xi_2 S^0 \\ 0 & \xi_1 S^0 - (\mu + \psi + \gamma) & \xi_2 S^0 \\ 0 & 0 & -\delta \end{pmatrix} e^{-\chi\tau}$$

$$\xi_1 = \left(\alpha_1 - \frac{\alpha_2 I^0}{m_1 + I^0}\right), \qquad \xi_2 = \left(\alpha_1 - \frac{\alpha_3 V^0}{m_1 + V^0}\right)$$

$$F(\chi) = \begin{vmatrix} \chi - (\xi_1 I^0 + \xi_2 V^0 + \psi) & (-\xi_1 S^0 + \mu)e^{-\chi\tau} & -\xi_2 S^0 e^{-\chi\tau} \\ \xi_1 I^0 + \xi_2 V^0 & \chi - (\xi_1 S^0 + (\xi_1 S^0 - (\mu + \psi + \gamma))e^{-\chi\tau}) & \xi_2 S^0 e^{-\chi\tau} \\ 0 & 0 & \chi - \left(\rho\left(1 - \frac{V}{K}\right) - \delta e^{-\chi\tau}\right) \end{vmatrix} = 0$$

### 3.1 Stability of Disease Free Equilibrium

The disease free equilibrium of the system is given as

$$E_0(S^0, I^0, V^0) = \left(\frac{\Lambda\rho(\rho m_1 + P)}{(\alpha_1 P + \psi\rho)(m_1\rho + P) - \alpha_3 P^2},\ 0,\ \frac{K}{\rho}(\rho - \psi - \delta)\right)$$

Where

$P = K(\rho - \psi - \delta)$,

$$\xi_1 = \alpha_1, \qquad \xi_2 = \frac{\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)}$$

$$A_0 = \begin{pmatrix} \frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)} - \psi & 0 & 0 \\ \frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)} & \alpha_1 \frac{\Lambda\rho(\rho m_1 + P)}{(\alpha_1 P + \psi\rho)(m_1\rho + P) - \alpha_3 P^2} & 0 \\ 0 & 0 & \psi + \delta \end{pmatrix}$$

$$A_1 = \begin{pmatrix} 0 & (\frac{-\alpha_1 \Lambda\rho(\rho m_1 + P)}{(\alpha_1 P + \psi\rho)(m_1\rho + P) - \alpha_3 P^2} + \mu)e^{-\chi\tau} & -\frac{\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)} \frac{\Lambda\rho(\rho m_1 + P)}{(\alpha_1 P + \psi\rho)(m_1\rho + P) - \alpha_3 P^2} e^{-\chi\tau} \\ 0 & (\frac{\alpha_1 \Lambda\rho(\rho m_1 + P)}{(\alpha_1 P + \psi\rho)(m_1\rho + P) - \alpha_3 P^2} - (\mu + \psi + \gamma))e^{-\chi\tau} & \frac{\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)} \frac{\Lambda\rho(\rho m_1 + P)}{(\alpha_1 P + \psi\rho)(m_1\rho + P) - \alpha_3 P^2} \\ 0 & 0 & (-\delta - \psi)e^{-\chi\tau} \end{pmatrix}$$

$$F(\chi)$$

$$= \chi^3 - \frac{\alpha_1 \Lambda\rho(\rho m_1 + P)}{(\alpha_1 P + \psi\rho)(m_1\rho + P) - \alpha_3 P^2}\chi^2 - \left(\frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)} + \psi\right)\chi^2$$

$$- (\delta + \psi)\chi^2 - \left(\frac{\alpha_1 \Lambda\rho(\rho m_1 + P)}{(\alpha_1 P + \psi\rho)(m_1\rho + P) - \alpha_3 P^2} - (\mu + \psi + \gamma)\right)e^{-\chi\tau}\chi^2 - (\delta + \psi)e^{-\chi\tau}\chi^2$$

$$- \left(\frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)}\right.$$

$$\left. + \psi\right)\frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)}\chi$$

$$+ \left(\frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)} + \psi\right)\frac{\alpha_1 \Lambda\rho(\rho m_1 + P)}{(\alpha_1 P + \psi\rho)(m_1\rho + P) - \alpha_3 P^2}\chi$$

$$+ \left(\frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)} + \psi\right)\left(\frac{\alpha_1 \Lambda\rho(\rho m_1 + P)}{(\alpha_1 P + \psi\rho)(m_1\rho + P) - \alpha_3 P^2}\right.$$

$$\left. - (\mu + \psi + \gamma)\right)e^{-\chi\tau}\chi + \left(\frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)} + \psi\right)(\delta + \psi)\chi$$

$$- \left(\frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)} + \psi\right)(\delta + \psi)e^{-\chi\tau}\chi$$

$$+ \frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)}\left(\frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)}\right.$$

$$\left. + \psi\right)(\delta + \psi)$$

$$- \frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)}\left(\frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)}\right.$$

$$\left. + \psi\right)(\delta + \psi)e^{-\chi\tau}$$

$$- \left(\frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)} + \psi\right)\frac{\alpha_1 \Lambda\rho(\rho m_1 + P)}{(\alpha_1 P + \psi\rho)(m_1\rho + P) - \alpha_3 P^2}(\delta + \psi)$$

$$+\left(\frac{(\alpha_1 m_1+(\alpha_1-\alpha_3)\frac{K}{\rho}(\rho-\psi-\delta))\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}+\psi\right)\left(\frac{\alpha_1\,\Lambda\rho(\rho m_1+P)}{(\alpha_1 P+\psi\rho)(m_1\rho+P)-\alpha_3P^2}\ -\right)(\delta+\psi)e^{-2\chi\tau}\ \ldots\ldots(3.3)$$

$$F(\chi)=\chi^3+\left\{\frac{\alpha_1\,\Lambda\rho(\rho m_1+P)}{\alpha_3P^2-(\alpha_1P+\psi\rho)(m_1\rho+P)}+\frac{(\alpha_1 m_1+(\alpha_1-\alpha_3)\frac{K}{\rho}(\rho-\psi-\delta))\frac{K}{\rho}(\rho-\psi-\delta)}{\frac{K}{\rho}(\psi+\delta)-m_1-K}-\delta\right\}\chi^2+$$
$$\left\{\left(\frac{(\alpha_1 m_1+(\alpha_1-\alpha_3)\frac{K}{\rho}(\rho-\psi-\delta))\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}+\psi\right)\frac{\alpha_1\,\Lambda\rho(\rho m_1+P)}{(\alpha_1P+\psi\rho)(m_1\rho+P)-\alpha_3P^2}+\left(\frac{(\alpha_1 m_1+(\alpha_1-\alpha_3)\frac{K}{\rho}(\rho-\psi-\delta))\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}+\right.\right.$$
$$\left.\left.\psi\right)(\delta+\psi\ )-\left(\frac{(\alpha_1 m_1+(\alpha_1-\alpha_3)\frac{K}{\rho}(\rho-\psi-\delta))\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}+\psi\right)\frac{(\alpha_1 m_1+(\alpha_1-\alpha_3)\frac{K}{\rho}(\rho-\psi-\delta))\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}\right\}\chi+$$
$$\frac{(\alpha_1 m_1+(\alpha_1-\alpha_3)\frac{K}{\rho}(\rho-\psi-\delta))\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}\left(\frac{(\alpha_1 m_1+(\alpha_1-\alpha_3)\frac{K}{\rho}(\rho-\psi-\delta))\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}+\psi\right)(\delta+\psi\ )-$$
$$\left(\frac{(\alpha_1 m_1+(\alpha_1-\alpha_3)\frac{K}{\rho}(\rho-\psi-\delta))\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}+\psi\right)\frac{\alpha_1\,\Lambda\rho(\rho m_1+P)}{(\alpha_1P+\psi\rho)(m_1\rho+P)-\alpha_3P^2}+\left\{\left(\left(\left((\mu+\psi+\gamma)-\right.\right.\right.\right.$$
$$\left.\left.\left.\frac{\alpha_1\,\Lambda\rho(\rho m_1+P)}{(\alpha_1P+\psi\rho)(m_1\rho+P)-\alpha_3P^2}\right)-(\delta+\psi)\right)\chi^2\ +\left(\left(\frac{(\alpha_1 m_1+(\alpha_1-\alpha_3)\frac{K}{\rho}(\rho-\psi-\delta))\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}+\right.\right.$$
$$\left.\left.\psi\right)\left(\frac{\alpha_1\,\Lambda\rho(\rho m_1+P)}{(\alpha_1P+\psi\rho)(m_1\rho+P)-\alpha_3P^2}\ -(\mu+\psi+\gamma)\right)-\left(\frac{(\alpha_1 m_1+(\alpha_1-\alpha_3)\frac{K}{\rho}(\rho-\psi-\delta))\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}+\psi\right)(\delta+\psi\ )\right)\chi+$$
$$\left.\frac{(\alpha_1 m_1+(\alpha_1-\alpha_3)\frac{K}{\rho}(\rho-\psi-\delta))\frac{K}{\rho}(\rho-\psi-\delta)}{\frac{K}{\rho}(\psi+\delta)-m_1-K}\left(\frac{(\alpha_1 m_1+(\alpha_1-\alpha_3)\frac{K}{\rho}(\rho-\psi-\delta))\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}+\psi\right)(\delta+\psi\ )\right\}e^{-\chi\tau}+$$
$$\left(\frac{(\alpha_1 m_1+(\alpha_1-\alpha_3)\frac{K}{\rho}(\rho-\psi-\delta))\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}+\psi\right)\left(\frac{\alpha_1\,\Lambda\rho(\rho m_1+P)}{(\alpha_1P+\psi\rho)(m_1\rho+P)-\alpha_3P^2}\ -\right)(\delta+\psi)e^{-2\chi\tau}\quad \ldots\ldots(3.4$$

Using the method of Theorem 2.4 in [4] and Theorem 4.8 in {3], which provides a general algorithm for determining stability of model with delay differential equations to determine the stability of (2.4) - (2.6)

**Theorem 1:** A steady state with characteristic equation

$$\chi^3+a_2\chi^2+a_1\chi+a_0+\{b_2\chi^2+b_1\chi+b_0\}e^{-\chi\tau}=0\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.5)$$

$A\ =\ a_2^2-b_2^2-2a_1, B=\ a_1^2-b_1^2+2b_2b_0-2a_2a_0$ and $C\ =\ a_0^2-b_0^2$

is stable in the absence of delay and becomes unstable with increasing delay if and only if A, B, and C are not all positive and

(i) $a_2+b_2>0\ , a_0+b_0>0,\ (a_2+b_2)(a_1+b_1)-(a_0+b_0)>0$

(ii) Either$C\ <0\ orC>0\ ,\ A^2-3B$

$$a_2 = \left\{\frac{\alpha_1 \Lambda\rho(\rho m_1 + P)}{\alpha_3 P^2 - (\alpha_1 P + \psi\rho)(m_1\rho + P)} + \frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{\frac{K}{\rho}(\psi + \delta) - m_1 - K} - \delta\right\}$$

$$a_1 = \left\{\left(\frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)} + \psi\right)\frac{\alpha_1 \Lambda\rho(\rho m_1 + P)}{(\alpha_1 P + \psi\rho)(m_1\rho + P) - \alpha_3 P^2} + \left(\frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)} + \psi\right)(\delta + \psi) - \left(\frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)} + \psi\right)\frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)}\right\}$$

$$a_0 = \frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)}\left(\frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)} + \psi\right)(\delta + \psi) - \left(\frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)} + \psi\right)\frac{\alpha_1 \Lambda\rho(\rho m_1 + P)}{(\alpha_1 P + \psi\rho)(m_1\rho + P) - \alpha_3 P^2}$$

$$b_2 = \left((\mu + \psi + \gamma) - \frac{\alpha_1 \Lambda\rho(\rho m_1 + P)}{(\alpha_1 P + \psi\rho)(m_1\rho + P) - \alpha_3 P^2}\right) - (\delta + \psi)$$

$$b_1 = \left(\frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)} + \psi\right)\left(\frac{\alpha_1 \Lambda\rho(\rho m_1 + P)}{(\alpha_1 P + \psi\rho)(m_1\rho + P) - \alpha_3 P^2} - (\mu + \psi + \gamma)\right) - \left(\frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)} + \psi\right)(\delta + \psi)$$

$$b_0 = \frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{\frac{K}{\rho}(\psi + \delta) - m_1 - K}\left(\frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)} + \psi\right)(\delta + \psi)$$

Analysing the characteristic equation (3.4) of the system (2.4) - (2.6) based on Theorem 1, we discovered that at the disease free equilibrium, the system is stable in the absence of delay and unstable with increasing delay under the following conditions

$$\left\{\frac{\alpha_1 \Lambda\rho(\rho m_1 + P)}{\alpha_3 P^2 - (\alpha_1 P + \psi\rho)(m_1\rho + P)} + \frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{\frac{K}{\rho}(\psi + \delta) - m_1 - K} > 2\delta + (\mu + \gamma) + \frac{\alpha_1 \Lambda\rho(\rho m_1 + P)}{\alpha_3 P^2 - (\alpha_1 P + \psi\rho)(m_1\rho + P)}\right\}$$

$$\frac{2(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)}\left(\frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)} + \psi\right)(\delta + \psi) > \left(\frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)} + \psi\right)\frac{\alpha_1 \Lambda\rho(\rho m_1 + P)}{(\alpha_1 P + \psi\rho)(m_1\rho + P) - \alpha_3 P^2}$$

$$\left(\frac{(\alpha_1 m_1+(\alpha_1-\alpha_3)\frac{K}{\rho}(\rho-\psi-\delta))\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}+\psi\right)\left\{\frac{2\alpha_1\Lambda\rho(\rho m_1+P)}{(\alpha_1 P+\psi\rho)(m_1\rho+P)-\alpha_3 P^2} - \frac{(\alpha_1 m_1+(\alpha_1-\alpha_3)\frac{K}{\rho}(\rho-\psi-\delta))\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}-(\mu+\psi+\gamma)\right\}$$

$$(a_2+b_2)(a_1+b_1)-(a_0+b_0)>0$$

$$\left\{\frac{\alpha_1\Lambda\rho(\rho m_1+P)}{\alpha_3 P^2-(\alpha_1 P+\psi\rho)(m_1\rho+P)}+\frac{(\alpha_1 m_1+(\alpha_1-\alpha_3)\frac{K}{\rho}(\rho-\psi-\delta))\frac{K}{\rho}(\rho-\psi-\delta)}{\frac{K}{\rho}(\psi+\delta)-m_1-K}-2\delta+(\mu+\gamma) + \frac{\alpha_1\Lambda\rho(\rho m_1+P)}{\alpha_3 P^2-(\alpha_1 P+\psi\rho)(m_1\rho+P)}\right\}\left(\frac{(\alpha_1 m_1+(\alpha_1-\alpha_3)\frac{K}{\rho}(\rho-\psi-\delta))\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)} + \psi\right)\left\{\frac{2\alpha_1\Lambda\rho(\rho m_1+P)}{(\alpha_1 P+\psi\rho)(m_1\rho+P)-\alpha_3 P^2}-\frac{(\alpha_1 m_1+(\alpha_1-\alpha_3)\frac{K}{\rho}(\rho-\psi-\delta))\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}-(\mu+\psi+\gamma)\right\}$$

$$-\frac{2(\alpha_1 m_1+(\alpha_1-\alpha_3)\frac{K}{\rho}(\rho-\psi-\delta))\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}\left(\frac{(\alpha_1 m_1+(\alpha_1-\alpha_3)\frac{K}{\rho}(\rho-\psi-\delta))\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}+\psi\right)(\delta+\psi) + \left(\frac{(\alpha_1 m_1+(\alpha_1-\alpha_3)\frac{K}{\rho}(\rho-\psi-\delta))\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}+\psi\right)\frac{\alpha_1\Lambda\rho(\rho m_1+P)}{(\alpha_1 P+\psi\rho)(m_1\rho+P)-\alpha_3 P^2}>0'$$

$$\left(\frac{(\alpha_1 m_1+(\alpha_1-\alpha_3)\frac{K}{\rho}(\rho-\psi-\delta))\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}\left(\frac{(\alpha_1 m_1+(\alpha_1-\alpha_3)\frac{K}{\rho}(\rho-\psi-\delta))\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}+\psi\right)(\delta+\psi) - \left(\frac{(\alpha_1 m_1+(\alpha_1-\alpha_3)\frac{K}{\rho}(\rho-\psi-\delta))\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}+\psi\right)\frac{\alpha_1\Lambda\rho(\rho m_1+P)}{(\alpha_1 P+\psi\rho)(m_1\rho+P)-\alpha_3 P^2}\right)^2$$

$$-\left(\frac{(\alpha_1 m_1+(\alpha_1-\alpha_3)\frac{K}{\rho}(\rho-\psi-\delta))\frac{K}{\rho}(\rho-\psi-\delta)}{\frac{K}{\rho}(\psi+\delta)-m_1-K}\left(\frac{(\alpha_1 m_1+(\alpha_1-\alpha_3)\frac{K}{\rho}(\rho-\psi-\delta))\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}+\psi\right)(\delta+\psi)\right)^2<0$$

### 3.2 Stability of Disease Endemic Equilibrium

At the endemic equilibrium point, the infection has spread everywhere in the system. Hence there is no susceptible population, that is, $S=0$

The endemic point is thus $E_d(S^0,I^0,V^0)=\left(0,\ \frac{\Lambda}{\mu},\ \frac{K}{\rho}(\rho-\psi-\delta)\right.$

$$A_0=\begin{pmatrix}-\xi_1 I^0-\xi_2 V^0-\psi & 0 & 0\\ \xi_1 I^0+\xi_2 V^0 & \xi_1 S^0 & 0\\ 0 & 0 & \rho\left(1-\frac{V}{K}\right)\end{pmatrix}$$

$$A_1=\begin{pmatrix}0 & -\xi_1 S^0+\mu & -\xi_2 S^0\\ 0 & \xi_1 S^0-(\mu+\psi+\gamma) & \xi_2 S^0\\ 0 & 0 & -\delta\end{pmatrix}e^{-\chi\tau}$$

$$\xi_1=,\left(\alpha_1-\frac{\alpha_2 I^0}{m_1+I^0}\right)=\left(\alpha_1-\frac{\alpha_2\frac{\Lambda}{\mu}}{+\frac{\Lambda}{\mu}}\right),\qquad \xi_2=\left(\alpha_1-\frac{\alpha_3 V^0}{m_1+V^0}\right)=\left(\alpha_1-\frac{\alpha_3\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}\right)$$

$$F(\chi) = \begin{vmatrix} \chi-(\xi_1 I^0+\xi_2 V^0+\psi) & \mu e^{-\chi\tau} & 0 \\ \xi_1 I^0+\xi_2 V^0 & \chi-((\mu+\psi+\gamma)e^{-\chi\tau}) & 0 \\ 0 & 0 & \chi-(\delta+\psi-(\delta+\psi)e^{-\chi\tau}) \end{vmatrix} = 0$$

$$x^3 - \left(\frac{\alpha_1\Lambda(\mu m_1+\Lambda)-\alpha_2\Lambda^2}{\mu(\mu m_1+\Lambda)} + \frac{\{\alpha_1 m_1+(\alpha_1-\alpha_2)\frac{K}{\rho}(\rho-\psi-\delta)\}\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)} + \psi + (\mu+\psi+\gamma)e^{-\chi\tau} + \delta + \psi - (\delta+\psi)e^{-\chi\tau}\right)x^2$$

$$+\Bigg(\left(\frac{\alpha_1\Lambda(\mu m_1+\Lambda)-\alpha_2\Lambda^2}{\mu(\mu m_1+\Lambda)} + \frac{\{\alpha_1 m_1+(\alpha_1-\alpha_2)\frac{K}{\rho}(\rho-\psi-\delta)\}\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)} + \psi\right)(\mu+\psi+\gamma)e^{-\chi\tau}$$
$$+ (\delta+\psi-(\delta+\psi)e^{-\chi\tau}) + (\delta+\psi)(\mu+\psi+\gamma)e^{-\chi\tau} - (\delta+\psi)(\mu+\psi+\gamma)e^{-2\chi\tau}$$
$$- \left(\frac{\alpha_1\Lambda(\mu m_1+\Lambda)-\alpha_2\Lambda^2}{\mu(\mu m_1+\Lambda)} + \frac{\{\alpha_1 m_1+(\alpha_1-\alpha_2)\frac{K}{\rho}(\rho-\psi-\delta)\}\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}\right)\mu e^{-\chi\tau}\Bigg)x$$
$$+ \left(\frac{\alpha_1\Lambda(\mu m_1+\Lambda)-\alpha_2\Lambda^2}{\mu(\mu m_1+\Lambda)} + \frac{\{\alpha_1 m_1+(\alpha_1-\alpha_2)\frac{K}{\rho}(\rho-\psi-\delta)\}\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}\right)\big((\delta+\psi)\mu e^{-\chi\tau}$$
$$- (\delta+\psi)e^{-2\chi\tau}\big)$$
$$-(\left(\frac{\alpha_1\Lambda(\mu m_1+\Lambda)-\alpha_2\Lambda^2}{\mu(\mu m_1+\Lambda)} + \frac{\{\alpha_1 m_1+(\alpha_1-\alpha_2)\frac{K}{\rho}(\rho-\psi-\delta)\}\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)} + \psi\right)(\mu+\psi+\gamma)e^{-\chi\tau}(\delta+\psi$$
$$- (\delta+\psi)e^{-\chi\tau}) = 0 \quad \dots.(3.6)$$

$$x^3+\left(\frac{\alpha_2\Lambda^2-\alpha_1\Lambda(\mu m_1+\Lambda)}{\mu(\mu m_1+\Lambda)}+\frac{\{\alpha_1 m_1+(\alpha_1-\alpha_2)\frac{K}{\rho}(\rho-\psi-\delta)\}\frac{K}{\rho}(\rho-\psi-\delta)}{\frac{K}{\rho}(\psi+\delta)-m_1-K}-2\psi-\delta\right)x^2+(\delta+\psi)\mathrm{x}$$

$$+\left\{(\mu+\gamma-\delta)x^2\right.$$

$$+\left(\left(\frac{\alpha_1\Lambda(\mu m_1+\Lambda)-\alpha_2\Lambda^2}{\mu(\mu m_1+\Lambda)}+\frac{\{\alpha_1 m_1+(\alpha_1-\alpha_2)\frac{K}{\rho}(\rho-\psi-\delta)\}\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}+\psi\right)(\mu+\psi+\gamma)-(\delta+\psi)\right.$$

$$\left.+(\delta+\psi)(\mu+\psi+\gamma)-\left(\frac{\alpha_1\Lambda(\mu m_1+\Lambda)-\alpha_2\Lambda^2}{\mu(\mu m_1+\Lambda)}+\frac{\{\alpha_1 m_1+(\alpha_1-\alpha_2)\frac{K}{\rho}(\rho-\psi-\delta)\}\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}\right)\mu\right)x$$

$$+\left(\frac{\alpha_1\Lambda(\mu m_1+\Lambda)-\alpha_2\Lambda^2}{\mu(\mu m_1+\Lambda)}+\frac{\{\alpha_1 m_1+(\alpha_1-\alpha_2)\frac{K}{\rho}(\rho-\psi-\delta)\}\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}\right)(\delta+\psi)\mu$$

$$\left.-\left(\left(\frac{\alpha_1\Lambda(\mu m_1+\Lambda)-\alpha_2\Lambda^2}{\mu(\mu m_1+\Lambda)}+\frac{\{\alpha_1 m_1+(\alpha_1-\alpha_2)\frac{K}{\rho}(\rho-\psi-\delta)\}\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}+\psi\right)(\mu+\psi+\gamma)(\delta+\psi)\right)\right\}e^{-\chi\tau}$$

$$+\left\{\left(\frac{\alpha_1\Lambda(\mu m_1+\Lambda)-\alpha_2\Lambda^2}{\mu(\mu m_1+\Lambda)}+\frac{\{\alpha_1 m_1+(\alpha_1-\alpha_2)\frac{K}{\rho}(\rho-\psi-\delta)\}\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}+\psi\right)(\mu+\psi+\gamma)(\delta+\psi)\right.$$

$$\left.-\left(\frac{\alpha_1\Lambda(\mu m_1+\Lambda)-\alpha_2\Lambda^2}{\mu(\mu m_1+\Lambda)}+\frac{\{\alpha_1 m_1+(\alpha_1-\alpha_2)\frac{K}{\rho}(\rho-\psi-\delta)\}\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}\right)((\delta+\psi))\right\}e^{-2\chi\tau}$$

$$=0\ldots\ldots\ldots\ldots\ldots..(3.7)$$

$$a_2=\left(\frac{\alpha_2\Lambda^2-\alpha_1\Lambda(\mu m_1+\Lambda)}{\mu(\mu m_1+\Lambda)}+\frac{\{\alpha_1 m_1+(\alpha_1-\alpha_2)\frac{K}{\rho}(\rho-\psi-\delta)\}\frac{K}{\rho}(\rho-\psi-\delta)}{\frac{K}{\rho}(\psi+\delta)-m_1-K}-2\psi-\delta\right)$$

$a_1=(\delta+\psi)$ , $a_0\ =0 b_2=(\mu+\gamma-\delta)$

$$b_1=\left(\frac{\alpha_1\Lambda(\mu m_1+\Lambda)-\alpha_2\Lambda^2}{\mu(\mu m_1+\Lambda)}+\frac{\{\alpha_1 m_1+(\alpha_1-\alpha_2)\frac{K}{\rho}(\rho-\psi-\delta)\}\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}+\psi\right)(\mu+\psi+\gamma)$$
$$-(\delta+\psi)+(\delta+\psi)(\mu+\psi+\gamma)$$
$$-\left(\frac{\alpha_1\Lambda(\mu m_1+\Lambda)-\alpha_2\Lambda^2}{\mu(\mu m_1+\Lambda)}+\frac{\{\alpha_1 m_1+(\alpha_1-\alpha_2)\frac{K}{\rho}(\rho-\psi-\delta)\}\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}\right)\mu+$$

$$b_0=\left(\frac{\alpha_1\Lambda(\mu m_1+\Lambda)-\alpha_2\Lambda^2}{\mu(\mu m_1+\Lambda)}+\frac{\{\alpha_1 m_1+(\alpha_1-\alpha_2)\frac{K}{\rho}(\rho-\psi-\delta)\}\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}\right)(\delta+\psi)\mu$$
$$-\left(\left(\frac{\alpha_1\Lambda(\mu m_1+\Lambda)-\alpha_2\Lambda^2}{\mu(\mu m_1+\Lambda)}+\frac{\{\alpha_1 m_1+(\alpha_1-\alpha_2)\frac{K}{\rho}(\rho-\psi-\delta)\}\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}+\psi\right)(\mu\right.$$
$$\left.+\psi+\gamma)(\delta+\psi)\right)$$

Using the method of Theorem 2.4 in [4] and Theorem 4.8 in {3], which provides a general algorithm for determining stability of model with delay differential equations to determine the stability of (2.4) - (2.6)

Analysing the characteristic equation (3.7) of the system (2,4) (2.6) based on Theorem 1, we discovered that at the endemic equilibrium, the system is stable in the absence of delay and unstable with increasing delay under the following conditions

$$\left(\frac{\alpha_2\Lambda^2-\alpha_1\Lambda(\mu m_1+\Lambda)}{\mu(\mu m_1+\Lambda)}+\frac{\{\alpha_1 m_1+(\alpha_1-\alpha_2)\frac{K}{\rho}(\rho-\psi-\delta)\}\frac{K}{\rho}(\rho-\psi-\delta)}{\frac{K}{\rho}(\psi+\delta)-m_1-K}-2\psi-\delta\right)+(\mu+\gamma-\delta)>0$$

$$\left(\frac{\alpha_1\Lambda(\mu m_1+\Lambda)-\alpha_2\Lambda^2}{\mu(\mu m_1+\Lambda)}+\frac{\{\alpha_1 m_1+(\alpha_1-\alpha_2)\frac{K}{\rho}(\rho-\psi-\delta)\}\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}\right)(\delta+\psi)\mu$$
$$>(\left(\frac{\alpha_1\Lambda(\mu m_1+\Lambda)-\alpha_2\Lambda^2}{\mu(\mu m_1+\Lambda)}+\frac{\{\alpha_1 m_1+(\alpha_1-\alpha_2)\frac{K}{\rho}(\rho-\psi-\delta)\}\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}+\psi\right)(\mu+\psi+\gamma)(\delta+\psi))$$

$$\Bigg\{\left(\frac{\alpha_1\Lambda(\mu m_1+\Lambda)-\alpha_2\Lambda^2}{\mu(\mu m_1+\Lambda)}+\frac{\{\alpha_1 m_1+(\alpha_1-\alpha_2)\frac{K}{\rho}(\rho-\psi-\delta)\}\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}+\psi\right)(\mu+\psi+\gamma)-(\delta+\psi)$$
$$+(\delta+\psi)(\mu+\psi+\gamma)$$
$$-\left(\frac{\alpha_1\Lambda(\mu m_1+\Lambda)-\alpha_2\Lambda^2}{\mu(\mu m_1+\Lambda)}+\frac{\{\alpha_1 m_1+(\alpha_1-\alpha_2)\frac{K}{\rho}(\rho-\psi-\delta)\}\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}\right)\mu+(\delta+\psi)\Bigg\}$$
$$-\left(\frac{\alpha_1\Lambda(\mu m_1+\Lambda)-\alpha_2\Lambda^2}{\mu(\mu m_1+\Lambda)}+\frac{\{\alpha_1 m_1+(\alpha_1-\alpha_2)\frac{K}{\rho}(\rho-\psi-\delta)\}\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}\right)(\delta+\psi)\mu$$
$$-(\left(\frac{\alpha_1\Lambda(\mu m_1+\Lambda)-\alpha_2\Lambda^2}{\mu(\mu m_1+\Lambda)}+\frac{\{\alpha_1 m_1+(\alpha_1-\alpha_2)\frac{K}{\rho}(\rho-\psi-\delta)\}\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}+\psi\right)(\mu+\psi+\gamma)(\delta+\psi))$$
$$-(\left(\frac{\alpha_1\Lambda(\mu m_1+\Lambda)-\alpha_2\Lambda^2}{\mu(\mu m_1+\Lambda)}+\frac{\{\alpha_1 m_1+(\alpha_1-\alpha_2)\frac{K}{\rho}(\rho-\psi-\delta)\}\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}\right)(\delta+\psi)\mu$$
$$-\left(\left(\frac{\alpha_1\Lambda(\mu m_1+\Lambda)-\alpha_2\Lambda^2}{\mu(\mu m_1+\Lambda)}+\frac{\{\alpha_1 m_1+(\alpha_1-\alpha_2)\frac{K}{\rho}(\rho-\psi-\delta)\}\frac{K}{\rho}(\rho-\psi-\delta)}{m_1+\frac{K}{\rho}(\rho-\psi-\delta)}+\psi\right)(\mu+\psi+\gamma)(\delta+\psi))\right)^2<0$$

***3.3 Global asymptotic stability (GAS) of the disease free equilibrium (DFE) of the model* (2.1) –(2.3)**

**Co**nsider the system (2.1) –(2.3)

$$F=\begin{pmatrix}\frac{\alpha_1 m_1-\alpha_2}{m_1} & \frac{\alpha_1 m_1-\alpha_3}{m_1}\\ 0 & 0\end{pmatrix}$$

$$V = \begin{pmatrix} \mu+\psi+\gamma & 0 \\ 0 & \varphi+\delta \end{pmatrix}$$

$$V^{-1} = \begin{pmatrix} \frac{1}{\mu+\psi+\gamma} & 0 \\ 0 & \frac{1}{\varphi+\delta} \end{pmatrix}$$

$$FV^{-1} = \begin{pmatrix} \frac{\alpha_1 m_1-\alpha_2}{m_1(\mu+\psi+\gamma)} & \frac{\alpha_1 m_1-\alpha_3}{m_1(\varphi+\delta)} \\ 0 & 0 \end{pmatrix}$$

Thus, the basic reproduction number is given by the spectral radius of $FV^{-1}$as

$$R_b = \frac{\alpha_1 m_1-\alpha_2}{m_1(\mu+\psi+\gamma)}$$

The left Eigen- vector of non-negative matrix $V^{-1}F$ is $w^T = \left(\frac{m_1(\mu+\psi+\gamma)}{\alpha_1 m_1-\alpha_3} R_b \quad 1\right)$

$$f(x,y) = \begin{bmatrix} \left(\alpha_1 - \frac{\alpha_2 I}{m_1+I}\right)SI + \left(\alpha_1 - \frac{\alpha_3 V}{m_1+V}\right)SV \\ \rho V\left(1-\frac{V}{K}\right)SI \end{bmatrix}$$

$for\ x = \begin{pmatrix} I \\ V \end{pmatrix}\ and\ y = S$

We observe that $f(x,y) \geq 0\ in\ \Gamma = \left\{(S,I,V)\ \epsilon\Re_+^3 : 0 \leq I+S \leq \frac{\Lambda\rho(\rho m_1+P)}{(\alpha_1 P+\psi\rho)(m_1\rho+P)-\alpha_3 P^2}\right\}$. If $S \leq S_0\ and\ f(0,y_0) = 0$.

Since $F \geq 0$ , $V^{-1} \geq 0$ and $f(x,y) \geq 0$ . By theorem 2.1 of [21], $\mathbb{Q} = \omega^T V^{-1}\mathrm{x}$ is the lyapunov function, where $w^T = \left(\frac{m_1(\mu+\psi+\gamma)}{\alpha_1 m_1-\alpha_3} R_b \quad 1\right)$ is the left Eigen- vector of non-negative matrix $V^{-1}F$.

Hence $\mathbb{Q} = \frac{m_1}{\alpha_1 m_1-\alpha_3} R_b I + \frac{V}{\varphi+\delta}$

Which a lyapunov function of the model (2.1) -(2.3)

The following theorem is needed to establish the Global Asymptotic Stability (GAS) of the disease free equilibrium of the model (2.1) -(2.3).

**Theorem 2:** The disease free equilibrium of the model (2.1) -(2.3) is Globally Asymptotically Stable in Γ if $R_b \leq 1$

Proof:

Let $\mathbb{Q} = \frac{m_1}{\alpha_1 m_1-\alpha_3} R_b I + \frac{V}{\varphi+\delta}$ be a lyapunov function of the model (2.1) -(2.3) on Γ with $R_b \leq 1$ and $f(x,y) \geq 0$

Then by differentiating $\mathbb{Q}$ along the solutions of (2.1) -(2.3) gives

$$\dot{\mathbb{Q}} = \omega^T V^{-1}\dot{\mathrm{x}}$$

$$\dot{\mathbb{Q}} = \omega^T V^{-1}(F-V)x - \omega^T V^{-1} f(x,y)$$

$$\dot{\mathbb{Q}} = (\mathrm{R}_b - 1)\left(\frac{m_1(\mu+\psi+\gamma)}{\alpha_1 m_1-\alpha_3} R_b\, I + \quad V\right) - \frac{m_1}{\alpha_1 m_1-\alpha_3} R_b\left(\left(\alpha_1 - \frac{\alpha_2 I}{m_1+I}\right)SI + \left(\alpha_1 - \frac{\alpha_3 V}{m_1+V}\right)SV\right) - \frac{1}{\varphi+\delta}\left(\rho V\left(1-\frac{V}{K}\right)SI\right)$$

Thus it follows that $\dot{\mathbb{Q}} \leq 0$ if $R_b \leq 1$. If $R_b = 1$, then $\dot{\mathbb{Q}} = 0$ if and only if $\mathrm{I} = \mathrm{V} = 0$ . Therefore every solution trajectory of equations in the model (2.1) -(2.3) converges to the largest compact invariant set $M = \{(S_0, I, V)\}$, and the only point in$M$ is the disease-free equilibrium. Then by LaSalle's invariant principle [22], $\chi_0$ is globally asymptotically stable in $\Gamma$ if $R_b \leq 1$. That is every solution trajectory of equations in the model (2.1) -(2.3) approaches $\chi_0 \;\; as\, t \;\; \rightarrow \infty$.

CONCLUSION

In this work, a vector-host deterministic mathematical model has been formulated to include the susceptible, infected and vector (mosquito) population at time $> 0$ . The model was then transformed to a vector-host model that incorporate discrete delay parameter, to investigate the impact of media coverage on the transmission dynamics of fowl pox with discrete delay. The model was analysed using methods which provides a general algorithm for determining stability of models with delay differential equations.At both disease free equilibrium and endemic equilibrium, it was observed that the system is stable in the absence of delay and unstable with increasing delay under the following established conditions: for Disease free equilibrium:

$$\left\{\frac{\alpha_1 \Lambda\rho(\rho m_1 + P)}{\alpha_3 P^2 - (\alpha_1 P + \psi\rho)(m_1\rho + P)} + \frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{\frac{K}{\rho}(\psi + \delta) - m_1 - K} > 2\delta + (\mu + \gamma) + \frac{\alpha_1 \Lambda\rho(\rho m_1 + P)}{\alpha_3 P^2 - (\alpha_1 P + \psi\rho)(m_1\rho + P)}\right\}$$

$$\frac{2(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)}\left(\frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)} + \psi\right)(\delta + \psi) > \left(\frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)} + \psi\right)\frac{\alpha_1 \Lambda\rho(\rho m_1 + P)}{(\alpha_1 P + \psi\rho)(m_1\rho + P) - \alpha_3 P^2}$$

$$\left(\frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)} + \psi\right)\left\{\frac{2\alpha_1 \Lambda\rho(\rho m_1 + P)}{(\alpha_1 P + \psi\rho)(m_1\rho + P) - \alpha_3 P^2} - \frac{(\alpha_1 m_1 + (\alpha_1 - \alpha_3)\frac{K}{\rho}(\rho - \psi - \delta))\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)} - (\mu + \psi + \gamma)\right\}$$

For Endemic equilibrium

$$\left(\frac{\alpha_2\Lambda^2 - \alpha_1\Lambda(\mu m_1 + \Lambda)}{\mu(\mu m_1 + \Lambda)} + \frac{\{\alpha_1 m_1 + (\alpha_1 - \alpha_2)\frac{K}{\rho}(\rho - \psi - \delta)\}\frac{K}{\rho}(\rho - \psi - \delta)}{\frac{K}{\rho}(\psi + \delta) - m_1 - K} - 2\psi - \delta\right) + (\mu + \gamma - \delta) > 0$$

$$\left(\frac{\alpha_1\Lambda(\mu m_1 + \Lambda) - \alpha_2\Lambda^2}{\mu(\mu m_1 + \Lambda)} + \frac{\{\alpha_1 m_1 + (\alpha_1 - \alpha_2)\frac{K}{\rho}(\rho - \psi - \delta)\}\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)}\right)(\delta + \psi)\mu > (\left(\frac{\alpha_1\Lambda(\mu m_1 + \Lambda) - \alpha_2\Lambda^2}{\mu(\mu m_1 + \Lambda)} + \frac{\{\alpha_1 m_1 + (\alpha_1 - \alpha_2)\frac{K}{\rho}(\rho - \psi - \delta)\}\frac{K}{\rho}(\rho - \psi - \delta)}{m_1 + \frac{K}{\rho}(\rho - \psi - \delta)} + \psi\right)(\mu + \psi + \gamma)(\delta + \psi))$$

Also in the work, we have carried out the construction of aLyapunov function for the Vector-Host model (2.1) - (2-3). The results indicated that the DFE is globally asymptotically stable when $R_b \leq 1$(i.e. every solution trajectory of the Vector-Host model (2.1) - (2-3) converges to the largest compact invariant set$M = \{(S_0, I, V)\}$and unstable when $R_b \geq 1$.

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