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MATHEMATICAL MODELLING ON KIDNAPPING WITH APPREHENDED KIDNAPPERS

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ABSTRACT

In this work, we develop a mathematical model on kidnapping by incorporating the concept of apprehended kidnappers in a system of ordinary differential equations describing the evolution and propagation of kidnapping as a crime in human society. It accounts for the interaction between kidnappers and vulnerable humans leading to their abduction for the main purpose of ransom payment. We establish the crime propagation number, CP_N*, in which a* $CP_N < 1$ *guarantees a kidnaping free state. The analysis and the numerical simulations of the model suggests that, increasing the apprehension rate of kidnappers by Security agents is a better and more effective way of ensuring a kidnapping free society.*

1. Introduction

Kidnapping is a crime prohibited by law. It is a violent crime committed by an individual with an illegal means of abducting a person without his/her consent for the purpose of ransom taking. The status of an individual attracts kidnapping and is increasing every day. With the increasing rate of kidnapping in the society, people now move with fear especially those with high financial, political, economic and social status in our society [1,2].In its earliest manifestation, the phenomenon of kidnapping took the form of child abduction for ransom [3]. Over the years, however, kidnapping has metamorphosed into sophisticated organize crime, with immense political and economic underpinning [4]. According to [5], kidnapping is not a new or emerging crime as some observers may want to hold. It has been around as an important criminal pathology of the contemporary society [6].

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In the work of[7], the authors aver that Kidnap victims that purportedly have a ‘kidnap value’ are usually identified by ‘catchers’ who work for a kidnap syndicate. They further observed that, many people think of kidnapping as something that is very rare and not something that could possibly happen to them because they do not have a kidnap value. The reality is that it is a huge risk that can happen to anyone and at any time.

The causes of kidnapping have been attributed to several factors by various authors. Dodo in [8], opined that people go into kidnapping due to ‘’ Quick-Money Syndrome’’. According to him, people see kidnapping as a means of getting rich quickly without much stress. Other causes of kidnapping include; changing value system of the society [9], unemployment [10],poverty[9], political indifference [8],terrorism [11].The authors in [7], proposed rehabilitation as a solution to the menace of kidnapping in the society. However, their model give room for the possibility of a onetime kidnaper who has been successfully rehabilitated back to the society to re-consider and go back to kidnapping when faced with similar economic situations before he ventured into kidnapping in the first place and remembering the gains of his/her kidnapping activities.

Hence, in this work, we present a mathematical model that recommends apprehension and subsequent elimination of the apprehended kidnappers as a likelysolution to the menace of kidnapping threatening the peaceful existence of our society. Five sections are considered in this work, with a brief introduction in section 1 and the formulation of the model in section 2. In section 3, we present the model analysis followed by numerical simulations and discussion in section 4. The paper was rounded up in section 5 with a brief discussion and conclusion.The total human population.$N(t)$, is divided into 4 classes namely, susceptible class, $G(t)$, kidnappers population Q(t), kidnappees population, $H(t)$, and apprehended kidnappers Population A(t)). State variables in the model are given in Table 1 and the movement between compartments is summarized in Figure 1.

Table 1 Variables and Definitions

Variables	**Description**
N	Total human population
G	Number of susceptible or vulnerable human Population
H	Kidnappees population
Q	Kidnappers Population
A	Apprehended kidnappers Population

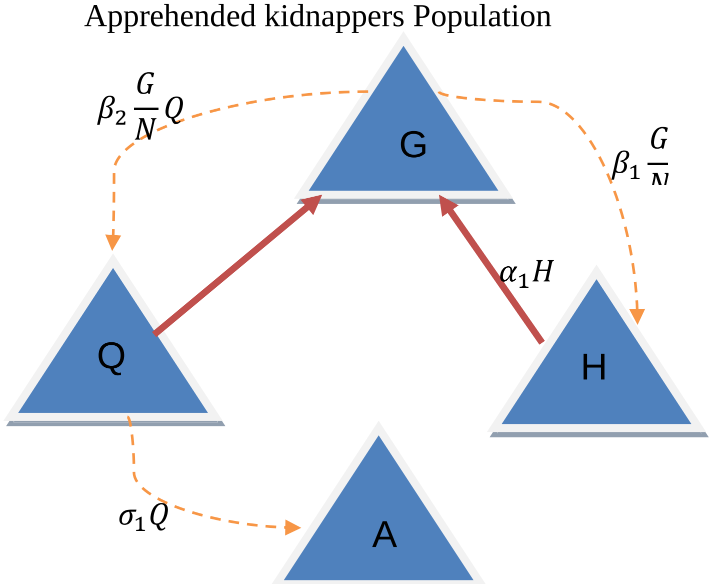

Figure 1: Schematic representation of kidnappers and vulnerable individuals interaction model

2. Model Construction.
In this work, we divided the total number of population into four components namely:
$N = G + Q + H + A$ (2.1)
vulnerable class, $G(t)$, kidnappeesclass, $H(t)$, kidnappers class Q(t), and apprehended kidnappers class $A(t)$.Allhuman beings in a given society could be kidnapped or couldbe introduced into kidnappingif they come in contact with kidnappers.We assume that the contact is necessitated by the willingness of kidnappers to gain some benefits through ransom or vulnerable humans being enticed by their kidnapper friends from the proceeds of kidnapping. Vulnerable individuals (G) moves into the Q compartment at a rate$\beta_2 \frac{G}{N} Q$ where $\frac{G}{N}$ is the probability that the contact is with a vulnerable human and β_2 is the rate constant. We also assume a kidnapping rate constant$\beta_1$such that the transition rate, $\beta_1 \frac{G}{N} Q$, of susceptible humans into the H class is proportional to the contact between kidnappers and vulnerable humans. Recruitment into the vulnerable compartment (G) is through natural birth, λN and rescued kidnapped victims, $\alpha_1 H,$ where $\lambda$and α_1 are per capita birth rate and rescue rate constants respectively with a correction term $\theta_3 N^2$, stopping the population from growing without limit in the absence of the disease, where θ_3 is per capita resource availability for the human population. We assume that both kidnappers and kidnappees die during rescure operations by security agents at a rate$\tau_1 HQ$ and $\tau_2 HQ$, where τ_1 and τ_2 are rate constants. We also assume that kidnappers are apprehended at a rate $\sigma_1 Q$ with σ_1 as a rate constant. We assume that apprehended kidnappers are sentence to life imprisonment as been enacted by Law [12].Hence, the A class is likened to serve as the recovered or removed class in the SIR model of infectious diseases.
The differential equations following the stated assumptions are outlined below:

$$\frac{dG}{dt} = \lambda_1 N + \alpha_1 H - \beta_1 \frac{G}{N} Q - \beta_2 \frac{G}{N} Q - \mu_1 G - \theta_3 N^2 \quad (2.2)$$

$$\frac{dH}{dt} = \beta_1 \frac{G}{N} Q - (\alpha_1 + \mu_1) H - \tau_1 HQ \quad (2.3)$$

$$\frac{dQ}{dt} = \beta_2 \frac{G}{N} Q - (\sigma_1 + \mu_1) Q - \tau_2 HQ \quad (2.4)$$

$$\frac{dA}{dt} = \sigma_1 Q - \mu_1 A \quad (2.5)$$

$$\frac{dN}{dt} = (\lambda_1 - \mu_1) N - (\tau_1 + \tau_2) HQ - \theta_3 N^2 \quad (2.6)$$

Equation (2.6) is obtained by adding equation (2.2) – (2.5).

2.1 Parameter Values and Nondimensionalisation
All the model parameters are listed in Table 2 below together with values taken from various sources.
Table 2. List of model parameters and their values.

parameters	**Description**	**Value**	**Unit**	**Source**
λ_1	Per captia birth rate	0.000104	Day^{-1}	[7,13]
μ_1	Per captia death rate	0.0000356	Day^{-1}	[7,13]
α_1	Rescure rate of Kidnapped Victims	0.098	Day^{-1}	[7,]
β_1	Kidnapping rate	1.58	Day^{-1}	[7,13]
β_2	Recruitment rate of kidnappers	0.305	Day^{-1}	[7,13]
τ_1	Kidnapping induced death of Kidnapped victims	1.18 x10^{-9}	$Human^{-1}Day^{-1}$	[7,13]
τ_2	Death rate of kidnappers due to operation by SecurityAgents	2.35 x10^{-9}	$Human^{-1}Day^{-1}$	[7,13]
σ_1	Apprehension rate of kidnappers	0.35	Day^{-1}	varied

We rescalethe system using, $G^* = \frac{G}{N}$, $H^* = \frac{H}{N}$, $Q^* = \frac{Q}{N}$, $A^* = \frac{A}{N}$, $t^* = \frac{t}{t_0}$, $N^* = \frac{N}{N_0}$, where the variables with the (*) are the nondimensionalvariables such that:

$G^* + H^* + Q^* + A^* = 1$

Then;

$$\frac{dG^*}{dt^*} = \frac{t_0}{N}\left[\frac{dG}{dt} - G^*\left(\frac{dN}{dt}\right)\right]$$
$$\frac{dH^*}{dt^*} = \frac{t_0}{N}\left[\frac{dH}{dt} - H^*\left(\frac{dN}{dt}\right)\right]$$
$$\frac{dQ^*}{dt^*} = \frac{t_0}{N}\left[\frac{dQ}{dt} - Q^*\left(\frac{dN}{dt}\right)\right] \quad (2.7)$$
$$\frac{dA^*}{dt^*} = \frac{t_0}{N}\left[\frac{dA}{dt} - A^*\left(\frac{dN}{dt}\right)\right]$$
$$\frac{dN^*}{dt^*} = \frac{t_0}{N_0}\left(\frac{dN}{dt}\right)$$

With appropriate substitution of the equation (2.7) into equations (2.2)-(2.6) gives the following.

$$\frac{dG^*}{dt^*} = t_0\lambda_1(1 - G^*) + t_0\alpha_1 H^* - t_0\beta_1 G^*Q^* - \beta_2 G^*Q^* + t_0N_0\tau_1 G^*H^*Q^*N^* + t_0N_0\tau_2 G^*H^*Q^*N^* - t_0N_0\theta_3(1 - G^*)N^*$$
$$\frac{dH^*}{dt^*} = t_0\beta_1 G^*Q^* - (\alpha_1 + \lambda_1)H^* - t_0N_0\tau_1 H^*Q^*N^* + t_0N_0\tau_1 H^{*2}Q^*N^* + t_0N_0\tau_2 H^{*2}Q^*N^* + t_0N_0\theta_3 H^*N^*$$
$$\frac{dQ^*}{dt^*} = t_0\beta_2 G^*Q^* - (\sigma_1 + \lambda_1)Q^* - t_0N_0\tau_2 H^*Q^*N^* + t_0N_0\tau_1 H^*Q^{*2}N^* + t_0N_0\tau_2 H^*Q^{*2}N^* + t_0N_0\theta_3 Q^*N^* \quad (2.8)$$
$$\frac{dA^*}{dt^*} = t_0\sigma_1 Q^* - t_0\lambda_1 A^* + t_0N_0\tau_1 A^*H^*Q^*N^* + t_0N_0\tau_2 A^*H^*Q^*N^* + t_0N_0\theta_3 A^*N^*$$
$$\frac{dN^*}{dt^*} = t_0(\lambda_1 - \mu_1)N^* - t_0N_0\tau_1 H^*Q^*N^{*2} - t_0N_0\tau_2 H^*Q^*N^{*2} + t_0\theta_3 N_0 N^2$$

Now, we define the following non-dimensional parameters as follows:

$$t_0 = \frac{1}{\sigma_1}, \lambda = \frac{\lambda_1}{\sigma_1}, a = \frac{N_0\beta_1}{\sigma_1}, b = \frac{N_0\beta_2}{\sigma_1}, d = \frac{N_0\tau_1}{\sigma_1}, e = \frac{N_0\tau_2}{\sigma_1}, \alpha = \frac{\alpha_1}{\sigma_1}, \mu = \frac{\mu_1}{\sigma_1}, f = \frac{\theta_3 N_0}{\sigma_1} \quad (2.9)$$

By substituting (3.9) into (3.8) and dropping the (*). We obtained the nondimensional system:

$$\frac{dG}{dt} = \lambda(1 - G) + \alpha H - (a + b)GQ + (d + e)GHQN + f(G - 1)N \quad (2.10)$$
$$\frac{dH}{dt} = aGQ - (\lambda + \alpha)H - dHQN + (d + e)H^2QN + fHN \quad (2.11)$$
$$\frac{dQ}{dt} = bGQ - (\lambda + 1)Q - eQHN + (d + e)HQ^2N + fQN \quad (2.12)$$
$$\frac{dA}{dt} = Q - \lambda A + (d + e)AHQN + fAN \quad (2.13)$$
$$\frac{dN}{dt} = (\lambda - \mu)N - (d + e)HQN^2 - fN^2 \quad (2.14)$$

3. Model Analysis

3.1. Establishing the Crime Propagation Number, C_{P_N}

The ‘Crime Propagation Number’ (CP_N) as used in [7, 13], can be defined as the expected number of kidnapping that would arise when a single kidnapper is introduced in a susceptible (kidnapping free)

society. Kidnapping cases will naturally die out even if it rises in the society if our $CP_N < 1$ and kidnapping cases would prevail in the society if$CP_N > 1$.
Using the next generation matrix approach [14, 15,16], we consider the equation

R'= FR-VR
Where$R' = \frac{dR}{dt}$,

F=$\begin{bmatrix} 0 & a & 0 \\ 0 & b & 0 \\ 0 & 1 & 0 \end{bmatrix}$, V=$\begin{bmatrix} h_1 & 0 & 0 \\ 0 & h_2 & 0 \\ 0 & 0 & \mu \end{bmatrix}$, R=$\begin{bmatrix} H \\ Q \\ A \end{bmatrix}$

Where F represents emergence of new kidnappers and kidnapped victims, and V represents movement of kidnap cases within the various classes and R is the kidnapping control vector. We assume a non-negative matrix $G = FV^{-1}$that gives a unique, positive and real eigenvalue. We solve for inverse (V^{-1}) of the matrix V

$$V^{-1} = \frac{1}{\mu h1h2}\begin{bmatrix} \mu h_2 & 0 & 0 \\ 0 & \mu h_1 & 0 \\ 0 & 0 & h_1h_2 \end{bmatrix}$$

where, $h_1 = \alpha + \mu$ and h_2= $1 + \mu$

Then, $G = FV^{-1} = \frac{1}{\mu h1h2}\begin{bmatrix} 0 & a & 0 \\ 0 & b & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} \mu h_2 & 0 & 0 \\ 0 & \mu h_1 & 0 \\ 0 & 0 & h_1h_2 \end{bmatrix}$

$$G = \frac{1}{\mu h1h2}\begin{bmatrix} 0 & a\mu h_1 & 0 \\ 0 & b\mu h_1 & 0 \\ 0 & \mu h_1 & 0 \end{bmatrix}$$

We now look for the max (σ), the largest eigenvalue such that $|G - \sigma I|$=0

$$\begin{vmatrix} -\sigma & \frac{a}{h2} & 0 \\ 0 & \frac{b}{h2} - \sigma & 0 \\ 0 & \frac{1}{h2} & -\sigma \end{vmatrix} = 0$$

$$\sigma\hat{}2\left(\frac{-\sigma h2 + b}{h2}\right) = 0$$

$\sigma = 0$ orσ=$\frac{b}{h2}$

$\therefore CP_N = \frac{b}{\mu+1}$ (3.1)

3.2. Positivity, Existence and Uniqueness of Solution

The model is described in the domain

$\Omega \in R^5 = \{G, Q, H, A, N$:G ≥ 0 , Q ≥ 0 , H ≥ 0 ,A ≥ 0,N> 0 , G +Q+H+A = 1} (3.2)

Suppose at t = 0 all variables are non-negative, then G(0)+Q(0)+H(0)+A(0) = 1. If G = 0, and all other variables are inΩ, then $\frac{dG}{dt}$ ≥ 0. This is also the case for all other variables in (2.11) − (2.13). If N = 0, then $\frac{dN}{dt} = 0$. But if N> 0 and assuming λ>μ, then with appropriate initial conditions, $\frac{dN}{dt}$> 0 for all values of t> 0. We note that the right-hand side of (2.10) − (2.14) is continuous with continuous partial derivatives, so solutions exist and are unique. The model is therefore mathematically and criminologically well posed with solutions in Ω for all t∈ [0, ∞)

3.3. Steady State Solution

It can easily be shown from the system that the kidnap free state is (G, Q,H, A) = (1, 0, 0, 0). In the absence of a kidnapper, $Q = H = 0$ and substituting this into the right hand side of (2.13) we obtain A = 0. Further substitution of the values of Q, H and A into (3.2) gives G = 1.
At the kidnapping free-state, all humans are entirely susceptible and we obtain from (3.14) the following logistic equation,

$$\frac{dN}{dt} = r_1 N - fN^2 \qquad (3.3)$$

With solution

$$N(t) = \frac{KN_O}{N_O + (K - N_O)e^{-r_1 t}} \qquad (3.4)$$

Where $r_1 = \lambda - \mu$ and $K = \frac{r_1}{f}$. As $t \to \infty, N(t) \to K,$ which is the carrying capacity of the environment.

3.4 Local Stability Analysis of the Disease-Free Equilibrium (E_0)

Now, let us consider the Jacobian Matrix of the system at the kidnapping-free state, $(G, Q, H, A) = (1,0,0,0)$

$$\begin{bmatrix} -\mu & -h_3 & \alpha & 0 \\ 0 & h_4 & 0 & 0 \\ 0 & a & -h_1 & 0 \\ 0 & 1 & 0 & -\mu \end{bmatrix} \qquad (3.5)$$

$h_3 = a + b$ and $h_4 = b - h_2 = h_2(CP_N - 1)$ from (3.1)

Thecharacteristics polynomial of (3.5) in terms of τ^* is given as:

$$(\mu + \tau^*)(\mu + \tau^*)(h_1 + \tau^*)(h_4 - \tau^*) = 0 \qquad (3.6)$$

Three of the eigenvalues are strictly negative and the fourth, which is expressed as
$\tau^* = (\mu + 1)(CP_N - 1)$, is also negative whenever the crime propagation number is less that unity.
Thus, if $CP_N < 1$, meaning $b < \mu + 1$ and the coefficients of the linear polynomial of (3.6) are all positive and non zero; so by the Descartes' rule of signs there is no positive real eigenvalue. Thus Routh Hurwitz stability condition as stated in [17] is satisfied. We observe that if $CP_N > 1$, $b > \mu + 1$, then there is at least one sign change and by the Descartes' rule of sign there exists at least one positive real eigenvalue. Hence we conclude that the kidnap free state is unstable if $CP_N > 1$. We note that $CP_N = 1$ is a bifurcation surface in the (μ, b) parameter plane.

3.5 Global Stability Analysis

Lemma 3.2. The kidnapping free equilibrium of the model equations (2.10) to (2.14) is globally asymptotically stable (GAS) if $R_0 < 1$ and in addition, if F_1 and F_2 holds.

Proof

We show that:

$(F_1) \frac{dX}{dt} = F(X, 0)$, E_0 is globally asymptotically stable (g.a.s.), and

$(F_2) \hat{G}(X, Y) = AY - G(X, Y) \geq 0 \; \forall \; (X, Y) \in \Omega$ holdsas used in [18, 19].

We write the modelequations (2.10) to (2.14) in the form

$$\frac{dX}{dt} = F(X, Y) = [\lambda(1 - G) + \alpha H - (a + b)GQ + (d + e)GHQN + f(G - 1)N]$$

$$\frac{dY}{dt} = G(X,Y) = \begin{bmatrix} bGQ - (\lambda+1)Q - eQHN + (d+e)HQ^2N + fQN \\ aGQ - (\lambda+\alpha)H - dHQN + (d+e)H^2QN + fHN \\ Q - \lambda A + (d+e)AHQN + fAN \end{bmatrix}$$

Where $X = (G)$and $Y = (Q, H, A)$,with the components of $X \in R$, denoting susceptible population and the components of $Y \in R^3$.

Now,

$$F(X,0) = [\lambda(1-G) + f(G-1)N]$$

$$\frac{dG}{dt} = \lambda(1-G) + (G-1)(\lambda-\mu)$$

$$\frac{dG}{dt} = -\mu G +$$

$$\frac{dG}{dt} + \mu G = \mu$$

Integrating factor (IF)= $e^{\int(\mu dt} = e^{\mu t}$, then;

$$G.e^{\mu t} = \int e^{\mu t}\mu\, dt$$

$$G.e^{\mu t} = \frac{1}{\mu}e^{ut}\mu + k$$

$$G(t) = 1 + ke^{-\mu t}$$

Thus,

$G(t) = 1 = G_0\ as\ t \to \infty$.

Hence, F_1 holds.

$$\frac{dY}{dt} = G(X,Y) = \begin{bmatrix} bGQ - (\lambda+1)Q - eQHN + (d+e)HQ^2N + fQN \\ aGQ - (\lambda+\alpha)H - dHQN + (d+e)H^2QN + fHN \\ Q - \lambda A + (d+e)AHQN + fAN \end{bmatrix}$$

$$A = \begin{bmatrix} bG - (\mu+1) & 0 & 0 \\ aG & -(\alpha+\mu) & 0 \\ 1 & 0 & -\mu \end{bmatrix}$$

$$AY = \begin{bmatrix} bGQ - (\mu+1)Q \\ aGQ - (\mu+\alpha)H \\ Q - \mu A \end{bmatrix}$$

$$\hat{G}(X,Y) = AY - G(X,Y)$$

$$= \begin{bmatrix} bGQ - (\mu+1)Q \\ aGQ - (\mu+\alpha)H \\ Q - \mu A \end{bmatrix} - \begin{bmatrix} bGQ - (\lambda+1)Q - eQHN + (d+e)HQ^2N + fQN \\ aGQ - (\lambda+\alpha)H - dHQN + (d+e)H^2QN + fHN \\ Q - \lambda A + (d+e)AHQN + fAN \end{bmatrix}$$

$$\therefore \hat{G}(X,Y) = \begin{bmatrix} -dQHN \\ -eQHN \\ -(d+e)QHAN \\ 0 \end{bmatrix}$$

$$\to \hat{G}(X,Y) \geq 0\ \forall\ (X,Y) \in \Omega\ iff\ e = d = 0$$

Thus conditions$F_1 and\ F_2$are satisfiedandthe model isglobally asymptoticallystablefor $e = d = 0$.

4. Numerical Simulations

We used MATLAB ODE45 routine solver for the numerical simulations with the graphical representations of the various compartments of kidnapping are given below:

The parameters used in the simulations are: $\lambda = 0.076, a = 0.028, b = 1.012, d = 0.04, e = 0.086, \alpha = 0.13, \mu = 0.036, f = 0.024$.

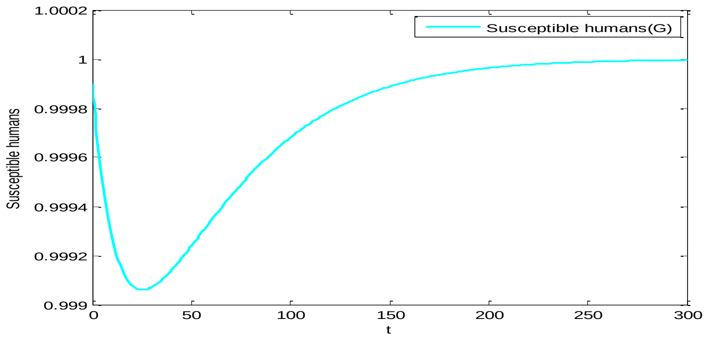

Fig1 Graph of the susceptible class

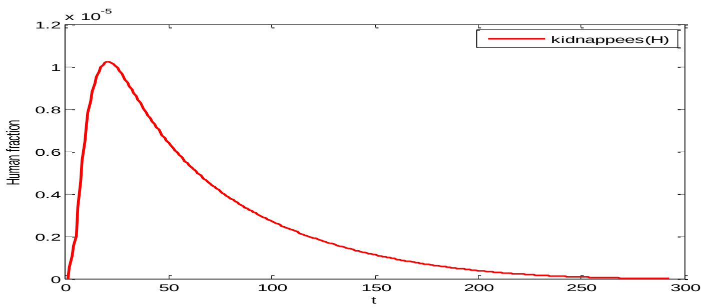

Fig 2 Result showing the effect of high apprehension rate on Kidnappees population

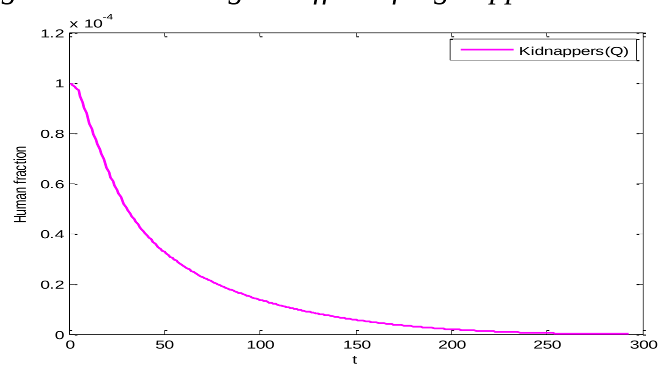

Fig 2 Result showing the effect of high apprehension rate on Kidnappers population

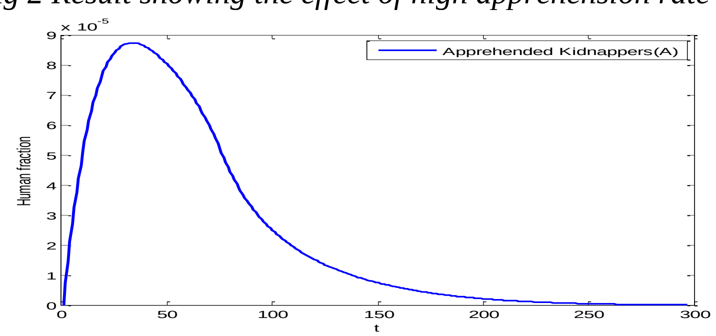

Fig3 Result showing the effect of high apprehension rate on Kidnappees population.

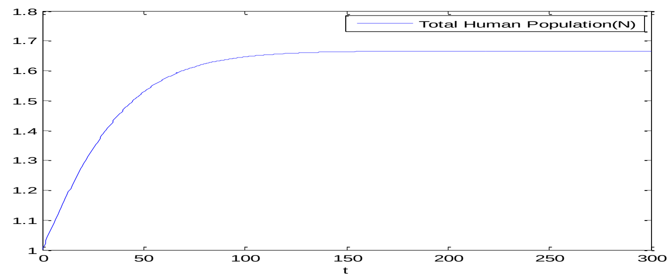

Fig4 Graph of Total Human Population

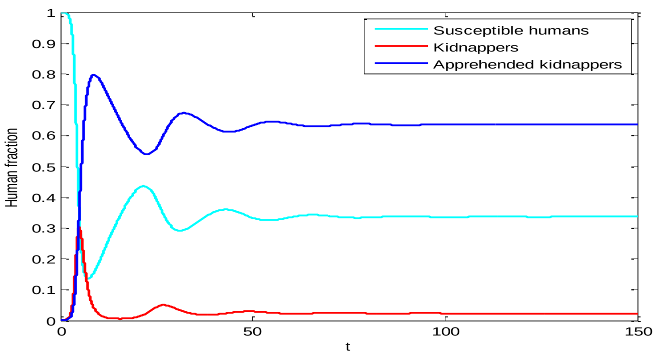

Fig5 Showing established kidnapping scenario when $CP_N > 1$. The parameter values used are the same as those above except that we have used *b = 3.08.*

5. Discussion and Conclusion:

We notice in Fig1 that, Susceptible human population drops sharply at the beginning before increasing. The initial drop in the population is as a result of the introduction of kidnappers into the Society, which leads to lot of vulnerable persons becoming victims of kidnapping. The later increase in the susceptible human population as shown above implies that the control of kidnapping activities in the society by Security Agents through apprehension and eventual sentencing of kidnappers to life imprisonment of kidnappers, would lead to decrease in kidnapping related activities in the society resulting in an increase in the susceptible population.

Fig2 shows that the more kidnappers are apprehended, the more kidnappers population decreases, which implies that, continuous apprehension of kidnappers by security agents would possibly cause the kidnappers population to die out as there would be no more kidnappers in the society to perpetuate the crime of kidnaping.

We notice from Fig3, an initial increase in the kidnappees population before dropping to zero. This increase is as a result of kidnapping related activities as kidnappers are introduced into the society. But, as Security agents swing into action by apprehending and jailing more kidnappers, Kidnappees population decreases and eventual drops to zero.The graph of the total human population as shown above is growing logistically as kidnappers population declines due to increasing number of apprehended kidnappers and less of kidnapping related deaths.

The analysis and simulations of the model shows that, kidnapping can be eradicated in the society, if the recruitment of kidnappers are discouraged through continuous apprehension and elimination of kidnappers by security agents.

To control the menace of kidnapping in the Society, ransom payment to secure release of kidnapped victims should be discouraged. Rather, there should be renewed efforts on the side of Government to recruit and train more security agents into the anti-kidnapping squad in other to apprehend more kidnappers. The kidnapping free equilibrium is locally asymptotically stable andglobally asymptotically stable in the absence kidnapping related death due to rescue operations.

We emphasize here that the crime propagation number is a veritable tool that may be leveraged on when considering measures of solving kidnapping problems. From our result, the crime propagation number can be reduced by reducing the nondimensional parameter, *b*. We note that the parameter, *b*, is the ratio of the recruitment rate of kidnappers to the apprehension rate of kidnappers in dimensional form. Thus, a possible control measure should be targeted at reducing the recruitment rate and increasing the apprehension rate of kidnappers. Fig1 to Fig4 show case where the crime propagation number is less than unity but variation of the parameter b in favour of kidnapping creates a bad scenario for which kidnapping becomes fully established. This is the case described in Fig5.

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