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STABILITY ANALYSIS ON THE DYNAMICS OF ATMOSPHERIC CO_2 UNDER THE INFLUENCE OF DEFORESTATION AND INDUSTRIAL AND NON-INDUSTRIAL EMISSIONS WITH NATURAL ABSORBENTS

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Abstract

Many mathematical models have been developed to investigate the dynamics atmospheric gases that contribute to global warming and atmospheric carbon dioxide is one of the leading gases. In this work, we developed mathematical model for the dynamics of atmospheric carbon dioxide (from human respiration, due to emission from industrial and non-industrial based sources of atmospheric CO_2, precipitation and particulate matters, forest biomass, secondary species formed from reaction of CO_2 with precipitation and particulate matter). In developing the model, we adopted compartmental modeling approach where a system of nonlinear differential equation for the dynamics of entities mentioned was developed. Existence and uniqueness of the solution of the model equation was proved as well as the positivity of the solution. Three feasible equilibrium points of the model equation were established and local stability analysis showed that the points are locally asymptotically stable (LAS). Global stability analysis of the equilibrium point was carried out using lyapunov method and the result showed that the equilibrium point is globally asymptotically stable (GAS). We carried out numerical simulation and the result shows that the concentration of atmospheric CO_2 increases with an increase in the rate of emission of CO_2 by industrial and non-industrial CO_2 emitting devices and deforestation.

***Keywords*:** Equilibrium point, Precipitate, Global warming, Particulate matter, Stability, Ecosystem, forest biomass, Deforestation and Greenhouse effect

1.0 INTRODUCTION

The atmosphere is composed of a mix of several different gases in differing amounts. The permanent gases whose percentages do not change from day to day are nitrogen, oxygen and argon. Nitrogen accounts for 78% of the atmosphere, oxygen 21% and argon 0.9%. Gases like carbon dioxide, nitrous oxides, methane, and ozone are trace gases that account for about a tenth of one percent of the atmosphere [1]. Natural sources of atmospheric CO_2 includes respiration from animals, death decayed plant and animal, volcanoes, hot springs and geysers and it is freed from carbonate rocks by dissolution of carbon in water and acids. Other source of atmospheric CO_2 includes; industrial emission of CO_2, fossil fuel contribution from CO_2 emitting devices and the indirect effect of deforestation for agriculture, commercial, housing and industrialization purposes [2].

Atmospheric carbon dioxide plays a vital role in the ecosystem. Owing to the fact that it contributes to greenhouse effect, it keeps the earth warm enough to evolve and sustain life as we know it. Animals, including humans owe their existence to green plants that use energy from sunlight to convert CO_2 and water molecules into carbohydrates, releasing oxygen into the atmosphere in the process. Plants get the carbon they need from CO_2 in the air, and they obtain other essential nutrients from the soil.

Though atmospheric CO_2 plays an essential role to ecosystem especially in the sustainability of lives, its increase in the atmosphere causes global warming which tend to be more disastrous to the entire ecosystem. Global warming has led to several undesirable consequences such as, poor air quality, rise in sea levels, melting of glaciers, decrease in rainfall,

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draught and heat waves [3-12]. Again, climate change influence the incidence diseases such as malaria, diarrhea and fever [13].

However, to curtail the above mention effects of increasing level of atmospheric CO_2, it reduction in the atmosphere has become the focus point of every scientist. Moreover, it can be removed from the atmosphere viz; absorption by plant and stored as forest biomass during photosynthesis, absorption due to reaction with precipitates such as rain and water vapour, snow and absorption due to exposed particulate matters that can interact with atmospheric CO_2.

Many researches have been conducted to understand the dynamics of atmospheric carbon dioxide in order to curtail the consequences of its increase and mathematical research approaches are not left out. Some of the researches conducted are as follows;

Naresh, et al [14] investigated the removal of gaseous pollutant and particulate matters in the atmosphere by means of precipitation and the result shows that the removal of gases pollutant is inversely proportional to the amount of precipitation obtained.

Misra and Verma [13] considered the influence of human population and forest biomass on the dynamics of atmospheric carbon dioxide (CO_2) gas and their result reveals that anthropogenic emission of CO_2 in the atmosphere increase as human population decline.

Shukla *et al*. [15] investigated the removal of carbon dioxide from the atmosphere by spraying external species above the sources of emissions as control strategies. Their result shows that concentration of atmospheric CO_2 is inversely proportional to the concentrations of liquid splash and particulate matter. The reactions of the external splashes are as follows;

$CO_2 + H_2O \rightarrow H_2CO_3$

$CO_2 + CaO \rightarrow CaCO_3$

Other researches includes; the role of cloud droplets on the removal of gaseous pollutants was also considered by Sundar and Naresh [16] and mathematical model for the removal of gaseous pollutant and particulate matters in the atmosphere by rain in the city [17].

Shukla *et al*. [18] focus on the depletion of forest resources by population and industrialization and their conservation by green belts plantation. In their study, a nonlinear differential equation was developed to analyze the forest depletion due to population expansion and industrialization. They considered greenbelt plantation as control strategy. They assumed that the growth rate of cumulative biomass density is proportional to the depletion of the cumulative biomass of the forest resources. The dependent variable used for the model was cumulative biomass of forest resources, human population, industrialization density and cumulative biomass of green belt plantation. Analysis shows that forest resources will go on extinction in the absence of conservation; however, with conservation, under certain conditions, they can be maintained very closely to the level before depletion.

Archana *et al.* [19] focused on modeling and analysis of the impact of awareness programs on rural population for conservation of forest biomass resources. A mathematical model equation with cumulative biomass density of resources, the population density in the rural areas which is divided into aware (educated) individuals on the need of conservation of forest resources and the cumulative density of public awareness programs by various sources including media was proposed. The equilibrium point of the model equation was established. Local and global stability of the equilibrium point was analyzed followed by obtaining the persistence of the equation and numerical simulation. The result shows that as the density of aware population increases, the cumulative biomass density of forest resources increases leading to sustainable conservation of resources.

Based on the available literatures it is obvious that none of the models considered the dynamics of atmospheric CO_2 under the influence of atmospheric precipitates and particulate matter as CO_2 natural absorbents considering the contributions of forest biomass, human population, emission from industrial and non-industrial emitting devices, deforestation and emission of CO_2 due to anthropogenic sources (such as bush burning, volcanic eruption, etc.) to the atmosphere. The research work by Misra and Verma [13] and Shukla et al. [15] will serve as the frame work of this study

2.0 METHODS AND MATERIALS

2.1 Model Assumptions

In forming the model, it is assumed that the rate of emission of CO_2 from natural means such as volcanic eruption, bush burning etc. is assumed to be constant, production of CO_2 by industries and non-industrial CO_2 emitting devices is assume to be constant and the rate of absorption of carbon dioxide by the atmosphere is assume to be proportional to the concentration of CO_2 emitted by industries and carbon emitting devises. Again, the rate of natural occurrence of particulate matters in atmosphere is assumed to be constant, the formation rate of atmospheric precipitate is assumed to be constant and finally, absorption rate of CO_2 by other means such as oceans and sea weeds before escaping to the atmosphere is proportional to the concentration of CO_2 produced by humans.

2.2 Notations

C_H (t) = Concentration of CO_2 emitted due to human respiration at time t
$C(t)$ = Concentration of atmospheric CO_2 at time t
$C_p(t)$ = Concentration of particulate matter at time t
$C_a(t)$ = Concentration of secondary species formed due to interaction of atmospheric CO_2 and precipitate at time t
$C_I(t)$ = Concentration of CO_2 emitted by industrial activities and non-industrial CO_2 emitting device only at time t
$C_{pa}(t)$ = Concentration of secondary species formed due to interaction of CO_2 with particulate matter at time t
$C_f(t)$ = Concentration of forest biomass at time t
$C_i(t)$ = Concentration of atmospheric precipitation at time t

σ = rate at which atmospheric precipitates fail to react with CO_2
ϕ = Rate at which naturally occurred particulate matter is being removed to form secondary species due to interaction with atmospheric CO_2
K=Carrying capacity of humans
K_1= Carrying capacity of forest biomass
δ_1 = Rate of deforestation
β_1 = Rate at which particulate matter fail to react with CO_2
ε = Rate at which atmospheric CO_2 fail to react with precipitates
θ_0 = Natural depletion rate of secondary species due to interaction of CO_2 with atmospheric precipitate
θ = Rate at which precipitate is formed in the atmosphere.
β_0 =Natural depletion rate of Atmospheric CO_2
α_1 =Rate of emission of CO_2 by industrial and non-industrial CO_2 emitting devices
β_2= Absorption rate of atmospheric CO_2 by forest biomass absorbents
α_0= Natural depletion rate of CO_2 produced by industries and non-industrial CO_2 emitting devices
τ = Production rate of CO_2 by industrial and non-industrial CO_2 emitting devices
γ =Rate at which particulate matters is naturally formed and exposed to the atmosphere
π_0= Natural depletion rate of naturally occurred particulate matter
λ_2 = Rate at which atmospheric CO_2 fail to react with particulate matter
α_2 = Removal rate of atmospheric CO_2 to form secondary species due to interaction of CO_2 and particulate matter
γ_0 =Natural depletion rate of secondary species form due to interaction of atmospheric CO_2 and particulate matter
π = Absorption rate of CO_2 emitted due to human respiration by other means (such as oceans and sea weeds) before escaping to the atmosphere
α = Logistic growth rate of forest biomass
β =Rate of CO_2 production due to human respiration
λ = Emission rate of CO_2 from natural anthropogenic sources (such as bush burning, volcanic eruption, etc.) to the atmosphere
δ_2 = Removal rate of atmospheric CO_2 to form secondary species due to interaction with atmospheric precipitates.
ε_0 = Natural depletion rate of atmospheric precipitates
λ_1 = Removal rate of particulate matter to form secondary species due to interaction with atmospheric CO_2

2.3 Derivation of the Model

The model is compartmentalized into eight sub-compartments, viz: Concentration of CO_2 due to human respiration (C_H), Concentration of CO_2 due to industrial and non-industrial emitting devices (C_I), Concentration of atmospheric CO_2 (C), Concentration of particulate matter (C_p), Concentration of atmospheric precipitates (C_i), Concentration of forest biomass (C_f), Concentration of secondary species formed due to interaction of particulate matter and atmospheric CO_2 (C_{pa}) and Concentration of secondary species formed due to the interaction of atmospheric precipitates and atmospheric CO_2 (C_a).

The dynamics of CO_2 can be described as follows; Concentration of CO_2 from human respiration (C_H) increases logistically at rate β and reduces due to other absorbing means (such as sea weeds, oceans, etc.) at a rate π, and escaping to the atmosphere at a rate ($1-\pi$). Concentration of CO_2 due to industrial and non-industrial CO_2 emitting device (C_I) increases at rate τ and reduces due to lost to atmosphere at rate α_1, and due to natural depletion at a rate α_0. Atmospheric CO_2 (C) increases due to the escaped proportion from human respiration and industrial and non-industrial CO_2 emitting devices, it also increases at rate λ due to natural source of emission (such as bush burning, volcanic eruption) and due to reaction failures of CO_2 with atmospheric precipitate and particulate matter at rates ε and λ_2 respectively. Atmospheric CO_2 also decreases due reaction with atmospheric precipitates and particulate matter at rates α_2 and δ_2 respectively, due to absorption by forest biomass and natural depletion at rates β_0 and β_2 respectively. Concentration of forest biomass (C_f) increases logistically at rate α and due to conversion of CO_2 by forest biomass for growth and development at a rate β_2, and decreases due to deforestation at a rate δ_1 The concentration of atmospheric precipitates (C_i) increases due to natural formation of atmospheric precipitates at constant rate θ and due to reaction failure of some proportion of the precipitate with atmospheric CO_2 at rate σ and reduces due to interaction of the formed precipitate with atmospheric CO_2 and natural depletion at rates λ_1 and ε_0 respectively. The concentration of particulate matter (C_p) increases at constant rate γ due to natural occurrence of particulate matter, and at rate β_1 due to reaction failure of proportion of particulate matter with atmospheric CO_2, it reduces as a result of interaction with atmospheric CO_2 and natural depletion at rates ϕ and π_0 respectively. Concentration of secondary species due to interaction of atmospheric CO_2 and precipitates (C_a) increases due to interaction of naturally formed precipitates with atmospheric CO_2 and reduces due to natural depletion at a rate θ_0, due to reaction failure of some proportion of naturally formed precipitates with CO_2. Finally, the concentration of secondary species (C_{pa}) due to interaction of atmospheric CO_2 with particulate matter increases as a result of interaction of atmospheric CO_2 with particulate matter and reduces due to reaction failure of some proportion of atmospheric CO_2 with particulate matter, and also reduces due to natural depletion at a rate of γ_0.

2.4 Model Formation

Based on the stated assumptions, a system of nonlinear ordinary differential equations on the dynamics of atmospheric carbon dioxide is given by;

$$\frac{dC_H}{dt} = \beta C_H\left(1-\frac{C_H}{K}\right) - (1-\pi)C_H - \pi C_H \tag{1}$$

$$\frac{dC}{dt} = \lambda + \alpha_1 C_I + (1-\pi)C_H - (1-\lambda_2)\alpha_2 CC_p - (1-\varepsilon)\delta_2 CC_i - \beta_2 CC_f - \beta_0 C \tag{2}$$

$$\frac{dC_f}{dt} = \alpha C_f\left(1-\frac{C_f}{K_1}\right) + \beta_2 CC_f - \delta_1 C_f \tag{3}$$

$$\frac{dC_a}{dt} = (1-\varepsilon)\delta_2 CC_i + (1-\sigma)\lambda_1 CC_i - \theta_0 C_a \tag{4}$$

$$\frac{dC_i}{dt} = \theta - (1-\sigma)\lambda_1 CC_i - \varepsilon_0 C_i \tag{5}$$

$$\frac{dC_{pa}}{dt} = (1-\lambda_2)\alpha_2 CC_p + (1-\beta_1)\phi CC_p - \gamma_0 C_{pa} \tag{6}$$

$$\frac{dC_p}{dt} = \gamma - (1-\beta_1)\phi CC_p - \pi_0 C_p \tag{7}$$

$$\frac{dC_I}{dt} = \tau - (\alpha_0 + \alpha_1)C_I \tag{8}$$

Note that $C(0) > 0$, $C_H(0) > 0$, $C_I(0) > 0$, $C_a(0) \geq 0$, $C_P(0) \geq 0$. $C_{pa}(0) \geq 0$ $C_f(0) > 0$ and $C_i(0) \geq 0$ and $\alpha_0 > \alpha_1$, $\delta_2 > \varepsilon$, $\alpha \geq \beta_2$, $\alpha > \lambda_2$, β_1, $\phi \geq \alpha_2$, $\delta_2 \geq \lambda_1$ and $\lambda_1 > \sigma$, ε

3.0 RESULT AND DISCUSSION

3.1 Results

3.1.1 Positivity of the solution

Lemma 1: Let the initial values of the variables be $\{C(0) \geq 0, C_H(0) \geq 0, C_I(0) \geq 0, C_a(0) \geq 0, C_P(0) \geq 0, C_{pa}(0) \geq 0, C_f(0) \geq 0 \text{ and } C_i(0) \geq 0\}$ in Ω, then solution set be $\{C(t)\ C_H(t)\ C_I(t)\ C_a(t)\ C_P(t)\ C_{pa}(t)\ C_f(t) \text{ and } C_i(t)\}$ is positive for all $t \geq 0$

Proof:

To prove positivity of the solution, we solve each equation with respect to the state variables and we have the following

$$C_H(t)=\frac{K(1-\beta)}{\beta\left(M(1-\beta)e^{(1-\beta)t}-1\right)}\geq 0 \quad \forall t>0, \tag{9}$$

$$C(t)=\frac{\lambda\left(1+\beta_0 Qe^{-\beta_0 t}\right)}{\beta_0}\geq 0 \quad \forall t>0 \tag{10}$$

$$C_f(t)=\frac{K_1(\alpha-\delta_1)}{\alpha\left(1+A(\alpha-\delta_1)e^{-(\alpha-\delta_1)t}\right)}\geq 0 \quad \forall t>0 \tag{11}$$

$$C_a(t)=Ue^{-\theta_0 t} \qquad \forall t>0 \tag{12}$$

$$C_i(t)=\frac{\theta\left(1+\varepsilon_0 Xe^{-\varepsilon_0 t}\right)}{\varepsilon_0} \qquad \forall t>0 \tag{13}$$

$$C_{pa}(t)=Ne^{-\gamma_0 t}\geq 0 \qquad \forall t>0 \tag{14}$$

$$C_p(t)=\frac{\gamma\left(1+\pi_0 Be^{-\pi_0 t}\right)}{\pi_0} \qquad \forall t>0 \tag{15}$$

$$C_I(t)=\frac{\tau\left(1+(\alpha_0+\alpha_1)De^{-(\alpha_0+\alpha_1)t}\right)}{\tau(\alpha_0+\alpha_1)} \quad \forall t>0 \tag{16}$$

Since the initial conditions C(0), $C_H(0)$, $C_I(0)$, $C_a(0)$, $C_P(0)$, $C_{pa}(0)$, $C_f(0)$ and $C_i(0)$ are positive for t = 0, Hence it is obvious that{C(t) $C_H(t)$ $C_I(t)$ $C_a(t)$ $C_P(t)$ $C_{pa}(t)$ $C_f(t)$ and $C_i(t)$} is positive for all t ≥ 0. Therefore proved

3.1.2 Equilibrium Points of the Model

The equilibrium points of the system of non-linear ordinary differential equation describing the dynamics were obtained by setting the derivatives of the model equation to zero. i.e.

$$\frac{dC_H}{dt}=\frac{dC}{dt}=\frac{dC_f}{dt}=\frac{dC_a}{dt}=\frac{dC_i}{dt}=\frac{dC_{pa}}{dt}=\frac{dC_p}{dt}=\frac{dC_I}{dt}=0 \tag{17}$$

Thus at equilibrium point, the system of equation becomes;

$$\beta C_H\left(1-\frac{C_H}{K}\right)-(1-\pi)C_H-\pi C_H=0 \tag{18}$$

$$\lambda+\alpha_1 C_I+(1-\pi)C_H-(1-\lambda_2)\alpha_2 CC_p-(1-\varepsilon)\delta_2 CC_i-\beta_2 CC_f-\beta_0 C=0 \tag{19}$$

$$\alpha C_f\left(1-\frac{C_f}{K_1}\right)+\beta_2 CC_f-\delta_1 C_f=0 \tag{20}$$

$$(1-\varepsilon)\delta_2 CC_i+(1-\sigma)\lambda_1 CC_i-\theta_0 C_a=0 \tag{21}$$

$$\theta-(1-\sigma)\lambda_1 CC_i-\varepsilon_0 C_i=0 \tag{22}$$

$$(1-\lambda_2)\alpha_2 CC_p+(1-\beta_1)\phi CC_p-\gamma_0 C_{pa}=0 \tag{23}$$

$$\gamma-(1-\beta_1)\phi CC_p-\pi_0 C_p=0 \tag{24}$$

$$\tau-(\alpha_0+\alpha_1)C_I=0 \tag{25}$$

It is obvious that the model equation has three feasible equilibrium points namely;

3.1.3 Equilibrium point in the absence of atmospheric precipitate, particulate matter and forest biomass

If C_i = 0, C_p = 0 and C_f = 0, the other variables of the model equation can be solved analytically and equilibrium point in the absence of atmospheric precipitate, particulate matter and forest biomass is given by;

$E_0=\left(C_H^0, C^0, 0,0,0,0,0, C_I\right)$ Where

$$C_H=\frac{K(\beta-1)}{\beta} \tag{26}$$

$$C_I=\frac{\tau}{(\alpha_0+\alpha_1)} \tag{27}$$

$$C=\frac{\beta\lambda(\alpha_0+\alpha)+\alpha_1\beta\tau+K(1-\pi)(\alpha_0+\alpha)(\beta-1)}{\beta\beta_0(\alpha_0+\alpha)} \tag{28}$$

3.1.4 Equilibrium point in the absence of atmospheric precipitate and particulate matter only

If $C_i = 0$ and $C_p = 0$ and $C_f \neq 0$ the equilibrium point in the absence of atmospheric precipitate and particulate matter is; $E_1 = \left(C_H^*, C^*, C_f^*, 0,0,0,0, C_I^*\right)$

Where

$$C_H = \frac{K(\beta - 1)}{\beta} \tag{29}$$

$$C_I = \frac{\tau}{(\alpha_0 + \alpha_1)} \tag{30}$$

$$C_1 = \frac{\left(-(\alpha_0+\alpha_1)(\alpha\beta\beta_2K_1 + \beta_0\alpha\beta - \beta_2\delta_1\beta K_1) + \sqrt{\begin{matrix}((\alpha_1+\alpha_0)(\alpha\beta\beta_2K_1 + \beta_0\alpha\beta - \beta_2\delta_1\beta K_1))^2 + \\ 4(\beta_2^2\beta K_1(\alpha_0+\alpha_1))(\alpha\beta\lambda(\alpha_0+\alpha_1) + \alpha_1\tau\alpha\beta + K\alpha(\beta-1)(1-\pi)(\alpha_0+\alpha_1))\end{matrix}}\right)}{2(\beta_2^2\beta K(\alpha_1+\alpha_0))}$$

or

$$C_2 = \frac{\left(-(\alpha_0+\alpha_1)(\alpha\beta\beta_2K_1 + \beta_0\alpha\beta - \beta_2\delta_1\beta K_1) - \sqrt{\begin{matrix}((\alpha_1+\alpha_0)(\alpha\beta\beta_2K_1 + \beta_0\alpha\beta - \beta_2\delta_1\beta K_1))^2 + \\ 4(\beta_2^2\beta K_1(\alpha_0+\alpha_1))(\alpha\beta\lambda(\alpha_0+\alpha_1) + \alpha_1\tau\alpha\beta + K\alpha(\beta-1)(1-\pi)(\alpha_0+\alpha_1))\end{matrix}}\right)}{2(\beta_2^2\beta K(\alpha_1+\alpha_0))}$$

And

$$C_f = \frac{\alpha K_1 + K_1\beta_2 C - \delta_1 K_1}{\alpha} \tag{31}$$

3.1.4 Equilibrium point in the presence of atmospheric precipitate, particulate matter and forest biomass

Solving for the whole variables of the model equation, we have the equilibrium in the presence of atmospheric precipitate, particulate matter and forest biomass $E_2 = \left(C_H^{**}, C^{**}, C_f^{**}, C_a^{**}, C_i^{**}, C_{pa}^{\ \ **}, C_p^{**}, C_I^{**}\right)$

Where

$$C_H = \frac{K(\beta - 1)}{\beta} \tag{32}$$

$$C_i = \frac{\theta}{\varepsilon_0 + (1-\sigma)\lambda_1 C} \tag{33}$$

$$C_p = \frac{\gamma}{\pi_0 + (1-\beta_1)\phi C} \tag{34}$$

$$C_a = \frac{\theta((1-\varepsilon)\delta_2 C + (1-\sigma)\lambda_1 C)}{\theta_0(\varepsilon_0 + (1-\sigma)\lambda_1 C)} \tag{35}$$

$$C_{pa} = \frac{\gamma(1-\lambda_2)\alpha_2 C + \gamma(1-\beta_1)\phi C}{\gamma_0((1-\beta_1)\phi C + \pi_0)} \tag{36}$$

$$C_I = \frac{\tau}{(\alpha_0 + \alpha_1)} \tag{37}$$

$$C_f = \frac{\alpha K_1 + K_1\beta_2 C - \delta_1 K_1}{\alpha} \tag{38}$$

And C is a polynomial of degree 4 and it is given as;

$$\begin{aligned}
&K_1\beta_2\beta\varepsilon_0\pi_0AXC^4 \\
&+\begin{pmatrix}\beta_2\alpha_1\beta K_1AX + \beta_0\alpha\beta AX - \beta_2\delta_1\beta K_1AX \\ + K_1\beta_2^2\beta AZ\end{pmatrix}C^3 \\
&+\begin{pmatrix}\beta_0\beta\alpha AZ - \beta_2\delta_1K_1\beta AZ + \beta\beta_2^2\varepsilon_0\pi_0K_1A \\ + \alpha_1\beta\beta_2K_1AZ \\ + F\theta A\alpha\beta(1-\sigma)\lambda_1 - DA\alpha\beta\gamma(1-\beta_1)\phi \\ - \alpha BAX - \alpha\beta\alpha_1X - \lambda A\alpha\beta X\end{pmatrix}C^2 \\
&+\begin{pmatrix}\alpha\beta\varepsilon_0\pi_0\beta_0A - \beta_2\delta_1K_1A\beta\varepsilon_0\pi_0 + \beta_2\alpha_1K_1A\beta\varepsilon_0\pi_0 \\ + F\theta A\alpha\beta\varepsilon_0 + DA\alpha\beta\gamma\pi_0 \\ - \alpha BAZ - \alpha_1\alpha\beta Z - \alpha\beta\lambda A\end{pmatrix}C \\
&-(\alpha\varepsilon_0\pi_0AB + \alpha\beta\alpha_1\varepsilon_0\pi_0 + \lambda\alpha\beta\varepsilon_0\pi_0A) = 0
\end{aligned} \tag{39}$$

Where

$A = \alpha_1 + \alpha_0$

$B = K(1-\pi)(\beta - 1)$

$D = (1-\lambda_2)\alpha_2$

$E = \pi_0 + (1-\beta_1)\phi C$

$F = (1-\varepsilon)\delta_2$

$G = \varepsilon_0 + (1-\sigma)\lambda_1 C$

$X = \phi\lambda_1(1-\beta_1)(1-\sigma)$

$Z = \varepsilon_0\phi(1-\beta_1) + \pi_0\lambda_1(1-\sigma)$

3.1.5 Stability Analysis

Proposition 1:

Let $\Omega = (C_H^0, C^0, 0,0,0,0,0, C_I^0)$ be the equilibrium point in the absence of atmospheric precipitate, particulate matter and forest biomass. Ω is locally asymptotically stable if $R_e(\mu_i) \le 0$ (real part) for all $i = 1,2,3,\ldots$ Where μ_i is the eigenvalues of the Jacobian matrix.

Proof

Linearizing the system of equation near the equilibrium point, we have the matrix;

$$\det|J-\mu I| = \begin{vmatrix} B-\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ (1-\pi) & -\beta_0-\mu & -A & 0 & -x & 0 & -m & 0 \\ 0 & 0 & N-\mu & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\theta_0-\mu & y & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -Z-\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\gamma_0-\mu & R & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -G-\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -E-\mu \end{vmatrix} = 0 \quad (40)$$

where

$x = (1-\varepsilon)\delta_2 C^0$

$A = \beta_2 C^0$

$B = \beta - \frac{2\beta C_H^0}{K} - 1$

$m = (1-\lambda_2)\alpha_2 C^0$

$N = \alpha + \beta_2 C^0 - \delta_1$

$y = (1-\varepsilon)\delta_2 C^0 + (1-\sigma)\lambda_1 C^0$

$Z = (1-\sigma)\lambda_1 C^0 + \varepsilon_0$

$G = (1-\beta_1)\phi C^0 + \pi_0$

$E = \alpha_0 + \alpha_1$

$R = (1-\lambda_2)\alpha_2 C^0 + (1-\beta_1)\phi C^0$

With eigenvalues as

$\mu_1 = -\beta_0$,

$\mu_2 = \alpha + \beta_2 C^0 - \delta_1$

$\mu_3 = -((1-\sigma)\lambda_1 C^0 + \varepsilon_0)$,

$\mu_4 = -((1-\beta_1)\phi C^0 + \pi_0)$,

$\mu_5 = -\theta_0$,

$\mu_6 = \beta - \frac{2\beta C_H^0}{K} - 1$,

$\mu_7 = -\gamma_0$

and $\mu_8 = -(\alpha_1 + \alpha_0)$

The equilibrium point $E_0 = (C_H^0, C^0, 0,0,0,0,0,C_I^0)$ is locally asymptotically stable provided that the inequality holds $\beta < \frac{2\beta C_H^0}{K} + 1$ and $\alpha + \beta_2 C^0 < \delta_1$

Proposition 2:

Let $\Omega = (C_H^*, C^*, C_f^*, 0,0,0,0,C_I^*)$ be the equilibrium point with forest biomass and absence of atmospheric precipitate and particulate matter. Ω is locally asymptotically stable if $R_e(\mu_i) \le 0$ (real part) for all $i = 1,2,3,\ldots$ Where μ_i is the eigenvalues of the Jacobian matrix.

Proof

Consider the jacobian matrix evaluated at the equilibrium point with forest biomass and absence of atmospheric precipitate and particulate matter;

$$\det|J-\mu I| = \begin{vmatrix} B-\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ (1-\pi) & -q-\mu & -A & 0 & -x & 0 & -m & 0 \\ 0 & r & F-\mu & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\theta_0-\mu & y & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -Z-\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\gamma_0-\mu & R & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -G-\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -E-\mu \end{vmatrix} = 0 \quad (41)$$

Where (in addition to equation (40))

$$F = \alpha - \frac{2\alpha C_f}{K_1} + \beta_2 C - \delta_1$$

$$r = \beta_2 C_f$$

$$q = \beta_2 C_f + \beta_0$$

Hence the eigenvalues of equation (38) is given by; $\mu_1 = -((1-\sigma)\lambda_1 C^0 + \varepsilon_0)$ $\mu_2 = -((1-\beta_1)\phi C^0 + \pi_0)$

$\mu_3 = -\theta_0$,

$\mu_4 = \beta - \frac{2\beta C_H^0}{K} - 1$,

$\mu_5 = -\gamma_0$

$\mu_6 = -(\alpha_1 + \alpha_0)$

The roots of the remaining quadratic expression are given by;

$$\mu_{7\&8} = \frac{-(q-F) \pm \sqrt{(q-F)^2 - 4(Ar - qF)}}{2} \quad (42)$$

Where individual root is

$$\mu_7 = \frac{-\left(\beta_2 C_f^* + \beta_0 - \alpha + \frac{2\alpha C_f^*}{K_1} - \beta_2 C^* - \delta_1\right) + \sqrt{\left(\beta_2 C_f^* + \beta_0 - \alpha + \frac{2\alpha C_f^*}{K_1} - \beta_2 C^* - \delta_1\right)^2 - 4\left(\beta_2^2 C^* C_f^* - (\beta_2 C_f^* + \beta_0)\left(\alpha - \frac{2\alpha C_f^*}{K_1} + \beta_2 C^* - \delta_1\right)\right)}}{2}$$

or

$$\mu_8 = \frac{-\left(\beta_2 C_f^* + \beta_0 - \alpha + \frac{2\alpha C_f^*}{K_1} - \beta_2 C^* - \delta_1\right) - \sqrt{\left(\beta_2 C_f^* + \beta_0 - \alpha + \frac{2\alpha C_f^*}{K_1} - \beta_2 C^* - \delta_1\right)^2 - 4\left(\beta_2^2 C^* C_f^* - (\beta_2 C_f^* + \beta_0)\left(\alpha - \frac{2\alpha C_f^*}{K_1} + \beta_2 C^* - \delta_1\right)\right)}}{2} \quad (43)$$

The equilibrium point $E_1 = (C_H^*, C^*, C_f^*, 0,0,0,0,C_I^*)$ is locally asymptotically stable provided that the following inequalities $\beta < \frac{2\beta C_H^*}{K} + 1$ and $\beta_2^2 C^* C_f^* < (\beta C^* + \beta_0)\left(\alpha - \frac{2\alpha C_f^*}{K_1} + \beta_2 C^* - \delta_1\right)$ holds

Proposition 3:
Let $\Omega = \left(C_H^{**}, C^{**}, C_f^{**}, C_a^{**}, C_i^{**}, C_{pa}{}^{**}, C_p^{**}, C_I^{**}\right)$ be the equilibrium point in the presence of atmospheric precipitate, particulate matter and forest biomass. Ω is locally asymptotically stable if $R_e(\mu_i) \le 0$ (real part) for all $i = 1,2,3, \ldots$ Where μ_i is the eigenvalues of the Jacobian matrix.

Proof

Consider the jacobian matrix evaluated at the equilibrium point presence of atmospheric precipitate, particulate matter and forest biomass;

$$\det|J - \mu I| = \begin{vmatrix} B-\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ (1-\pi) & -t-\mu & -A & 0 & -x & 0 & -m & 0 \\ 0 & r & F-\mu & 0 & 0 & 0 & 0 & 0 \\ 0 & s & 0 & -\theta_0-\mu & y & 0 & 0 & 0 \\ 0 & -p & 0 & 0 & -Z-\mu & 0 & 0 & 0 \\ 0 & v & 0 & 0 & 0 & -\gamma_0-\mu & R & 0 \\ 0 & w & 0 & 0 & 0 & 0 & -G-\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -E-\mu \end{vmatrix} = 0 \qquad (44)$$

Note that the values of B, A, x, m, y, Z, R, G and E are the same as in equation (41). Where

$F = \alpha - \dfrac{2\alpha C_f^{**}}{K_1} + \beta_2 C^{**} - \delta_1$

$r = \beta_2 C_f^{**}$

$t = (1-\sigma)\alpha_2 C_p^{**} + (1-\varepsilon)\delta_2 C_i^{**} + \beta_2 C_f^{**} + \beta_0$

$s = (1-\varepsilon)\delta_2 C_i^{**} + (1-\sigma)\lambda_1 C_i^{**}$

$p = (1-\sigma)\lambda_1 C_i^{**}$

$v = (1-\lambda_2)\alpha_2 C_p + (1-\beta_1)\phi C_p$

$w = (1-\beta_1)\phi C_p$

With eigenvalues as

$\mu_1 = -\theta_0$

$\mu_2 = \beta - \dfrac{2\beta C_H^0}{K} - 1$

$\mu_3 = -\gamma_0$,

$\mu_4 = -(\alpha_1 + \alpha_0)$ and the polynomial

$$\mu^4 + P_1\mu^3 + P_2\mu^2 + P_3\mu + P_4 = 0 \qquad (45)$$

Where
$P_1 = [(F-t)+(G+Z)]$
$P_2 = [Ft + (F-t) + (F-t)(G+Z) + GZ - Ar + px - wm]$
$P_3 = [Ft(G+Z) + GZ(F-t) - Ar(G+Z) - px(F-G) + wm(F-Z)]$
$P_4 = [GZFt - ArGZ - pxGF + wmFZ]$

Then
$$D_3 = \begin{pmatrix} P_1 & P_3 & 0 \\ 1 & P_2 & P_4 \\ 0 & P_1 & P_3 \end{pmatrix}$$

The roots μ_5 μ_6 μ_7 and μ_8 will have negative real parts provided that $P_1P_2P_3 > P_1^2P_4 + P_3^2$ (Routh Hurwitz criteria) holds. Therefore, the equilibrium point $E_2 = \left(C_H^{**}, C^{**}, C_f^{**}, C_a^{**}, C_i^{**}, C_{pa}{}^{**}, C_p^{**}, C_I^{**}\right)$ is locally asymptotically stable provided that $\beta < \dfrac{2\beta C_H^*}{K} + 1$.

3.1.6. Global Stability

We will carry out global stability analysis of the equilibrium point in the presence of forest biomass, liquid splash and particulate matter $E_2 = \left(C_H^{**}, C^{**}, C_f^{**}, C_a^{**}, C_i^{**}, C_{pa}{}^{**}, C_p^{**}, C_I^{**}\right)$, by choosing lyapunov candidate. However, let us consider some definitions and theorems.

Definition 3.1 (Trajectory of a system)
Consider the system of differential equation of the form $\frac{dy}{dt} = f(y(t))$ for all $t \geq 0$, if a vector $y(t)$ satisfies the system, then $y(t)$ is called the trajectory or solution of the system of differential equation.
Definition 3.2 (Dissipation Energy of a system)
Consider the system of differential equation; $\frac{dy}{dx} = f(y(t))$ with trajectory $y(t)$ for all $t \geq 0$, then there exists a function $V(y(t))$ called **dissipation energy** of the system along the trajectory given by;

$$\overset{*}{V}(y(t)) = \left(\frac{\partial V}{\partial y}\right) \times \left(\frac{dy}{dt}\right) \quad \text{(chain rule)} \tag{46}$$

Definition 3.3 (positive definite function)
Let $V(y(t))$ be a function. $V(y(t))$ is called a positive definite function (PDF) if the following axioms are satisfied

i. $V(y(t)) > 0$ for all values of $y \neq 0$
ii. $V(y(t)) = 0$ if and only if y = 0
iii. all sublevel sets of V are bounded last condition equivalent to $V(y(t)) \rightarrow \infty$ as $y \rightarrow \infty$

Definition 3.4 (negative definite function)
Let $V(y(t))$ be a function. $V(y(t))$ is called a negative definite function (NDF) if the following axioms are satisfied

i. $V(y(t)) < 0$ for all values of $y \neq 0$
ii. $V(y(t)) = 0$ if and only if y = 0
iii. all sublevel sets of V are bounded last condition equivalent to $V(y(t)) \rightarrow \infty$ as $y \rightarrow \infty$

Definition 3.5 (Lyapunov candidates and functions of a system)
A function $V(y(t))$ (dissipation energy) is called lyapunov candidate if $V(y(t))$ is locally positive definite, and if the derivative $\overset{*}{V}(y(t))$ along the trajectory of the system $\frac{dy}{dt} = f(y(t))$ is locally negative definite, then $V(y(t))$ is called Lyapunov function.
Theorem 3.0 (Globally stability theorem)
Consider a system $\frac{dy}{dt} = f(y(t))$ and suppose there exist a lyapunov function $V(y(t))$ such that

i. $V(y(t))$ is positive definite
ii. $\overset{*}{V}(y(t))$ is negative definite in the entire space

The only solution of $\frac{dz}{dt} = f(z)$, $\overset{*}{V}(z) = 0$ is z(t) = 0 for all t, where z (t) is the equilibrium point then the system $\frac{dy}{dt} = f(y(t))$ is globally asymptotically stable
To prove global stability analysis of the equilibrium point in the presence of atmospheric precipitate, particulate matter and forest biomass, consider a positive definite composite lyapunov candidate of the form

$$V = m_1\left(C_H - C_H^* - C_H^* \log\frac{C_H}{C_H^*}\right) + \frac{1}{2}m_2\left(C - C^*\right)^2 + m_3\left(C_f - C_f^* - C_f^* \log\frac{C_f}{C_f^*}\right) + \frac{1}{2}m_4\left(C_a - C_a^*\right)^2$$
$$+\frac{1}{2}m_5\left(C_i - C_i^*\right)^2 + \frac{1}{2}m_6\left(C_{pa} - C_{pa}^*\right)^2 + \frac{1}{2}m_7\left(C_p - C_p^*\right) + \frac{1}{2}m_8\left(C_I - C_I^*\right)^2 \tag{47}$$

where $(C_H - C_H^*), (C - C^*), (C_I - C_I^*), (C_a - C_a^*), (C_f - C_f^*), (C_p - C_p^*), (C_{pa} - C_{pa}^*)$ and $(C_i - C_i^*)$ are perturbations from the equilibrium point. It is obvious that V is radially unbounded since $V \rightarrow \infty$ as $C_H(t), C(t), C_I(t), C_a(t), C_f(t), C_p(t), C_{pa}(t)$ and $C_i(t) \rightarrow \infty$.
Taking the derivative of equation (47) with respect to time and linearizing the system (i.e. equation 1-8) around the equilibrium, we get

$$
\begin{aligned}
\overset{*}{V} &= m_1\beta\frac{\left(C_H - C_H^*\right)^2}{C_H} - m_1\left(\frac{2\beta C_H}{K} + 1\right)\frac{\left(C_H - C_H^*\right)^2}{C_H} \\
&- m_2\left((1-\lambda_2)\alpha_2 C_p + (1-\varepsilon)\delta_2 C_i + \beta_2 C\right)\left(C - C^*\right)^2 - m_2\beta_0\left(C - C^*\right)^2 \\
&+ m_3(\alpha + \beta_2 C)\frac{\left(C_f - C_f^*\right)^2}{C_f} - m_3\left(\frac{2\alpha C_f}{K_1} + \delta_1\right)\frac{\left(C_f - C_f^*\right)^2}{C_f} - m_4\theta_0\left(C_a - C_a^*\right)^2 \\
&- m_5\left((1-\sigma)\lambda_1 C + \varepsilon_0\right)\left(C_i - C_i^*\right)^2 - m_6\gamma_0\left(C_{pa} - C_{pa}^*\right)^2 \\
&- m_7\left((1-\beta_1)\phi C + \pi_0\right)\left(C_p - C_p^*\right)^2 - m_8(\alpha_0 + \alpha_1)\left(C_I - C_I^*\right)^2 \\
&- \left(m_5(1-\sigma)\lambda_1 C_i + m_2(1-\varepsilon)\delta_2 C\right)\left(C - C^*\right)\left(C_i - C_i^*\right) \\
&- \left(m_7(1-\beta_1)\phi C + m_2(1-\lambda_2)\alpha_2 C\right)\left(C - C^*\right)\left(C_p - C_p^*\right) + \left(m_3\beta_2 - m_2\beta_2 C\right)\left(C - C^*\right)\left(C_f - C_f^*\right) \\
&+ m_2(1-\pi)\left(C_H - C_H^*\right)\left(C - C^*\right) + m_4\left((1-\sigma)\lambda_1 C_i + m_2(1-\varepsilon)\delta_2 C_i\right)\left(C - C^*\right)\left(C_a - C_a^*\right) \\
&+ m_6(1-\lambda_2)\alpha_2 C_p\left(C - C^*\right)\left(C_{pa} - C_{pa}^*\right) + m_4\left((1-\sigma)\lambda_1 C + m_2(1-\varepsilon)\delta_2 C\right)\left(C_i - C_i^*\right)\left(C_a - C_a^*\right) \\
&+ m_6\left((1-\beta_1)\phi C + m_2(1-\lambda_2)\alpha_2 C\right)\left(C_p - C_p^*\right)\left(C_{pa} - C_{pa}^*\right)
\end{aligned} \tag{48}
$$

Hence $E_2 = \left(C_H^{**}, C^{**}, C_f^{**}, C_a^{**}, C_i^{**}, C_{pa}{}^{**}, C_p^{**}, C_I^{**}\right)$ is globally asymptotically stable within the region of attraction Ω provided that the inequalities conditions are satisfied

$$[m_5(1-\sigma)\lambda_1 C_i + m_2(1-\varepsilon)\delta_2 C]^2 < \frac{1}{3}m_2 m_5 \beta_0 \varepsilon_0 \tag{49}$$

$$[m_7(1-\beta_1)\phi C + m_2(1-\lambda_2)\alpha_2 C]^2 < \frac{1}{3}m_2 m_7 \beta_0 \pi_0 \tag{50}$$

$$m_2(1-\pi)^2 < \frac{2}{3}m_1\beta\beta_0 \tag{51}$$

$$m_4[(1-\sigma)\lambda_1 C_i + (1-\varepsilon)\delta_2 C_i]^2 < \frac{2}{3}m_2\beta_0\theta_0 \tag{52}$$

$$m_6[(1-\lambda_2)\alpha_2 C_p]^2 < \frac{2}{3}m_2\beta_0\gamma_0 \tag{53}$$

$$m_4[(1-\sigma)\lambda_1 C + (1-\varepsilon)\delta_2 C]^2 < \frac{2}{3}m_5\theta_0\varepsilon_0 \tag{54}$$

$$m_6[(1-\beta_1)\phi C + m_2(1-\lambda_2)\alpha_2 C]^2 < \frac{2}{3}m_7\gamma_0\pi_0 \tag{55}$$

3.1.7 Numerical Simulations

We carried out two phase of numerical simulation of our model to back up our analytical results. It is simulated as follows:

i. We show numerically the dynamics of atmospheric CO_2 under the influence of atmospheric precipitate, particulate matter and forest biomass considering the contributions of human population expansion, deforestation and emission of CO_2 by industrial and non-industrial CO_2 emitting devices over a period of 7 years

ii. We show numerically the effects of deforestation and emission of CO_2 by industrial and non-industrial CO_2 emitting devices on the concentration of atmospheric carbon dioxide by varying their parameters respectively.

We used MATLAB R2018a Version and the values of the parameters and variables in the table below:

5.0 NUMERICAL EXPERIMENTS

Table 5.1: Numerical values of the parameters

Symbols	Values	Reference
σ	0.02	[15]
ϕ	0.03	Assumed
K	1000	[13]
K_1	2000	[13]
δ_1	0.025, 0.05 and 0.075	Assumed
β_1	0.02	[15]
ε	0.005	Assumed
θ_0	0.18	[15]
θ	0.7	[15]
β_0	1.0	[15]
α_1	0.03, 0.05 and 0.08	Assumed
β_2	0.0001	[13]
α_0	0.03	Assumed
τ	0.05	Assumed
γ	0.02	[15]
π_0	0.005	Assumed
λ_2	0.003	Assumed
α_2	0.6	[15]
γ_0	0.12	[15]
π	0.01	Assumed
α	0.2	[13]
β	0.2	[13]
λ	0.0025	Assumed
δ_2	0.3	[15]
ε_0	0.7	[15]
λ_1	0.2	[15]

Table 5.2: Numerical values of the Variables

Symbols	Descriptions	Normalized values	Reference
$C_H(t)$	Concentration of CO_2 from human respiration at time t	0.6	Assumed
$C(t)$	Concentration of atmospheric CO_2 at time t	1.35	[15]
$C_p(t)$	Concentration of particulate matter at time t	1.07	[15]
$C_a(t)$	Concentration of secondary species formed due to interaction of atmospheric CO_2 and liquid splash at time t	1.36	[15]
$C_I(t)$	Concentrations of CO_2 due to industrial activities and non-industrial CO_2 emitting device only at time t	0.25	Assumed
$C_{pa}(t)$	Concentration of secondary species formed from reaction of CO_2 with particulate matter at time t	1.81	[15]
$C_f(t)$	Concentration of forest biomass at time t	0.50	Assumed
$C_i(t)$	Concentration of liquid splash at time t	0.91	[15]

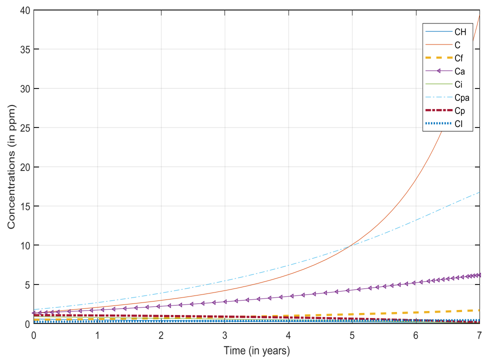

Figure 3.1: Graph showing the natural dynamics of atmospheric carbon dioxide (in ppm)

Some parameters were varied to create two scenarios; viz;

i. Simulation result for the effects of varying the rate of CO_2 emission by industrial and non-industrial means (α_1=0.03, α_1=0.3 and α_1=0.8),

ii. Simulation result for the effects of varying deforestation rate (δ_1=0.025 δ_1=0.5 and δ_1=0.8) on the dynamics of atmospheric CO_2

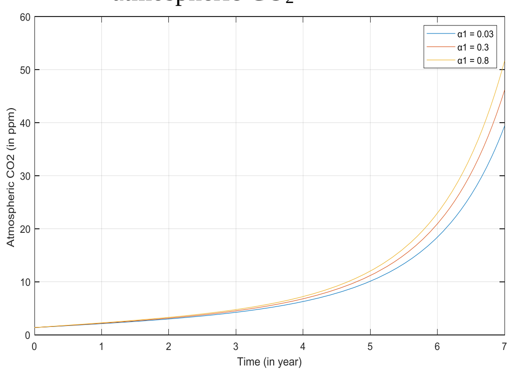

Figure 3.2: Graph showing the effect of varying rates of CO_2 emission by industrial and non-industrial CO_2 emitting devices on the dynamics of atmospheric CO_2

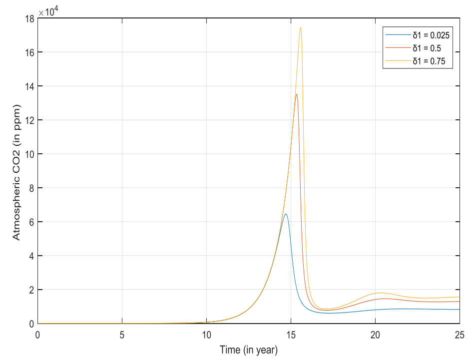

Figure 3.3: Graph showing the effect of varying rates of deforestation on the dynamics of atmospheric CO_2

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3.2 Discussion

From figure 3.1, the result showed that CO_2 produced from human population will continue to decrease over a period of 7 years. This is because CO_2 produce by human will be absorbed by plants and other absorbents and the remaining will always empty itself in the atmosphere to be C(t). It can also be due to high mortality of humans as a result of the effect of climate change on human population with time. The concentration of forest biomass increases rapidly over 12 years and drops till it maintains a steady state afterwards. Furthermore, the result showed that the concentrations of secondary species formed due to the interaction between atmospheric precipitate and particulate matter increases gradually with time. This may be due to more deposit of their respective secondary species formed after interaction. Again the result showed that the concentration of particulate matter and precipitate increased initially due to abundance of atmospheric CO_2, and it gradually drops as it is being used up during reaction with C(t) till it tends to zero in the atmosphere with time. This explains the occurrence of acid rain that happens occasionally. Finally the result showed that the concentration of CO_2 by industrial and non-industrial CO_2 emitting devices increases over time. This may be due to emission from industries and non-industrial CO_2 emitting devices.

Again, in figure 3.2, the result showed that the concentration of atmospheric CO_2 increases respectively with corresponding increase in emitted CO_2 by industrial and non-industrial CO_2 emitting devices over time despite the influence of precipitates, particulate matter and other natural absorbents.

In figure 3.3, the result showed that an increase in the rate of deforestation (due to population expansion, industrialization and urbanization) causes an increase in the concentration of atmospheric CO_2 even when 0.02 rate of particulate matter and 0.7 rate of atmospheric precipitate and 0.03 rate of emission of CO_2 by industrial and non-industrial CO_2 emitting devices are considered. This may be due to the fact that the amount of forest biomass is being reduced by deforestation.

4.0 CONCLUSION

Carbon dioxide in the atmosphere plays a vital role in the sustainability of lives on the earth surface especially in plant kingdom and striking a balance in the surface temperature of the earth (greenhouse effect) [20]. However the effect of the rise of CO_2 in the atmosphere has become of great concern to many, hence, researches have been conducted and some are still ongoing. Our study incorporates the contributions of deforestation (due to industrialization and urbanization) and emission of CO_2 by industrial and non-industrial CO_2 emitting devices bearing in mind the influence of atmospheric precipitates and particulate matter as natural absorbents. The result has shown that the more deforestation, the more accumulation of atmospheric CO_2 concentration. The result also showed that carbon dioxide emitted by industrial and non-industrial devices is also one of the leading contributing factors to the increase in atmospheric CO_2.

5.0 REFRENCES

[1] Ababio O.Y, *New Schools Chemistry for Senior Secondary Schools,* Africana first publisher, 3rd edition; Lagos, Nigeria, 2003.

[2] Matthews, E., Global vegetation and land-use - new high-resolution data-bases for climate studies. *Journal of Climate and Applied Meteorology,* **22,** 474-487, 1983.

[3] Cazenave, A. How fast are the ice sheets melting? *Science,* 314: 1251-1252, 2006.

[4] IPCC. Climate change 2007: *The Fourth Assessment Report (AR4) of the Intergovernmental Panel on Climate Change*. IPCC, 2007.

[5] Moench M. *Adapting to climate change and the risks associated with other natural hazards: Methods for moving from concepts to actions. In: Working with Winds of Change. 13-48,* Prevention Consortium, Institute for Social and Environmental Transition, Nepal, 2007.

[6] Robinson BA, Robinson NE, Willie S. Environmental effects of increased atmospheric carbon dioxide, *Journal of American physicians and Surgeons*, 12: 79-90, 2007.

[7] Ramanathan V, Feng Y. Air pollution, greenhouse gases and climate change: Global and regional perspectives. *Atmospheric Environment,* 43: 37-50, 2009.

[8] Zhang H. and Wang Z. Advances in the Study of Black Carbon Effects on Climate. *Advances in climate change research.* 2(1):23-30, 2011.

[9] Gao J, Chai F, Wang T, Wang W. Particle number size distribution and new particle formation: New characteristics during the special pollution control period in Beijing. *Journal of Environmental Sciences,* 24(1): 14-21, 2012.

[10] Wu SH, Zhang W.J. Current status, crisis and conservation of coral reef ecosystems in China. Proceedings of the International Academy of Ecology and Environmental *Sciences*, 2(1): 1-11, 2012.

[11] Wu S.H, Pan T, He S.F. Climate change risk research: A case study on flood disaster risk in China. *Advances in Climate Change Research,* 3(2): 92-98, 2012.

[12] Zhang J, Zhang W.J. Controversies, development and trends of biofuel industry in the world. *Environmental Skeptics and Critics*, 1(3): 48-55, 2012.

[13] Misra A. K., and Verma M., A mathematical model to study the dynamics of carbon dioxide gas in the atmosphere. *Appl. Math. Comp*. 219 8595–8609, 2013.

[14] Naresh R., Sundar S., Shukla J. B., Modeling the removal of gaseous pollutants and particulate matters from the atmosphere of a city, *Nonlinear analysis RWA*. 8(1) 337–344, 2007.

[15] Shukla J.B., Ashish K.M., Shyam S., & Ram N., Modeling the removal of CO_2 from the atmosphere by spraying external species above the sources of emission: A mechanism to reduce global warming. *International e-journal of mathematical modeling and analysis of complex system: Nature and society* 1(1) 7-12, 2015.

[16] Sundar S. and Naresh R., Role of cloud droplets on the removal of gaseous pollutants from the atmosphere: A nonlinear model. *Int. J. Appl. Math. Comp.* 3(4) 274–282, 2011.

[17] Shukla J. B., Misra A. K., Sundar S., Naresh R., Effect of rain on removal of a gaseous pollutant and two different particulate matters from the atmosphere of a city. *Math. Comp. Model.* 48(5-6) 832–844, 2008.

[18] Shukla J.B., Rachana P., Manju A. & Yasuhiro T., Modeling the depletion of forest resources by population and industrialization and their conservation by green belt plantation. *International e-journal of mathematical modeling and analysis of complex system: Nature and society* 3(1) 7-36, 2017.

[19] Archana, S. B., Shukla, J. B., Rachana, P. & Manju, A., Modeling and analysis of the impact of awareness programs on rural population for conservation of forest resources. *International e-journal of mathematical modeling and analysis of complex system: Nature and society,* 3(1) 66-87, 2017.

[20] Mahlman J. D., Science and non-science concerning human-caused climate warming, *Annual reviewed energy Environs*: 23, 83 – 105, 1998.