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STRUCTURE AND PROPERTIES OF APPROXIMATE FUNCTIONS FROM COLLAPSING BOUNDARIES AND THE ANALOGY OF HUMAN RELATIONSHIPS

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Abstract

The paper explores structure and properties of pairs and faster pairs of approximate functions from collapsing boundaries of a positive series. It relates results from Mathematics of approximate functions to human relationship using analogies and suitable keys. It likens the mathematical search for faster pairs to breaking friendship boundaries and setting new boundaries or seeking new friends towards satisfying desired goals or timely expectations. It asserts among other things: Every collapsing boundary friend is a product of a compliant and non-compliant part, Amenable friends of collapsing boundaries can be converted in some activities, children of friends may not be friends of one's children, if they become friends, it is possible to have closer relationship than their parents. Analysis of the error term led to an accidental discovery from a simulation calculation that if the error term tend to zero as (n) tends to infinity, the square root functions are 1-subtractive square root of rational number equivalence. The discovery prompted further investigation on the conditions for 1-subtractiveness, its role in the convergence of square root function inclusions in error terms of the approximate functions, and a general proof that the square root function inclusions in error terms of pairs of approximate functions from collapsing boundaries are 1-subtractive square root of rational number equivalence. Thus 1-subtactiveness is a useful characterization of square root function inclusions in error terms of approximate functions from collapsing boundaries. The paper also illustrate how to use the concept of one subtractiveness to construct approximate of a given function of (n) when n is large.

***Keywords*:** Approximate Compliance, Conversion, Perspective, Attitude, Faster Pairs, collapsing boundaries.

1 Introduction

Pairs of approximate functions from collapsing boundaries has desirable stable characteristics of good approximation. The paper begins with literature review of collapsing boundary functions of the (1-n-k) exponential integral and its associated approximate functions. It explores the structure and properties of the approximate functions and applies the Mathematics of the approximate functions to human relationship using analogies and suitable keys. It uses the property of square root inclusions in error term of approximate functions from collapsing boundaries to construct large number approximations of a given function. To facilitate the clarity of discuss, the authors make the following definitions:

(i) The (1-n-k) exponential integral is a sub-class of (p-n-k) exponential integral defined by:

$$\int_o^x x^{pn-1}\, e^{kx^n} dx$$

When p = 1, p, n are whole numbers and k is a real number.

(ii) A collapsing boundary friend is a friend that has had a broken relationship previously or is currently violating individual or relationship boundaries

(iii) The square root of a rational number ($^a/_b$) is one(1)-subtractive if $\sqrt{\frac{a}{b}} = \frac{a-1}{b-1}$. When the square root of ($^a/_b$) is 1-

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subtractive, we say that $\sqrt{\frac{a}{b}}$ is a 1-subtractive square root of rational number equivalence of $\frac{a-1}{b-1}$.

2 Literature review of collapsing boundaries of the (1-n-k) exponential integral and its associated approximate functions.

Unit points of the (1-n-k) exponential integral are simply values for which the class of integrals;

$$\int_o^x x^{pn-1} e^{kx^n} dx = 1$$

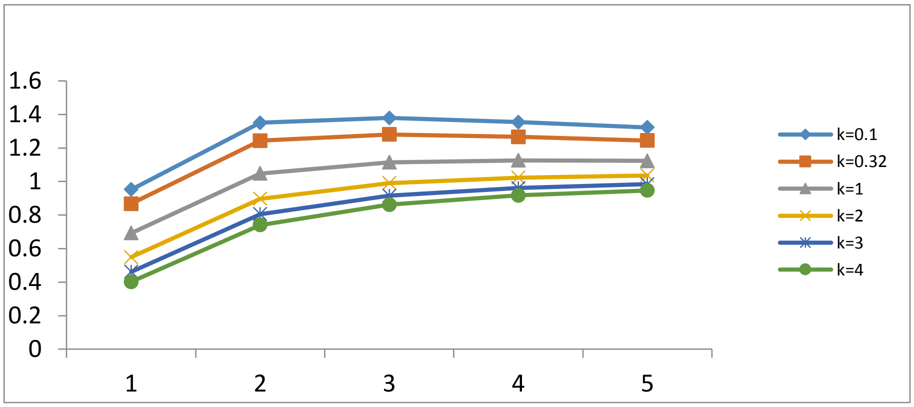

Fig 1: The graph of unit point of the p – n – k class exponential integral when p = 1 for values of k= 0.1, 0.32, 1, 2, 3, 4, and n = 1 to 5

Source: Dosomah, Audu and Oriakhi [1]

Fig 1 is a graph of unit points of the (1-n-k) exponential integral. Appreciating the beauty of the graph and the closeness of approach of the graph to one another prompted an investigation of the functions by Taylor series expansion towards finding an explanation for the pattern of the graph. That is, the constraining functions pushing the graphs towards one another, making it bounded between 0.4 and 1.4, and closer to one another as (n) tends to infinity. The investigation led to discovery of the series:

$$\frac{1}{m}\sum_{n=4}^{\infty}\frac{m^{2n}}{2n-1} + \left(1-\frac{1}{m}\right)\sum_{n=4}^{\infty}\frac{m^{2n}}{2^n} - \sum_{n=4}^{\infty}\frac{m^{2n}}{2n}$$

Associated with unit points of the (1-n-k) exponential integral.

The series has a slow sum to infinity less than 10 when n is one billion.

The study found that a finite unit point (x_{n+1}) is the (n+1) root of a sum of four finite terms and a bounded series of slow sum to infinity associated with it.

That is : $x_{n+1} = \sqrt[n+1]{\frac{1}{K_{n+1}}(S+P)}$ (1)

$P = \frac{m(2-m)}{2-m^2} - \frac{m^3}{6} - \frac{m^5}{20} - \frac{m^6}{24}$ and m = $\frac{K_{n+1}(n+1)}{1+2k_{n+1}}$.

Where (S) is the series associated with unit points.

Considering the elasticity of values of the series in relation to other terms of the unit point equation (1) obtained from simulation results, the authors opined that the series is a moderator constraining points to fit the requirement of unit points as an explanation for the observed boundedness and pattern of the graph of unit points. A study of positivity conditions of the series in its interval of convergence $(0 < m < \sqrt{2})$, Show that the series has three positivity intervals: $0 < m < 1$, $1 < m < \frac{2n}{2n-1}$ and $\frac{2n}{2n-1} < m < \sqrt{2}$. The interval $1 < m < \frac{2n}{2n-1}$ had an obvious collapsing boundary indicated by equality of its upper and lower limits. That is,

$$\lim_{n\to\infty} 1 = 1 \text{ and } \lim_{n\to\infty}\frac{2n}{2n-1} = 1$$

Calculating sum to infinity of the series required finding a k_{n+1} equivalent of positivity conditions on m from the series (m-k) relation m = $\frac{k_{n+1}(n+1)}{1+2k_{n+1}}$, using m as mean of boundary values of (m) in the intervals. It was observed by the authors [2] in the interval $1 < m < \frac{2n}{2n-1}$, that the k_{n+1} equivalent of m gave two functions:

$\frac{1+\sqrt{n+1}}{n+1}$ and $\frac{2n+\sqrt{4n^2+2n(n+1)(2n-1)}}{(n+1)(2n-1)}$

Approximately equal to14 decimal places at one billion. Since the calculation required finding an average of K_{n+1} from boundary values of m, the coincidence of values within limits of calculation (2 significant figures in the work)meant it was impossible to find a K_{n+1} between boundary values to satisfy the positivity conditions in the interval $1 < m < \frac{2n}{2n-1}$. Since $m > 0$ we cannot get K_{n+1} to make $m > 1$, the available option, $m < 1$ was observed in the simulation result of Dosomah, Audu and Oriakhi [2]. That is, the coincidence of boundary values in intervals with collapsing boundaries may result in reversal of (m) inequality direction to satisfy positivity conditions. In other words, one in a boundary situation in one interval still experiencing difficulties satisfying positivity conditions in that interval, may cross the boundary to be in another interval thus collapsing its previous boundary. Further research efforts by Dosomah, Audu and Edosomwan [3] led to clearer understanding of the process, construction of more collapsing boundaries and more pairs of approximate functions from positivity intervals with obvious and non-obvious collapsing boundaries and development of principles for the construction: if a convergent series has a transforming relationship and at least one interval of positivity, if solving the inequality of the transforming relationship on collapsing boundary points for which the series is positive, gives a quadratic inequality with real roots, then collapsing boundaries of the series can be used to construct pairs of approximate functions.

Dosomah,Audu and Edosomwan [3] posed a question of possibility for constructing approximate functions of limit zero as (n) tend to infinity that satisfy the following conditions:

i. From 1000 to one billion in steps of multiplier 10000, at least the first two digits from the first non-zero digit to the right of decimal point of the first function are respectively equal to the digits in corresponding position of the second function.

ii. The number of equal consecutive corresponding digits in the first set of consecutive corresponding digits to the right decimal point in both function is non-decreasing with increase in (n) in the given range

iii. At (n) equal one billion the function are equal to at least 14 decimal places"

They demonstrated the existence of such approximate functions by constructing them from collapsing boundaries of the series associated with unit points.

3 Structure and properties of some approximate functions from collapsing boundaries.

Sequel to Dosomah, Audu and Edosomwan [3] on construction of pairs approximate functions from collapsing boundaries, the functions

$$A(n)= \frac{n}{(n+1)(n+0.1)} \left\{1 + \sqrt{1+\frac{(n+1)(n+0.1)}{n}}\right\}$$

$$B(n)= \frac{2n}{(n+1)(2n+0.1)} \left\{1 + \sqrt{1+\frac{(n+1)(2n+0.1)}{2n}}\right\}$$

Were obtained from the construction.

Table 1: A simulation result of the function A(n) and B(n)

N	A(n)	B(n)	Decimal Equivalent
1000	0.032620079	0.03262092	6
10000	0.010099938	0.010099964	7
100000	0.003172275969	0.003172276764	8
1000000	0.001000999949	0.001000999974	10
10000000	0.0003163277644	0.0003163277652	11
100000000	0.00010000000000	0.00010000000000	14
1000000000	0.0000316237766	0.0000316237766	14

The function A(n) and B(n) are of the forms:

$A(n)= a_1 \times a_2$ where $a_1= \frac{n}{(n+1)(n+0.1)}$ and $a_2= 1+ \sqrt{1+\frac{(n+1)(n+0.1)}{n}}$

$B(n)= b_1 \times b_2$ where $b_1= \frac{2n}{(n+1)(2n+0.1)}$ and $b_2= 1+ \sqrt{1+\frac{(n+1)(2n+0.1)}{2n}}$

Table 2: A simulation result of a_1 and b_1.

N	**a_1**	**b_1**	**Decimal equivalence**
1000	$9.989011089 \times 10^{-4}$	$9.989510514 \times 10^{-4}$	(7)
10000	$9.998900111 \times 10^{-5}$	$9.998950105 \times 10^{-5}$	(9)
100000	$9.999890001 \times 10^{-6}$	$9.999895001 \times 10^{-6}$	(11)
1000000	9.999989×10^{-7}	9.9999895×10^{-7}	(13)
10000000	9.9999989×10^{-8}	$9.99999895 \times 10^{-8}$	(15)
100000000	$9.99999989 \times 10^{-9}$	$9.999999895 \times 10^{-9}$	(16)
1000000000	$9.99999999 \times 10^{-10}$	$9.99999999 \times 10^{-10}$	(18)

source: Dosomah and Omorogbe [5].

Table 3: A simulation result of a_2 and b_2.

N	**a_2**	**b_2**	**Significant figures**
1000	32.65596468	32.65517414	(5)
10000	101.0104995	101.0102495	(6)
100000	317.2310864	317.2310073	(7)
1000000	1001.00105	1001.001025	(8)
10000000	3163.277992	3163.277984	(8)
100000000	10001.0001	10001.0001	Unlimited
1000000000	31623.77663	31623.77663	Unlimited

Source: Dosomah and Omorogbe [5]

The following faster pairs of approximate functions were obtained by differentiation from Dosomah and Omorogbe [3].

$$C(n)= \frac{(n^2-0.1)}{2n(n+1)^2(n+0.1)^2}\left(\frac{n^2+3.1n+0.1}{\sqrt{1+\frac{(n+1)(n+0.1)}{2n}}}+2n\right)$$

And

$$D(n)= \frac{(2n^2-0.1)}{2n(n+1)^2(2n+0.1)^2}\left(\frac{2n^2+6.1n+0.1}{\sqrt{1+\frac{(n+1)(2n+0.1)}{2n}}}+4n\right)$$

Table: A simulation result of C(n) and D(n) from1000 to one billion in steps of multiplier 10000.

n	**C(n)**	**D(n)**	**Decimal Equivalence**
1000	$-1.680677528 \times 10^{-5}$	$-1.680806429 \times 10^{-5}$	8
10000	$-5.0999002867 \times 10^{-7}$	$-5.099941372 \times 10^{-7}$	13
100000	$-1.591136238 \times 10^{-8}$	$-1.591137434 \times 10^{-8}$	13
1000000	$-5.009999228 \times 10^{-10}$	$-5.009999604 \times 10^{-10}$	16
10000000	$-1.582138806 \times 10^{-11}$	$-1.582138818 \times 10^{-11}$	18
100000000	$-5.000999995 \times 10^{-13}$	$-5.000999548 \times 10^{-13}$	19
1000000000	$-1.58123883 \times 10^{-14}$	$-1.58123883 \times 10^{-14}$	22

Source: Dosomah and Omorogbe [5]

Each pair of approximate functions from collapsing boundary is a product of two functions. Corresponding functions in the product tend to each other with a progressive non-decreasing equivalence as n tends to infinity. That is, if A(n) and B(n) are pairs of approximate functions from collapsing boundary, A and B are of the forms:

$A(n) = a_1(n) \times a_2(n)$, $B(n) = b_1(n) \times b_2(n)$

Such that:

$a_1(n)$ tend to $b_1(n)$ i.e $a_1 \to b_1$

$a_2(n)$ tend to $b_2(n)$ i.e $a_2 \to b_2$

$A(n) \to B(n)$ i.e $a_1 \times a_2 \to b_1 \times b_2$

Each pair of a_1 and b_1, a_2 and b_2 has progressive non-decreasing equivalence in decimal places and significant figures .Using our discretion to choose, from the perspective of preference for higher equivalence, we see that a_1 and b_1 has progressive non-decreasing equivalence in decimal places, a_2 and b_2 has progressive non-decreasing equivalence in significant figures.

The pair of product functions $a_1 \times a_2 = A(n)$ and $b_1 \times b_2 = B(n)$ has progressive non-decreasing equivalence in decimal places with a contraction of decimal equivalence compared to the pair a_1 and b_1. Thus there are approximate functions such that if one of a pair that has a progressive non-decreasing equivalence in decimal places is multiplied by a corresponding one of a pair that has a progressive non-decreasing equivalence in significant figures, it can result in a pair of approximate functions with a progressive non-decreasing equivalence in decimal places (conversion) with a reduction (contraction) of decimal equivalence compared to the converter. It also show that there are approximate function such that if one of a pair of approximate functions that satisfy a given condition is multiplied by a corresponding one of another pair of approximate function, that do not satisfy the given condition, it can result in pair of approximate product functions that satisfy the condition (conversion). For Example, a_1 and b_1 satisfy the given conditions of Dosomah, Audu and Edosomwan [3] in the literature review of this paper. a_2 and b_2 do not satisfy the condition. The pair of product functions $a_1 \times a_2$ and $b_1 \times b_2$ satisfy the conditions.

The question is, will these properties be transferred to faster pairs obtained by differentiation? Simulation results indicate that some of the properties are not transferable. This indicates that these properties may be peculiar to functions constructed directly from collapsing boundaries. Thus the non-transferability of some properties may be due to loss of some stabilizing terms in a secondary process for example, differentiation of constants may eliminate some stabilizing terms of the function characteristics.

Simulation results show that the faster pairs

$$\frac{(n^2-0.1)}{2n(n+1)^2(n+0.1)^2}\left[\frac{n^2+3.1n+0.1}{\sqrt{1+\frac{(n+1)(n+0.1)}{n}}}+2n\right]$$

And $\frac{(2n^2-0.1)}{2n(n+1)^2(2n+0.1)^2}\left[\frac{2n^2+6.1n+0.1}{\sqrt{1+\frac{(n+1)(2n+0.1)}{2n}}}+4n\right]$ are products but has no corresponding product terms tending to one another.

That is if $c_1=\frac{(n^2-0.1)}{2n(n+1)^2(n+0.1)^2}$, $c_2=\frac{n^2+3.1n+0.1}{\sqrt{1+\frac{(n+1)(n+0.1)}{n}}}+2n$

$d_1=\frac{(2n^2-0.1)}{2n(n+1)^2(2n+0.1)^2}$, $d_2=\frac{2n^2+6.1n+0.1}{\sqrt{1+\frac{(n+1)(2n+0.1)}{2n}}}+4n$

$c_1 \times c_2$ tend to $d_1 \times d_2$ but c_1 does not tend to d_1, c_2 does not tend to d_2, c_1 does not tend to c_2 and d_1 does not tend to d_2

Note also that a_1, b_1 a pair of corresponding internal product terms of A(n) and B(n) is another faster pair of approximate functions equal to at least 18 decimal places at one billion with no first non-zero digit to the right of decimal point alternation. The non – alternation indicates more stability.

Limited and unlimited equivalence of approximate functions

Some functions may attain unlimited equivalence faster than others. For example, the significant equivalence of a_2 and b_2 increase with (n) from 1000 to 1000000, it is equal at 10000000 and from 100000000, we observe unlimited equivalence. On the other hand, a_1 and b_1 increase in decimal equivalence from 1000 to 100000000 and at one billion it attain unlimited equivalence with no prior equality of limited decimal equivalence.

4 Human relationship and analogy of pairs and faster pairs of approximate functions from collapsing boundaries.

Three major under-pinning for the application of Mathematics of approximation to human relationship is:

i. Character assessment for relationship acceptance are often approximations of true character based on estimates of expressed character in a period of observation.
ii. Mis-management of perspective differences due to upbringing, life experiences and influence from association cause mis-understanding in relationship and desirable changes may not be automatic.
iii. One in a boundary situation in one interval still experiencing difficulties satisfying positivity conditions in that interval may cross the boundary to be in another interval and thus collapse its previous boundary.

There are different kinds of human relationship: Acquaintance, friends, marriage partners e.t.c. Each of the different types of human relationship have different levels. A relationship may move from one type or level to another. For example acquaintance may become friends and marriage spouses. Let us take friendship as an example, Friendship begins with identification of some timely likeable character traits and a desire for closeness. If evaluation of the periodic character traits of the friend is unsatisfactory beyond some critical tolerance level, the desire for change of state may lead to setting new boundaries or breaking relationship and seeking new friends of one's desire. Thus, change of state may involve boundary collapse. The search for new friends is in anticipation of expectation of desires for faster closeness in relationship towards achieving desired goals. Thus breaking relationship and seeking new friends is likened to a search for faster pairs of approximate functions from collapsing boundaries.

Key to Analogy
Take values of the approximate functions as per unit index of character and expectation from relationship. The greater the character and expectation from a relationship, the smaller the per unit index. Take values of (n) as levels of relationship, take the derivative of the approximate functions as children of collapsing boundaries. Note that the greater the number of digit equivalence of the approximate functions, the smaller the difference between them. Smaller differences in values of a pair of approximate function at a given level of relationship indicate closer relationship at that level.

Approximate compliance, conversion and contraction
If two pairs of approximate functions have a product structure of the form:
$f_1 = a_1 \times a_2$ tend to $f_2 = b_1 \times b_2$ we say that f_1 is approaching compliance with f_2 simply stated f_1 comply with f_2. If the first part product term comply with digit equivalence of Type (1),the second part product term comply with digit equivalence of Type (2) and the product functions f_1 and f_2 comply with digit equivalence of Type (1), we say that the association of a_1 and b_1 has converted a_2 and b_2 to a compliance of a_1 and b_1. Here, the pair of approximate functions (a_1 b_1) is the converter. Type (1) may be decimal places, Type (2) may be significant figures or Type (1) may be equal to Type (2) and be one of decimal places or significant figures.

Individual boundaries and relationship boundaries
Actions or activities that are detrimental to peace, sustainable development or acceptable practice of a partner in a relationship exceeding the tolerance level of the partner, collapse the partner boundaries of the relationship. Lingering un-resolved disputes in collapsing partner boundaries can collapse the boundaries of a relationship when one of the partners exit the relationship to seek another human pair in search of better relationship to satisfy desired goals or timely expectation of the exit partner (faster human pairs from collapsing boundaries)

Using Analogies to interprete results from the pairs and faster pairs in terms of Human Relationship.
From the structure of approximate functions from collapsing boundaries, an approximate collapsing boundary function is of the form $a_1 \times a_2$ tending to $b_1 \times b_2$ such that a_1 tends to b_1, a_2 tends to b_2 but a_2 does not tend to a_1 and b_1. Thus a_1 comply with b_1 but a_2 does not comply with b_1 and a_1. Regarding closeness in values of the approximate functions as indicative of friendship, we obtain:

i. Every collapsing boundary friend is a product of a compliant and non-compliant part.
Noting that the part product term (a_1) of $a_1 \times a_2$ and b_1 of $b_1 \times b_2$ are such that a_1 tend to b_1 faster than $a_1 \times a_2$ tend to $b_1 \times b_2$, and that faster pairs from internal part products do not have first digit to the right of decimal point alternation unlike "external" faster pairs from collapsing boundaries, we have:

ii. Within each collapsing boundary friend is a possibility for greater friendship depending on attitude to non-compliant part and when collapsing boundary friends make internal adjustments towards faster pairs, their closeness in relationship may be faster and more stable than a closeness resulting from an external search for faster pairs.
Noting that there are approximate functions such that if one of a pair that satisfy a given result is multiplied by another one of a pair that do not satisfy the given result it can result in a pair of approximate functions that satisfy the result with a reduction in number of digit equivalence. In terms of human relationship, it mean that:

iii. Amenable friends of collapsing boundaries can be converted in some activities. A pair of friends of collapsing boundary are amenable, if one of them have an influence on the other in some activity. The conversion is in direction of greater influence and the degree of compliance of the product relationship may be less than the degree of compliance of the stronger influence due to adjustment consideration for conversion. Noting that faster pairs from collapsing boundaries obtained by differentiation may lose some characteristics of their parent function and are of the form $c_1 \times c_2$ and $d_1 \times d_2$ such that c_1 does not tend to d_1 and c_2 does not tend to d_2 but $c_1 \times c_2$ tend to $d_1 \times d_2$ at a faster rate than the parent approximate functions A(n) and B(n) that was differentiated. Regarding the derivatives as children, in terms of human relationship, it means that:

iv. Children of friends may have nothing in common and may lose some characteristics of their parents but if they become friends, it is possible to have closer relationship than their parents. In other words, children of friends may not be friends of one's children but if they become friends, it is possible to have closer relationship than their parents. For example, it is possible that children of friends can marry one another and enjoy a flourishing relationship.
Noting the properties of steady approach of the approximate functions can result in limited or unlimited equivalence. In terms of human relationship, it means faithful commitment to progressive non-decreasing relationship can result in limited or unlimited equivalence. However, the fastness of a pair is not a guarantee for an earlier (in terms of levels) attainment of unlimited equivalence. That is, a less faster pair can attain unlimited equivalence earlier than a faster pair.

5 Square Root Function Inclusion in Error Term of Collapsing Boundary Functions and 1-Subtractive Square Root of Rational Number Equivalence

Error term of the Approximate functions
Error term obtained by difference of two functions is indicative of closeness of the function values, accuracy of approximation and stability of convergence of the approximation. It is a well-known fact that for good approximation, the error term should tend to zero as n tend to infinity. Analysis of the error term of the approximate function show that the error term is a sum of a rational function of limit zero as n tends to infinity and a difference of square root functions that tend to 1 – subtractive square root of rational number equivalence as n tend to infinity.

The Error term E(n) of the Pair of approximate functions A(n) and B(n) is given by:

E(n) = B(n) – A(n)

$$=\frac{2n}{(n+1)(2n+0.1)}\left[1+\sqrt{1+\frac{(n+1)(2n+0.1)}{2n}}\right]-\frac{n}{(n+1)(n+0.1)}\left[1+\sqrt{1+\frac{(n+1)(n+0.1)}{n}}\right]$$

$$=\frac{2n}{(n+1)(2n+0.1)}-\frac{n}{(n+1)(n+0.1)}+\frac{2n}{(n+1)(2n+0.1)}\left[\sqrt{1+\frac{(n+1)(2n+0.1)}{2n}}\right]$$

$$-\frac{n}{(n+1)(n+0.1)}\left[\sqrt{1+\frac{(n+1)(n+0.1)}{n}}\right]$$

$$=\frac{2n(n+0.1)-n(2n+0.1}{(n+1)(n+0.1)(2n+0.1)}+\frac{2n}{(n+1)(2n+0.1)}\left[\sqrt{1+\frac{(n+1)(2n+0.1)}{2n}}\right]$$

$$-\frac{n}{(n+1)(n+0.1)}\left[\sqrt{1+\frac{(n+1)(n+0.1)}{n}}\right]$$

$$=\frac{2n^2+0.2n-2n^2-0.1n}{(n+1)(n+0.1)(2n+0.1)}+\frac{2n}{(n+1)(2n+0.1)}\left[\sqrt{1+\frac{(n+1)(2n+0.1)}{2n}}\right]$$

$$-\frac{n}{(n+1)(n+0.1)}\left[\sqrt{1+\frac{(n+1)(n+0.1)}{n}}\right]$$

$$=\frac{0.1n}{(n+1)(n+0.1)(2n+0.1)}+\frac{2n}{(n+1)(2n+0.1)}\left[\sqrt{1+\frac{(n+1)(2n+0.1)}{2n}}\right]$$

$$-\frac{n}{(n+1)(n+0.1)}\left[\sqrt{1+\frac{(n+1)(n+0.1)}{n}}\right]$$

Since the Error term is greater than or equal to zero, simplifying the inequality gives:

$$=\frac{0.1n}{(n+1)(n+0.1)(2n+0.1)}+\frac{2n}{(n+1)(2n+0.1)}\left[\sqrt{1+\frac{(n+1)(2n+0.1)}{2n}}\right]$$

$$-\frac{n}{(n+1)(n+0.1)}\left[\sqrt{1+\frac{(n+1)(n+0.1)}{n}}\right]$$

≤ 0

That is,

$$\frac{n}{(n+1)(n+0.1)}\left[\sqrt{1+\frac{(n+1)(n+0.1)}{n}}\right]-\frac{2n}{(n+1)(2n+0.1)}\left[\sqrt{1+\frac{(n+1)(2n+0.1)}{2n}}\right]$$

$$\leq \frac{0.1n}{(n+1)(n+0.1)(2n+0.1)}$$

Research Notes for the concept of 1-subtractiveness in the error term

simulation result of the error term when n is 1000 is

$$E(1000)=\frac{100}{1001\times 1000.1\times 2000.1}+\frac{100010.5005\sqrt{10012.001}-10011.001\sqrt{10011.5005}}{10011.001\times 10010.5005}$$

Since the part sum of the error term $\frac{100}{1001\times 1000.1\times 2000.1}$ tend to zero as n tend to infinity, and the error term tends to zero,

$\frac{10010.5005\sqrt{10012.001}-10011.001\sqrt{10011.5005}}{10011.001\times 10010.5005}$ must tend to zero as n tends to infinity.

i.e. 10010.5005 $\sqrt{10012.001}$ - 10011.001 $\sqrt{10011.5005}$ tend to zero as n tends to infinity.

Setting 10010.5005 $\sqrt{10012.001}-10011.001\ \sqrt{10011.5005}=0$ gives

10010. 5005 $\sqrt{10012.001}$=10011. 001$\sqrt{10011.5005}$

i.e. $\frac{\sqrt{10012.001}}{\sqrt{10011.5005}}=\frac{10011.001}{10010.5005}$

observe that: 10012.001 – 1 =10011.001

10011.5005 – 1 = 10010.5005

This observation leads naturally to the question of studying the conditions for 1 – subtractive square root of rational number equivalence. That is, equivalent systems of the form.

$\frac{\sqrt{a}}{\sqrt{b}} = \frac{a-1}{b-1}$ that is $\sqrt{\frac{a}{b}} = \frac{a-1}{b-1}$ and its relaionship to approximation

A proof that the square root functions in Error terms of collapsing boundary functions are 1 – subtractive

The square root function inclusion in error terms of the pair of approximate function from collapsing boundaries in Dosomah, Audu and Edosomwan [3] is:

$$\frac{1}{(n+1)(n+0.1)}\sqrt{1+\frac{(n+1)(n+0.1)}{n}} - \frac{2n}{(n+1)(2n+0.1)}\sqrt{1+\frac{(n+1)(2n+0.1)}{2n}}$$

$$\frac{n(2n+0.1)\sqrt{1+\frac{(n+1)(n+0.1)}{n}} - 2n(n+0.1)\sqrt{1+\frac{(n+1)(2n+0.1)}{2n}}}{(n+1)(n+0.1)(2n+0.1)}$$

The square root function inclusion will tend to Zero as n tends to infinity if

$$n(2n+0.1)\sqrt{1+\frac{(n+1)(n+0.1)}{n}} - 2n\,(n+0.1)\sqrt{1} + \sqrt{1+\frac{(n+1)(2n+0.1)}{2n}} = 0$$

i.e. $\frac{\sqrt{1+\frac{(n+1)(2n+0.1)}{2n}}}{\sqrt{1+\frac{(n+1)(n+0.1)}{n}}} = \frac{n\,(2n+0.1)}{2n\,(n+0.1)} = \frac{(2n+0.1)}{2(n+0.1)}$

Also, $1+\frac{(n+1)(2n+0.1)}{2n} - 1 = \frac{(n+1)(2n+0.1}{2n}$

$$1+\frac{(n+1)(n+0.1)}{n} - 1 = \frac{(n+1)(+0.1)}{n}$$

$$\frac{1+\frac{(n+1)(2n+0.1)}{2n}-1}{\frac{1+(n+1)(n+0.1)}{n}-1} = \frac{(n+1)(2n+0.1)}{2n} \div \frac{(n+1)(n+0.1)}{n}$$

$$= \frac{(n+1)(2n+0.1)}{2n} \times \frac{n}{(n+1)(n+0.1)} = \frac{(2n+0.1)}{2(n+0.1)}$$

Thus the square root function inclusion in error term of the pair of approximate functions A(n), B(n) constructed from the sub-interval $\left(\frac{2n}{2n+0.2} < m < \frac{2n}{2n+0.1}\right)$ of positivity and convergence $(0 < m < 1)$ of the series associated with (1-n-k) exponential integral is $1-$ subtractive.

Similarly, the pairs of approximate functions:

$$C(n) = \frac{2n}{(n+1)(2n-0.1)}\left[1+\sqrt{1+\frac{(n+1)(2n-0.1)}{n}}\right]$$

And

$$D(n) = \frac{n}{(n+1)(n-0.1)}\left[1+\sqrt{1+\frac{(n+1)(n-0.1)}{n}}\right]$$

Constructed from collapsing boundaries of sub-interval $\frac{2n}{2n-0.1} < m < \frac{2n}{2n-0.2}$ of positivity and convergence $\left(1 < m < \frac{2n}{2n-1}\right)$ of the series and

$$E(n) = \frac{2n\sqrt{2}}{(n+1)(2n-0.1)}\left[1+\sqrt{1+\frac{\sqrt{2}(n+1)(2n-0.1)}{4n}}\right]$$

$$F(n) = \frac{\sqrt{2}}{(n+1)}\left[1+\sqrt{1+\frac{\sqrt{2}(n+1)}{2}}\right]$$

Constructed from sub-interval $\frac{2n\sqrt{2}}{2n-0.1} < m < \sqrt{2}$ of positivity and convergence of $\left(\frac{2n}{2n-1} < m < \sqrt{2}\right)$ of the series were all found to be 1 – subtractive.

This indicates that $1-$ subtractiveness is a useful characterization of square root function inclusions in error terms of approximate functions constructed from the collapsing boundaries of Dosomah, Audu and Edosomwan [3]

The condition for $1-$ subtractiveness

If $\frac{\sqrt{a}}{\sqrt{b}} = \frac{a-1}{b-1}$ either a=b or ab=1

Proof; $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

$$\sqrt{\frac{a}{b}} = \frac{a-1}{b-1}$$

Square both sides

$\frac{a}{b} = \frac{(a-1)^2}{(b-1)^2}$
$a(b-1)^2 = b\,(a-1)^2$
$a(b^2 - 2b + 1) = b(a^2 - 2a + 1))$
$ab^2 - 2ab + a = ba^2 - 2ab + b$
$ab^2 - ba^2 = b - a$
$ab(b - a) = b - a$
$ab(b - a) - (b - a) = 0$
$(b - a)(ab - 1) = 0$
$b - a = 0$ or $ab - 1 = 0$
$b = a$ or $ab = 1$
Therefore, the numerator and denominator of a 1- subtractive square root of rational number equivalence are either equal or reciprocals of each other.

Application of 1 – subtractiveness to approximations in the large
Given a function F(n) of n, to construct a large number approximate of F(n) using 1- subtractiveness.
Method
Find a 1- subtractive expression of the form $\frac{\sqrt{a}}{\sqrt{b}} - \frac{(a-1)}{(b-1)}$ such that a tends to b as n tends to infinity.
Add the 1- subtractive expression to the function and simplify.
Example
Find a large number approximate of the function $F(n) = \frac{n^2+3}{3(n^2+1)}$ using 1−subtractiveness.
Solution:
Consider the expression $\sqrt{\frac{2n+0.1}{2(n+0.1)}}$
Check that;

$$\lim_{n\to\infty} 2n + 0.1 = \lim_{n\to\infty} 2(n+0.1)$$

1 – subtractive expression is $\sqrt{\frac{2n+0.1}{2(n+0.1)}} - \frac{(2n+0.1-1)}{2(n+0.1)-1}$

$$\sqrt{\frac{2n+0.1}{2(n+0.1)}} - \frac{(2n-0.9)}{(2n-0.8)}$$

Adding this expression to the function $F(n) = \frac{n^2+3}{3(n^2+1)}$ gives

$$\frac{n^2+3}{3(n^2+1)} + \sqrt{\frac{2n+0.1}{2(n=0.1)}} - \frac{(2n-0.9)}{(2n-0.8)}$$

$$= \frac{n^2+3}{3(n^2+1)} - \frac{(2n-0.9)}{2n-0.8} + \sqrt{\frac{2n+0.1}{2(n+0.1)}}$$

$$= \frac{(n^2+3)(2n-0.8) - 3(n^2+1)(2n-0.9)}{3(n^2+1)(2n-0.8)} \sqrt{\frac{2n+0.1}{2(n+0.1)}}$$

$$G(n) = \frac{0.9n^2 + 0.3 - 4n^3}{6(n^2+1)(n-0.4)} + \sqrt{\frac{2n+0.1}{2(n+0.1)}}$$

Table 5: Simulation results of F(n) and G(n) from n=1000 to one billion in steps of multiplier 10000 is as follows:

(n)	F(n)	G(n)	Decimal Equivalence
1000	0.333334	0.333192289	3
10000	0.33333334	0.3333191729	4
100000	0.3333333333	0.3333319167	5
1000000	0.3333333333	0.3333331917	6
10000000	0.3333333333	0.3333333192	7
100000000	0.3333333333	0.3333333318	8
1000000000	0.3333333333	0.3333333332	9

Looking at the simulation results in Table (1) and Table (5), observe that the approximate function A(n), B(n) constructed from collapsing boundaries is faster in number of decimal digit equivalence than the approximate function constructed using 1- subtractiveness only.

6. Conclusion.

The paper explores the structure and properties of approximate functions from collapsing boundaries of a series associated with unit points of the (1-n-k) exponential integral. It applies results from the structure and properties to human relationship using analogies and suitable keys. The paper found that the square root function inclusion in the error terms of the 3 pairs of approximate function constructed from 3 intervals of positivity and convergence of the series were all 1- subtractive. The paper studies the conditions for 1-subtractiveness and applies it to construct approximates of a given function of (n) in the large that is when tends to infinity.

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