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DETERMINATION OF THE CHARACTERISTICS OF KEY COMPONENTS OF A QUEUING MODEL USING THE PROBABILITY FUNCTION OF ITS STEADY STATE.

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Abstract

In this paper, we review existing queuing models in literature and the general structure of the model. Basic components of a queue model are then analyzed using the steady state probability function to determine various characteristics. This includes queue length, variance of a queue, expected number of customers in the queue and expected waiting time. The three states of a queuing system which includes transient, steady and exponential states are discussed and analyzed. Consequently, the steady state is derived to show the interrelationship between the other components.

***Keywords*:** customer, queue, steady state, service.

1.0: Introduction

The queuing theory was developed by A. K. Erlang, a Danish Engineer, who took up the problem on congestion of telephone traffic in 1903 [1]. According to [2], waiting lines or queuing problems arise due to two reasons: (i) Too much demand on a facility leading to an excess of waiting time. (ii) There is too less demand, in which case there are too many facilities. As stated in [3], queuing lines have been applied to various aspects of business situations where customers are involved; this includes restaurants, banks, petrol pumps, patients in clinics, etc. The key elements of a queuing system are listed in [4], which includes; customers or arriving unit that require some service to be performed, queue which is the number of customers to be served and the service channel which is the system performing the service to the customer.

As remarked in [5], arrival distribution represents the pattern in which the number of customers arrive at the system. Arrival may also be represented by the inter – arrival time, which is the period between two successive arrivals. The rate at which customers arrive to be served represents the number of customers arriving per unit of time. Random arrival is when customer arrival has no fixed pattern. Service (Departure) Distribution represents the pattern in which the number of customers leave the system. Random arrivals, departure and service time are usually described by the exponential probability distribution, [6]. According to [7], a population in queuing system is said to be finite if there is a maximum number of customers in the system, while an infinite population implies that there is no definite number of customers in the system.

The three states of a queuing system are reported in [8], which includes transient, steady and exponential states. If the behavior of the system varies with time, it is said to be transient state. A queuing system is said to be in steady state condition if its behavior does not change with time, while an exponential state is when a queuing system builds up to infinity.

A single – channel queuing system is defined in [9] as one in which there is a random arrival time and a random service time at a single station. In [10], arrival and departure of customers in a queue system occur randomly, hence their mathematical models are formulated based on the following assumptions:

Assumption 1: Given $N(t) = n$, the current probability distribution of the remaining time until the next arrival is exponential with parameter λ_n $(n = 0, 1, 2, \ldots)$.

Assumption 2: Given $N(t) = n$, the current probability distribution of the remaining time until the next departure (service completion) is exponential with parameter μ_n $(n = 0, 1, 2, \ldots)$.

Assumption 3: The random variable of assumption 1 (the remaining time until the next arrival) and the random variable of assumption 2 (the remaining time until the next departure) are mutually of independent. The next transition in the state od the process is either $n = n+1$ (a single arrival) or $n = n-1$ (a single departure) depending on whether the former or later random variable is small.

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The most common decision that needed to be made when designing a queuing system as contained in [11], include: (a) number of servers at a service facility (b) efficiency of the servers, (c) number of service facilities (d) amount of waiting space in the queue, (e) any priorities for different categories of customers and the two primary considerations in making these kind of decisions typically are (i) the cost of service capacity provided by the queuing system and (ii) the consequences of making the customers wait in the queuing system. Providing too much service capacity causes excessive costs, providing too little causes excessive waiting. Therefore, the goal is to find an appropriate trade – off between the service cost and the amount of waiting.

In [13], a queuing model on Toll Gate with the aim of decongesting traffic on the highways was developed while [14], a queuing model on patients' waiting time in an ante – natal care clinic to determine the number of doctors required so that a given percentage of pregnant women do not exceed a given waiting time and the number of expectant mothers in the queue do not surpass a given threshold was developed. According to [15], a queuing system can be described by its input or arrival process, its queue discipline and its service mechanism.

In this paper, we use the single – channel Poisson arrivals with Exponential Service Infinite – Finite population Model (M/M/I) to analyze the queue mean and variance including the states in line with the model described in [15].

2.0: Mathematical Notations and Symbols

(a) M = Markovian (Poison) arrival or departure distribution (or exponential interarrival or service time distribution),

(b) E_k = Erlangian or gamma interarrival of service time distribution with parameter k,

(c) GI = general independent arrival distribution,

(d) G = general departure distribution,

(e) D = deterministic inter – arrival or service times.

Others are:

FCFS = first come, first served,

LCFS = last come, first served,

SIRO = service in random order,

GD = general service discipline.

(M/E/1): (FCFS/N/∞) represents Poison arrival (exponential inter-arrival), Erlangian departure, single, first come, first served discipline, maximum allowable customers N in the system and infinite population model. (N = finite)

2.1: Model 1. Single – Channel Poisson Arrival with Exponential Service Infinite – Population Model [(M/M/1): (FCFS/∞/∞)]

Let us consider a single – channel system with Poison arrivals and exponential service time distribution. Both the arrivals and service rates are independent of the number of customers in the waiting line. Arrivals are handled on 'first come, first served' basis. Also the arrival rate λ is less than the service rate μ.

The following mathematical notations (symbols) will be used in connection with queuing models:

n = number of customers in the system (waiting line + service facility) at time t.

λ = mean arrival rate (number of arrival per unit of time).

μ = mean service rate per busy server (number of customers served per unit of time)

λdt = probability that an arrival enters the system between t and $t+dt$ time interval i.e., within time interval dt.

$1-\lambda dt$ = probability that no arrival enters the system within interval dt plus higher order terms in dt.

μ = mean service rate per channel.

μdt = probability of one service completion between t and $t+dt$ time interval i.e., within time interval dt.

$1-\mu dt$ = probability of no service rendered during the interval dt plus higher order terms in dt

P_n = steady state probability of exactly n customers in the system.

$P_n(t)$= transient state probability of exactly n customers in the system at time t, assuming the system started its operation at time zero.

$P_{n-1}(t)$ = transient state probability of having $n+1$ customers in the system at time t.

$P_{n-1}(t)$ = transient state probability of having $n-1$ customers in the system at time t.

$P_n(t+dt)$ = probability of having n units in the system at time $t+dt$.

L_q = expected (average) number of customers in the queue.

L_s = expected number of customers in the queue (waiting + being served).

W_q = expected waiting time per customer in the queue (expected time a customer keeps waiting in the line).

W_s = expected time a customer spends in the system.

L_n = expected number of customers waiting in line excluding those times when the line is empty i.e., expected number in non – empty queue (expected number of customers in a queue that is formed from time to time).

W_n = expected time a customer waits in line if he has to wait at all i.e., expected time in the queue for non – empty queue.

Since the mean arrival rate is constant over time, it follows that the probability of an arrival between time t and ds is $\lambda.\ dt$.
Thus probability of an arrival in time $dt = \lambda.dt$ (1)
The following characteristics of Poisson distribution are written here without proof:

Probality of n arrivals in time t = $\frac{(\lambda t)^n . e^{-\lambda t*}}{n!}$ (2)

Probability density function of inter-arrival time (time interval between two consecutive arrivals)
$= \lambda\ .\ e^{-\lambda t}$ (3)

Finally, poisson distribution assumes that the time period dt is very small so that $(dt)^2$, $(dt)^3$, etc. and can be ignored.
Mean service rate μ is also assumed to be constant over time and independent of number of units already serviced, queue length or any other random property of the system. Thus probability that service is completed between t and $t + dt$, provided that the service is continuous

$= \mu dt.$ (4)

Under the condition of continuous service, the following characteristics of exponential distribution are written, without proof

Probability of n complete services in time t = $\frac{(\mu t)^n . e^{\mu t}}{n!}$ (5)

Probability density function (p.d.f.) of inter-service time, i.e., time between two consecutive services
$= \mu\ .\ e^{\mu t}$ (6)

To determine the properties of the single channel system, it is necessary to find an expression for the probability of n customers in the system at time t i.e., $P_n\ (t)$ is known, the expected number of customers in the system expression for $P_n(t)$, we shall first find the expression for $P_n(t + dt)$.
The probability of n units (customers) in the system at time $t + dt$ can be determined by summing up probabilities of all the ways this event could occur. The event can occur in four mutually exclusive and exhaustive ways.

Table 1: Probability of n units (customers) in the system at time *t +dt.*

Event	*No. of units at time t*	*No. of arrivals in time dt*	*No. of services in time dt*	*No. of units at time t + dt*
1	*n*	*0*	*0*	*n*
2	*n + 1*	*0*	*1*	*n*
3	*n – 1*	*1*	*0*	*n*
4	*n*	*1*	*1*	*n*

Now we compute the probability of occurrence of each of the event, remembering that the probability of a service or arrival is μdt or λdt and $(dt)^2$

∴ Probability of event 1 = Probability of having n units at time t
x Probability of no arrivals
x Probability of no services
$= P_n(t).\ (1 - \lambda dt)\ (1 - \mu dt)$
$= P_n(t)\ [1 - \lambda dt - \mu dt + \lambda\mu(dt)^2]$
$= P_n\ (t)[1 - \lambda dt - \mu dt].$

Similarly, probability of event $2 = P_{n+1}(t)\ .\ (1 - \lambda dt).\ (\mu dt)$
$= P_{n+1}\ (t)\ [\mu dt],$

Probability of event 3 $= P_{n+1}\ [\lambda dt].\ (1 - \mu dt) = p_{n-1}(t)\ [\lambda dt],$

Probability of event 4 $= P_n\ (t)\ .\ (\lambda dt)\ (\mu.dt)$
$= P_n(t)\ .\ [\lambda.\ \mu\ (dt)^2] = 0.$

Note that other events are not possible because of the small value of dt that causes $(dt)^2$ to approach zero (as in event 4).
Since one and only one of the above events can happen, we can obtain $P_n(t + dt)$ {where $n > 0$} by adding the probabilities of above four events.

$\therefore P_n(t+dt) = P_n(t)[1 - \lambda dt - \mu dt] + P_{n+1}(t)[\mu dt] + P_{n-1}(t)[\lambda dt] + 0$
or $P_n(t+dt) = P_n(t) - P_{n+1}(t)[\mu dt] + P_{n-1}(t)[\lambda dt]$
or $\frac{P_n(t+dt) - P_n(t)}{dt} = -(\lambda + \mu) \cdot P_n(t) + \mu \cdot p_{n+1}(t) \cdot + \lambda P_{n-1}(t).$
Taking the limit when dt → 0, we get the following differential equation which gives the relationship between P_n, P_{n+1} at any time t, mean arrival rate λ and mean service rate μ:
$\frac{d}{dt}[P_n(t)] = \lambda P_{n-1}(t) + \mu P_{n+1}(t) - (\lambda + \mu)P_n(t),\ where\ n > 0$ ………………(7)
After solving for $P_n(t+dt)$ where $n > 0$, it is necessary to solve for $P_n(t+dt)$ where n = 0 i.e., to solve for $P_0(t+dt)$. If $n=0$, only two mutually exclusive and exhaustive events can occur as shown in table 2.

Table 2: Arrivals for mutually exclusive and exhaustive events

Event	*No. of units at time t*	*No. of arrivals in time dt*	*No. of services in time dt*	*No. of units at time t + dt*
1	0	0	-	0
2	1	0	1	0

∴ Probability of event 1 = $P_0(t) \times (1 - \lambda\, dt) \times 1$
Probability of event 2=$P_1(t) \times (1 - \lambda\, dt) \times (\mu\, dt)$.
Note that if no units were in the system, the probability of no service would be
1. Probability of having no unit in the system at time $t + dt$ is given by summing up the probabilities of the above two events.

$$\therefore \quad P_o(t+dt) = P_o(t) \cdot (1 - \lambda dt) + P_1(t) \cdot (\mu dt)(1 - \lambda dt)$$
$$= P_o(t) - P_o(t) \cdot (\lambda dt) + P_1(t) \cdot (\mu dt)$$
or $P_o(t+dt) - P_o(t) = -P_o(t) \cdot (\lambda dt) + P_1(t)(\mu dt)$

or $\frac{P_{0(t+dt)} - P_0(t)}{dt} = \mu . Po(t) - \lambda p_0(t).$
When $dt \to 0$, the differential equation which indicates the relationship between probabilities p_0 and p_1 at any time t, mean arrival rate λ and mean service rate μ, is
$\frac{d}{dt}[P_0(t)] = \mu P_1(t) - \lambda Po(t),\quad where\ n = 0$ (8)
Equations (7) and (8) provide relationships involving the *probability density function* $p_n(t)$ for all values of n but still we do not know the value of $p_n(t)$.
Assuming the steady state condition for the system, when the probability of having n units (customers) in the system becomes independent of time, we get.
$p_n(t) = p_n,\ \frac{d}{dt}[p_n(t)] = 0.$
Therefore, for a steady state system the differential equations (7) and (8) reduce to difference equations (9) and 10)
$0 = \lambda p_{n-1} + \mu . p_{n+1} - (\lambda + \mu)p_{n,}\ where\ n > 0,$ (9)
$0 = \mu p_1 - \lambda p_{0,}\ where\ n = 0.$ (10)
From equation (10), we have
$p_1, \quad \frac{\lambda}{\mu} p_0$
Putting n = 1 in equation (9), we have
$0 = \lambda p_0 + \mu p_2 - (\lambda + \mu)p_1$
$\therefore p_2 = \frac{\lambda+\mu}{\mu} p_1 \frac{\lambda}{\mu} p_0$
$= \frac{\lambda+\mu}{\mu}\left(\frac{\lambda}{\mu} p_0\right) - \frac{\lambda}{\mu} p_0$
$= \frac{\lambda}{\mu} p_0 \left[\frac{\lambda+\mu}{\mu} - 1\right]$
or
$= p_2 = \left(\frac{\lambda}{\mu}\right)^2 . p_{0.}$
similarly, for n = 2, equation (9) gives
$p_3 = \left(\frac{\lambda}{\mu}\right)^3 . p_{0.}$

For n = n, we get $P_n = \left(\frac{\lambda}{\mu}\right)^n . p_0$. where n ≥ 0 (11)

Equation (11) gives P_n in terms of P_o λ and μ must be obtained. The easiest way to do this is to recognize that the probability that the channel is busy is the ratio of the arrival rate and service rate$\left(\frac{\lambda}{\mu}\right)$. Thus p_0 is 1 minus this ratio.

i.e.

$P_o = 1 - \frac{\lambda}{\mu}$. (12)

Hence

$P_o = \left(\frac{\lambda}{\mu}\right)^n . \left(1 - \frac{\lambda}{\mu}\right)$ (13)

Having known the value of p_n we can find the various characteristics of the system.

1. Expected number of units in the system (waiting + being served), L, is obtained by using the definition of an expected value

$E(x) = \sum_{t=0}^{i=\infty} x_i p_i$

$\therefore L_s = \sum_{n=0}^{n=\infty} n p_n$

$or\ L_s = \sum_{n=0}^{\infty} n \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right)$

$= \left(1 - \frac{\lambda}{\mu}\right) \sum_{n=0}^{\infty} n \left(\frac{\lambda}{\mu}\right)^n$

$= \left(1 - \frac{\lambda}{\mu}\right) \left[0\left(\frac{\lambda}{\mu}\right)^0 + 1\left(\frac{\lambda}{\mu}\right) + 2\left(\frac{\lambda}{\mu}\right)^2 + 3\left(\frac{\lambda}{\mu}\right)^3 + \dots\right]$

$= \left(1 - \frac{\lambda}{\mu}\right) \left[0 + \left(\frac{\lambda}{\mu}\right)^0 + 2\left(\frac{\lambda}{\mu}\right)^2 + 3\left(\frac{\lambda}{\mu}\right)^3 + \dots\right]$

The series within brackets is an infinite series of the form 0, a, $2a^2$, $3a^3$, …., xa^x … For such an infinite series, if a is a constant and less than one, the sum is given by the formula.

$S_\infty = \frac{a}{(1-a)^2}$

$\therefore L_s = \left(1 - \frac{\lambda}{\mu}\right)\left[\frac{\lambda/\mu}{(1-\lambda\,\mu)^2}\right] = \frac{\lambda/\mu}{1-\lambda\,\mu} = \frac{\lambda}{\lambda - \mu}$

2. Expected number of units in the queue L_4 = Expected number of units in the system – Expected number in service number in service (single server).

$\therefore L_4 = L_s - \frac{\lambda}{\mu} = \frac{\lambda}{\lambda - \mu} - \frac{\lambda}{\mu} = \lambda\left[\frac{\mu - \mu + \lambda}{\mu(\mu - \lambda)}\right]$.

$\therefore L_4 = \lambda^2/\mu(\mu - \lambda)$.

Note that the expected number in service is 1 times the probability that the service channel is busy *i.e.,* $1.\frac{\lambda}{\mu}$.

3. Expected time per unit in the system (expected time a unit spends in the system)

$W_s = \frac{\text{Expected number of units in the system}}{Arrival\ rate}$

$= \frac{L_s}{\lambda} = \frac{\lambda}{(\mu - \lambda)\lambda}$

$\therefore W_s = \frac{1}{\mu - \lambda}$

4. Expected waiting time per unit in the queue W_q = Expected time in system – time in service.

$W_q = W_s - \frac{1}{\mu}$

$= \frac{1}{\mu - \lambda} - \frac{1}{\mu} = \frac{\mu - \mu + \lambda}{\mu(\mu - \lambda)}$

$\therefore\quad W_q = \frac{\lambda}{\mu(\mu - \lambda)}$

5. Variance of queue length:

By definition we have

$Var(n) = [E(n)]^2$

$= \sum_{n=1}^{\infty} n^2 p_n - \left[\sum_{n=1}^{\infty} n p_n\right]^2$

$= \sum_{n=1}^{\infty} n^2 . \left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^n - [L_s]^2$

$= \left(1 - \frac{\lambda}{\mu}\right) \sum_{n=1}^{\infty} n^2 . \left(\frac{\lambda}{\mu}\right)^n - \left(\frac{\lambda}{\mu - \lambda}\right)^2$

$= \left(1 - \frac{\lambda}{\mu}\right)\left[1 . \frac{\lambda}{\mu} + 2^2 . \left(\frac{\lambda}{\mu}\right)^2 + 3^2 \left(\frac{\lambda}{\mu}\right)^3 + \dots\right] - \left(\frac{\lambda}{\mu - \lambda}\right)^2$

$= \frac{\lambda}{\mu}\left(1 - \frac{\lambda}{\mu}\right)\left[1 + 2^2 \frac{\lambda}{\mu} + 3^2 \left(\frac{\lambda}{\mu}\right)^2 + \dots\right] - \left(\frac{\lambda}{\mu - \lambda}\right)^2$

Let $S = 1 + 2^2 \frac{\lambda}{\mu} 3^2 . \left(\frac{\lambda}{\mu}\right)^2 + \dots = 1 + 2^2 p + 3^2 p^2 + \dots$ $\left(\therefore p = \frac{\lambda}{\mu}\right)$

Integrating both sides w.r.t. p from 0 to p, we have

$$\int_0^p S.dp = \int_0^p (1 + 2^2p + 3^2p^2 + \ldots)dp = [p + 2p^2 + 3p^3 + \ldots]\,dp$$

$= p\,(1 + 2p^2 + 3p^3 + \ldots = p(1+2p+3p^2 + \ldots)$

$= p\,.\,\frac{1}{(1-p)^2} = \frac{p}{(1-p)^2}.$

Now differentiating both sides w.r.t. p, we have,

$S = \frac{1}{(1-p)^2} + p.(-2)\,.(1-p)^{-3}(-1) = \frac{1}{(1-p)^2} + \frac{2P}{(1-p)^3}\ \frac{1+P}{(1-p)^3} = \frac{1+\lambda/\mu}{(1-\lambda/\mu)^3}$

$\therefore\ Var(n) = \frac{\lambda}{\mu}\left(1-\frac{\lambda}{\mu}\right).\frac{\lambda}{\mu}\frac{\left(1-\frac{\lambda}{\mu}\right)}{\left(1-\frac{\lambda}{\mu}\right)^3} - \left(\frac{\lambda}{\mu-\lambda}\right)^2.$

∴ Variance of queue length

$$= \frac{\lambda/\mu(1+\lambda/\mu)}{(1-\lambda/\mu)^2} - \frac{\frac{\lambda}{\mu^2}^2}{\left(1-\frac{\lambda}{\mu}\right)^2}\frac{\frac{\lambda}{\mu}}{\left(1-\frac{\lambda}{\mu}\right)^2} \tag{14}$$

The following additional formulae are written here without proof:

(i) Average length of non – empty queue (length of queue that is formed from time to time), Ln $=\frac{\mu}{\mu-\lambda}$.

(ii) Average waiting time in non-empty queue (average waiting time of an arrival who waits), $W_n = \frac{1}{\mu-\lambda}$,

(iii) Probability density function of waiting time (excluding service) distribution

$= \lambda\left(1-\frac{\lambda}{\mu}\right)e^{-(\mu-\lambda)t}$

$$= \left[\begin{matrix}\frac{\lambda}{\mu}\left(\mu - \lambda e^{-(\mu-\lambda)t}, t > 0\right) \\ \lambda\left(1-\frac{\lambda}{\mu}\right), t = 0.\end{matrix}\right[$$

(iv) Probability density function of waiting + service time distribution

$$= (\mu-\lambda)e^{-(\mu-\lambda)t} \tag{15}$$

(v) Probability of queue length being greater than or equal to n, the number of customers,

$$= p(\geq n) = \left(\frac{\lambda}{\mu}\right)^n \tag{16}$$

Conclusion

In this paper, we have presented a queue model with single – channel Poisson Arrival with exponential server. To determine the properties of the single channel system, we have to derive the probability of n customers in the system at time t (i.e., $P_n(t)$). Probability of events 1 – 4 were derived using differential equation as limit dt→0 which gives the relationship between P_n, P_{n+1} at any time t, mean arrival rate λ and mean service rate μ. The known value of the probability function P_n was then used to obtain the characteristic value for the queue length, queue variance and the expected number of units in the queue.

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