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PERTURBATIONS IN CORIOLIS AND CENTRIFUGAL FORCES ON THE POSITIONS AND STABILITY OF ELLIPTICAL EQUILIBRIUM POINTS IN ROBE'S RESTRICTED THREE-BODY PROBLEM UNDER OBLATE-TRIAXIAL PRIMARIES

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Abstract

The paper examine the general effect of the small perturbations in the Coriolis and centrifugal forces, the buoyancy force of the fluid of the first primary as oblate spheroid and the triaxiality of the second primary on the position and stability of the elliptical equilibrium points in the Robe's restricted Three-Body problem (R3BP). With the use of the synodic coordinate system, we obtained the equations of motion. The analysis of the effects of these perturbations on the positions and stability of their elliptical equilibrium points taking into consideration all the components of the pressure field when the second primary moves in an elliptical orbit around the first primary revealed that the points on the ellipse lying within the first primary are the elliptical equilibrium points. It is observed that these are stable for all values of, $D > 0$, $\sigma_1 > \sigma_2$, $-1 \leq \cos\theta \leq 1$ ***and*** $-1 < \sin\theta < 1$. ***We conclude that the perturbing forces does not change the stability of the equilibrium points.***

Keywords: Coriolis and centrifugal forces, elliptical equilibrium points, stability of equilibrium points.

1. Introduction

The Three-Body Problem (3BP) is defined in terms of three heavenly bodies with arbitrary masses attracting one another according to Newtonian law of gravitation, and is free to move in space. But only ten integrals out of the eighteen first order, coupled, nonlinear differential equations that govern the motion of 3BP are known [1]. Thus, the equations of motion of the 3BP are unsolvable analytically [2]. In an attempt to solve the 3BP; Lanrange introduced restricted three-body problem (R3BP) in which one of the bodies is assumed to encompass infinitesimal mass. The R3BP investigate the motion of the infinitesimal mass moving under the gravitational effects of two finite bodies called primaries, which move in circular orbits around the center of mass on the account of their mutual attraction and the infinitesimal mass does not influence the motion of the primaries [1].

The new-fangled type of R3BP, known as Robe's R3BP [3], in which the first primary of mass m_1 is a rigid spherical shell, filled with homogenous, incompressible fluid of density ρ_1, and the second mass m_2 is a small point outside the shell and moving around m_1 in a Keplerian orbit, the infinitesimal mass m_3 is a small sphere of density ρ_3, moving inside the shell and is subject to attraction of m_2 and the buoyancy force due to the fluid and that the radius of m_3 is assumed to be infinitesimal. The major analysis is the linear stability of the equilibrium points obtained in two cases; the orbit of m_2 around m_1 is circular in the first case and elliptic in the second case.

The Robe's R3BP have been studied extensively with the introduction of obaleteness and triaxiality; [4] studied the combine effect of small perturbations on the Coriolis and centrifugal forces on the position of the equilibrium point on the Robe's model and considered the case with same densities$(i.e\ \rho_1 = \rho_3)$. They assumed that the pressure field of the fluid

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ρ_1 has spherical symmetry around the centre of the shell and considered just only one of the three components of the pressure field, because of the gravitational field of the fluid itself. [5] Considered all these components of the pressure field, but assumed the hydrostatic equilibrium figure of the first primary as a Roche ellipsoid, and examined the existence and linear stability of the equilibrium points. [6] obtained the conditions for the existence of an infinite number of equilibrium points and their linear stability, but assume the hydrostatic equilibrium figure of the bigger primary as an oblate spheroid instead of a Roche ellipsoid, while [7] generalised the model by taking both primaries as oblate spheroids.

The analysis on Robe's Circlar Restricted Three-Body Problem (CR3BP) with oblate- triaxial rigid body system, shows the existence of an equilibrium point near the center of the primary, another equilibrium point on the line joining the center of the primaries, and set up conditions for which the points can be stable [8]. The most interesting and distinguishable result of their study are the existence of elliptical points and their stabilities [9]. The study of the effect of small perturbations in Coriolis and centrifugal forces on the axial equilibrium points and their stability, revealed that the positions of the axial equilibrium points are influence by only the centrifugal force because the force function is dependent on it, and the magnitude of this force shift the first equilibrium point $(p_1, 0, 0)$ to the positive side of the horizontal axis, but the second axial equilibrium point $(x_{11} + p_1, 0, 0)$ is swing away from the origin towards the negative side of the horizontal axis due to the combine effect of the triaxiality and the centrifugal force. The stability of these points was as discussed; the first point is unstable and the second point is conditionally stable [8]. [9], [10],[11]..

The present paper will examine the combined effect of small perturbations in the Coriolis and centrifugal forces, the full buoyancy force of the fluid of the first primary as oblate spheroid and the triaxiality of the second primary on the position and stability of the elliptical equilibrium points in the Robe's restricted Three-Body problem. We propose to examine and analyze the effects of Coriolis and centrifugal forces on the positions and stability of their elliptical equilibrium points, taking into consideration all the components of the pressure field when the second primary moves in an elliptical orbit around the first primary.

The model under consideration will have numerous applications in various astronomical problems as it provide hint to understanding the problems of small oscillations of the Earth's core in the gravitational field of Earth-Moon system, the stability of the centre of the Earth as an equilibrium points of the Robe's problem and the effects of perturbations in the motions of the non-natural satellites in the Earth-Moon vicinity.

The arrangement of this paper is in five sections; this section is the introduction, the equations of motions is in the next section, followed by the position of the elliptical equilibrium point and their stability in section three and four respectively, while the conclusion is drawn in section five.

2. Equations of motion

Suppose m_1 and m_2 be the masses of the first and second primary respectively and let ρ_1 be the density of the homogenous incompressible fluid of the first primary, while the second primary which posses a triaxial rigid body of mass m_2 moves in an elliptical orbit around the first primary, and the infinitesimal body of mass m_3 with density ρ_3 moves inside the first primary. We adopt a synodic coordinate system $0x_1\,x_2\,x_3$ with the origin at the center of mass m_1. $0x_1$ Points towards m_2 and $0x_1\,x_2$ coincide with the equatorial plain of m_1. The equations of motion of the infinitesimal mass m_3 in this coordinate system as in [8] and [9] is given as

$$\ddot{x}_1 - 2\varphi n\dot{x}_2 = U_{x_1},$$
$$\ddot{x}_2 + 2\varphi n\dot{x}_1 = U_{x_2}, \qquad (2.1)$$
$$\ddot{x}_3 = U_{x_3},$$

Where

$$U = V + \frac{n^2\omega}{2}\left[\left(x_1 - \frac{m_2 R}{m_1 + m_2}\right)^2 + x_2^2\right],$$

$$V = B + B' - \frac{\rho_1}{\rho_3}\left[B + B' + \frac{n^2\omega}{2}\left\{\left(x_1 - \frac{m_2 R}{m_1 + m_2}\right)^2 + x_2^2\right\}\right],$$

$$B = \pi G\rho_1[I - A_1x_1^2 - A_1x_2^2 - A_3x_3^2], \qquad \varphi = 1 + \varepsilon, \quad \omega = 1 + \varepsilon', \qquad \varepsilon \ll 1, \ \varepsilon' \ll 1,$$

$$B' = \frac{Gm_2}{[(R - x_1)^2 + x_2^2 + x_3^2]^{1/2}} + \frac{Gm_2(2\sigma_1 - \sigma_2)}{2[(R - x_1)^2 + x_2^2 + x_3^2]^{3/2}} - \frac{3Gm_2(\sigma_1 - \sigma_2)x_2^2}{2[(R - x_1)^2 + x_2^2 + x_3^2]^{5/2}} - \frac{3Gm_2\sigma_1x_3^2}{2[(R - x_1)^2 + x_2^2 + x_3^2]^{5/2}},$$

$$I = 2a_1^2A_1 + a_2^2A_2, \qquad n^2 = \frac{G(m_1 + m_2)}{R^2}\left(1 + \frac{3}{2}\alpha + \frac{3}{2}(2\sigma_1 - \sigma_2)\right),$$

$$A_1 = a_1^2 a_2 \int_0^\infty \frac{du}{\Delta(a_1^2+u)}, \quad A_2 = a_1^2 a_2 \int_0^\infty \frac{du}{\Delta(a_2^2+u)}, \quad \Delta^2 = (a_1^2+u)^2(a_2^2+u)$$

$$\alpha = \frac{a_1^2 - a_3^2}{5R^2}, \quad \sigma_1 = \frac{a^2-c^2}{5R^2}, \quad \sigma_2 = \frac{b^2-c^2}{5R^2}, \quad \sigma_i \ll 1\ (i=1,2), \quad A \ll 1$$

Here, V is the potential that explains the combined forces upon the infinitesimal mass, B denotes the gravitational potential due to the fluid mass, and B' is the potential due to the triaxial body. R is the distance between the primaries and G is the gravitational constant while n *and* I are the mean motion and the polar moment of inertia respectively. $A_i (i = 1,2)$ are the index symbols and σ_1, σ_2 characterize triaxiality of the second primary with semi axes a, b, c.

The units are taken such that the sum of the masses and the distance between the primaries are unity, and the units of time is selected such that $G = 1$ and R = 1. Then, the system of equations (2.1) in the dimensionless form is given as

$$\ddot{x}_1 - 2\varphi n \dot{x}_2 = U_{x_1},$$
$$\ddot{x}_2 + 2\varphi n \dot{x}_1 = U_{x_2}, \qquad (2.2)$$
$$\ddot{x}_3 = U_{x_3},$$

where

$$U = D\left[\pi\rho_1\{\mathrm{I} - A_1(x_1^2+x_2^2) - A_2x_3^2\} + \frac{\mu}{r_1} + \frac{\mu(2\sigma_1-\sigma_2)}{2r_1^3} - \frac{3\mu(\sigma_1-\sigma_2)x_2^2}{2r_1^5} - \frac{3\mu\sigma_1x_3^2}{2r_1^5} + \frac{n^2\omega[(x_1-\mu)^2+x_2^2]}{2}\right],$$

$$n^2 = 1 + \frac{3}{2}\alpha + \frac{3}{2}(2\sigma_1-\sigma_2), \quad \mu = \frac{m_2R}{m_1+m_2} < 1 \text{ and } D = \left(1-\frac{\rho_1}{\rho_3}\right), \quad r_1 = [(1-x_1)^2 + x_2^2 + x_3^2]^{1/2}.$$

The system (2.2) gives the equations of motion of the infinitesimal mass m_3 under the framework of the Robe's problem taking into account the influences of the full buoyancy force of the fluid, oblateness, triaxiality and gravitational attractions of the primary bodies. They are different from those of [8] but fully coincide with those of [9].

3. Position of the equilibrium points.

Equilibrium points are the points upon which the velocity and the acceleration of the infinitesimal body are zero. These points are the solutions of the equations $U_{x_1} = 0, \quad U_{x_2} = 0, \ U_{x_3} = 0.$ That is to say,

$$U_{x_1} = D\left[-2\pi\rho_1A_1x_1 + \frac{\mu(1-x_1)}{r_1^3} + \frac{3\mu(2\sigma_1-\sigma_2)(1-x_1)}{2r_1^5} - \frac{15\mu(\sigma_1-\sigma_2)(1-x_1)x_2^2}{2r_1^7} - \frac{15\mu\sigma_1(1-x_1)x_3^2}{2r_1^7} + n^2\omega(x_1 - \mu)\right] = 0,$$

$$U_{x_2} = Dx_2\left[-2\pi\rho_1A_1 - \frac{\mu}{r_1^3} - \frac{3\mu(2\sigma_1-\sigma_2)}{2r_1^5} - \frac{3\mu(\sigma_1-\sigma_2)}{r_1^5} + \frac{15\mu(\sigma_1-\sigma_2)x_2^2}{2r_1^7} + \frac{15\mu\sigma_1x_3^2}{2r_1^7} + n^2\omega\right] \qquad (3.1)$$

$$U_{x_3} = Dx_3\left[-2\pi\rho_1A_2 - \frac{\mu}{r_1^3} - \frac{3\mu(2\sigma_1-\sigma_2)}{2r_1^5} - \frac{3\mu\sigma_1}{r_1^5} + \frac{15\mu(\sigma_1-\sigma_2)x_2^2}{2r_1^7} + \frac{15\mu\sigma_1x_3^2}{2r_1^7}\right] = 0.$$

3.2. Elliptical equilibrium points

These points are the solution of the equations of system (3.1) with $x_1 \neq 0$ and $x_2 \neq 0,$ but $x_3 = 0.$ They lie in the $x_1, x_2 -$ plane where the x_1 *and* x_1 coordinates are the solution of the system of the equations (3.2) below.

$$-2\pi\rho_1A_1x_1 + \frac{\mu(1-x_1)}{r_2^3} + \frac{3\mu(2\sigma_1-\sigma_2)(1-x_1)}{2r_2^5} - \frac{15\mu(\sigma_1-\sigma_2)(1-x_1)x_2^2}{2r_2^7} + n^2\omega(x_1-\mu) = 0$$

and (3.2)

$$-2\pi\rho_1A_1 - \frac{\mu}{r_2^3} - \frac{3\mu(2\sigma_1-\sigma_2)}{2r_2^5} - \frac{3\mu(\sigma_1-\sigma_2)}{r_2^5} + \frac{15\mu(\sigma_1-\sigma_2)x_2^2}{2r_2^7} + n^2\omega = 0,$$

where $r_2 = [(1-x_1)^2 + x_2^2]^{1/2}.$ (3.3)

From (3.2), we have

$$2\pi\rho_1A_1 = \frac{3\mu(\sigma_1-\sigma_2)(x_1-1)}{r_2^5} + \left[1+\varepsilon' + \frac{3}{2}\alpha + \frac{3}{2}\alpha(2\sigma_1-\sigma_2)\right](1-\mu) \qquad (3.4)$$

and

$$1-\frac{1}{r_2^3}+\varepsilon'+\frac{3}{2}\alpha+\frac{3(2\sigma_1-\sigma_2)}{2}-\frac{3(2\sigma_1-\sigma_2)}{2r_2^5}-\frac{3(\sigma_1-\sigma_2)x_1}{r_2^5}+\frac{15(\sigma_1-\sigma_2)x_2^2}{2r_2^7}=0 \quad (3.5)$$

In the absence of the perturbation in the centrifugal force, oblateness and triaxiality, *i.e.* $\varepsilon'=\alpha=\sigma_1=\sigma_2=0$, we have $r_2=1$. But when perturbation in the centrifugal force, oblateness and triaxiality are present, then r_2 will change slightly by ε_2, such that

$$r_2=1+\varepsilon_2, \qquad \varepsilon_2\ll 1 \quad (3.6)$$

Substitute equation (3.6) into (3.5) and neglect the second and higher power of $\varepsilon',\varepsilon_2,\alpha,\ \sigma_1,\sigma_2$ as well as their products gives

$$\varepsilon_2=-\frac{1}{2}\alpha-\frac{1}{2}\varepsilon'+(\sigma_1-\sigma_2)x_1-\frac{5}{2}(\sigma_1-\sigma_2)x_2. \quad (3.7)$$

Substitute equation (3.7) into (3.6), we have

$$r_2=1-\frac{1}{2}\alpha-\frac{1}{2}\varepsilon'+(\sigma_1-\sigma_2)x_1-\frac{5}{2}(\sigma_1-\sigma_2)x_2. \quad (3.8)$$

Now, from equations (3.3) and (3.8), we have

$$x_1^2-2x_1(1+\sigma_1-\sigma_2)+x_2^2[1+5(\sigma_1-\sigma_2)]+\alpha+\frac{2}{3}\varepsilon'=0. \quad (3.9)$$

If $\sigma_1>\sigma_2$, equation (3.9) would be an ellipse centred at $(1+\sigma_1-\sigma_2,0)$, with the foci located at $\left(1+\sigma_1-\sigma_2\pm\sqrt{5(\sigma_1-\sigma_2)+(1/3)\varepsilon'},\ 0\right)$, the eccentricity is $\sqrt{5(\sigma_1-\sigma_2)+(1/3)\varepsilon'}$ and the lactus recta is given by $2[1-(1/2)\alpha-(2/3)\varepsilon'-4(\sigma_1-\sigma_2)]$. The coefficient normalizing factor q of this ellipse is given

$$q=16\left[\alpha+\frac{2}{3}\varepsilon'-1+3(\sigma_1-\sigma_2)\right]. \quad (3.10)$$

By the use of equation (3.10), we obtain the distance between the centre of the and foci points (either of the two) as $\sqrt{5(\sigma_1-\sigma_2)}$, the length of the semi major axis as$[1-(1/2)\alpha-(1/3)\varepsilon'+(\sigma_1-\sigma_2)]$ and the length of the semi minor axis as $[1-(1/2)(\alpha+\varepsilon')-(3/2)(\sigma_1-\sigma_2)]$. The general coordinates of a point on the ellipse are obtained by the use of the eccentricity angle θ, they are

$$x_1=1+\sigma_1-\sigma_2+\left(1-\frac{1}{2}\alpha-\frac{1}{3}\varepsilon'+\sigma_1-\sigma_2\right)\cos\theta \quad (3.11)$$

$$x_2=\left[1-\frac{1}{2}\alpha-\frac{1}{2}\varepsilon'-\frac{3}{2}(\sigma_1-\sigma_2)\right]\sin\theta \quad (3.12)$$

Substitute equations (3.11) and (3.12) into (3.8) and ignoring the product of $\alpha,\ \varepsilon',\sigma_1\ and\ \sigma_2$, we get

$$r_2=1-\frac{1}{2}\alpha-\frac{1}{3}\varepsilon'-\frac{1}{2}(\sigma_1-\sigma_2)(3-2\cos\theta-5\cos^2\theta). \quad (3.13)$$

The points on the ellipse lying within the first primary are the elliptical equilibrium points, and they are different from those of [8], because of the great influence of only the centrifugal force. In the absence of the perturbation in the centrifugal force, oblateness and triaxiality; the result of these points fully coincideswith those of [6].

4.1 Stability of the elliptical equilibrium points

In order to investigate the stability of the orbit in the vicinity of the equilibrium points, we assume a small displacement method by displacing the coordinates of the infinitesimal body and linearizing the equations of motion around the equilibrium points with coordinates (x_{10},x_{20},x_{30}). We displace the third body to the position $x_1=x_{10}+\xi,\ x_2=x_{20}+\eta$, $x_3=x_{30}+\zeta$, where $\xi,\eta\ and\ \zeta$ are small displacement in the coordinates of the equilibrium point (x_{10},x_{20},x_{30}) of the infinitesimal mass respectively. Taking only the linear terms, we obtain the variational equations corresponding to these equations of motion as

$$\begin{aligned}
\ddot{\xi}-2n\varphi\dot{\eta}&=(U^0_{x_1x_1})\xi+(U^0_{x_1x_2})\eta+(U^0_{x_1x_3})\zeta,\\
\ddot{\eta}+2n\varphi\dot{\xi}&=(U^0_{x_1x_2})\xi+(U^0_{x_2x_2})\eta+(U^0_{x_2x_3})\zeta, \qquad (4.1)\\
\ddot{\zeta}&=(U^0_{x_1x_3})\xi+(U^0_{x_2x_3})\eta+(U^0_{x_3x_3})\zeta.
\end{aligned}$$

The superscript 0 indicates that the partial derivatives are calculated at the equilibrium points $(x_1,\ x_2)$ under consideration. Hence, the variational equations of system (4.1) recast to the form

$$\ddot{\xi} - 2n\varphi\dot{\eta} = (U^*_{x_1x_1})\xi + (U^*_{x_1x_2})\eta,$$
$$\ddot{\eta} + 2n\varphi\dot{\xi} = (U^*_{x_1x_2})\xi + (U^*_{x_2x_2})\eta, \quad (4.2)$$
$$\ddot{\zeta} = (U^*_{x_3x_3})\zeta, \quad (4.3)$$

where,

$$U^*_{x_1x_1} = 3D\mu\left[(\sigma_1-\sigma_2)(1+2\cos\theta) + \cos^2\theta\left\{1+\frac{3}{2}\alpha+\varepsilon'-3\sigma_1+\frac{11}{2}\sigma_2-5(\sigma_1-\sigma_2)(\cos\theta-\cos^2\theta)\right\}\right],$$
$$U^*_{x_1x_2} = 3D\mu\sin\theta\left[(\sigma_1-\sigma_2)+\cos\theta\left\{1+\frac{3}{2}\alpha+\frac{5}{6}\varepsilon'-\frac{1}{2}\sigma_1+3\sigma_2-5(\sigma_1-\sigma_2)(\cos\theta-\cos^2\theta)\right\}\right],$$
$$U^*_{x_2x_2} = 3D\mu\sin^2\theta\left[1+\frac{3}{2}\alpha+\frac{2}{3}\varepsilon'+7\sigma_1-\frac{9}{2}\sigma_2-5(\sigma_1-\sigma_2)(\cos\theta-\sin^2\theta)\right],$$
$$U^*_{x_3x_3} = -D\left[2\pi\rho_1A_2 + \mu\left\{1+\frac{3}{2}\alpha+\varepsilon'+\frac{3}{2}(2\sigma_1+\sigma_2)-3(\sigma_1-\sigma_2)\cos\theta\right\}\right].$$

The motion of the infinitesimal body along the $x_3 - axis$ is stable, since equation (4.3) of the variational equations does not depend on the system (4.2) and the solution is solely imaginary. The characteristic equation of motion corresponding to system (4.2) written as

$$\lambda^4 - 4\varphi n^2 + U^*_{x_1x_1} + U^*_{x_2x_2}\lambda^2 + U^*_{x_1x_1} + U^*_{x_2x_2} - (U^*_{x_1x_1})^2 = 0$$

is a quadratic equation in λ^2, and the roots are

$$\lambda_1^2 = \frac{1}{2}\left\{U^*_{x_1x_1} + U^*_{x_2x_2} - 4\varphi n^2 + \sqrt{(U^*_{x_1x_1}+U^*_{x_2x_2}-4\varphi n^2)^2 - 4\left(U^*_{x_1x_1}+U^*_{x_2x_2}-(U^*_{x_1x_1})^2\right)}\right\}$$
$$\lambda_2^2 = \frac{1}{2}\left\{U^*_{x_1x_1} + U^*_{x_2x_2} - 4\varphi n^2 - \sqrt{(U^*_{x_1x_1}+U^*_{x_2x_2}-4\varphi n^2)^2 - 4\left(U^*_{x_1x_1}+U^*_{x_2x_2}-(U^*_{x_1x_1})^2\right)}\right\}.$$

The elliptical points are stable for all values of D> 0, $\sigma_1 > \sigma_2$, $-1 \le \cos\theta \le 1$ and $-1 < sin\,\theta < 1$ since the sum of the roots is negative and their product is positive, *i.e.*

$$\lambda_1^2 + \lambda_2^2 = 3D\mu\left\{1+\frac{3}{2}\alpha+3\sigma_1-\frac{1}{2}\sigma_2-\backslash 3(\sigma_1-\sigma_2)\cos\theta+\frac{1}{3}\varepsilon'(2+\cos^2\theta)-4\left(1+\varepsilon+\frac{3}{2}\alpha+\frac{3}{2}(\sigma_1-\sigma_2)\right)\right\} < 0,$$
$$\lambda_1^2 \times \lambda_2^2 = 9D^2\mu^2(\sigma_1-\sigma_2)\sin^2\theta > 0.$$

The stability behavior of this model differs from those of [8] due to the presence of the terms in the Coriolis and the centrifugal forces.

5. Conclusion

Considering the hydrostatic equilibrium figure of the bigger primary as an oblate spheroid, the shape of the smaller primary as a triaxial rigid body, and taking into account the full buoyancy force of the fluid acting on the infinitesimal body and the existence of perturbations in the Coriolis and centrifugal forces, we establish the equations of motion of the infinitesimal mass m_3 under the framework of the Robe's R3BP which fully coincide with those of [9].

The analysis of the position of the elliptical equilibrium points reviewed the points on the ellipse laying within the first primary are the elliptical equilibrium points, and they are different from those of [8]. It is observed that the foci, eccentricity, length of the semi major and minor axes and the general coordinate of a point on the ellipse are greatly influenced by only the centrifugal force, but the centre of the ellipse $(1+\sigma_1-\sigma_2, 0)$ and the distance between the centre and foci points (either of the two) $\sqrt{5(\sigma_1-\sigma_2)}$ fully coincide with those of [8].

The stability behavior of elliptical equilibrium points is different from those of [8] as a result of the presence of the terms in the Coriolis and the centrifugal forces in the points, and are stable for all values of D> 0, $\sigma_1 > \sigma_2$, $-1 \le \cos\theta \le 1$ and $-1 < sin\,\theta < 1$. We conclude that the perturbing forces does not change the stability of the equilibrium points.

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