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ANALYSIS OF DISTANCE CONSTRAINED VEHICLE ROUTING PROBLEM FOR THE ABUJA POST OFFICE

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Abstract

The vehicle routing problem (VRP) is the problem of finding the optimal routes of delivery or collection from one or several depots to a number of warehouses or customers.In this paper, the mail delivery of the Nigerian Postal Services (NIPOST) is modelled as a VRP in order to address the problem of delay in mail delivery occurring frequently in NIPOST. The Abuja Post Office is used as a case study and the Model of a related literature is applied with modifications to solve the problem. Appropriate data was obtained from Abuja Post Office and the data is used in coming up with the distance matrix. The problem is then solved with the help of Lexi-search algorithm.

Keywords: Vehicle Routing problem, Lexi-search algorithm, distance constraint, optimal and Mail delivery.

1. Introduction

The vehicle routing problem (VRP) can be defined as a problem of finding the optimal routes of delivery or collection from one or several depots to a number of cities or customers, while satisfying some constraints by [1]. Collection of household waste, gasoline delivery trucks, goods distribution, street cleaning, distribution of commodities, design telecommunication, transportation networks, school bus routing, snow plough and mail delivery are some of the areas where VRP is used.

The Abuja Post Office with a total of 22 Post Offices/Postal Agencies under it for the intra-city mail and parcel delivery currently uses three Toyota Hilux vans for mail and parcel clearance. Three routes are plied by the three Vehicles, one of them clears the mails and parcels around New karu axis, another one visits all the Post Offices on Airport road and terminates at Gwagwalada while the third clears mails around Bwari.

On one of the two routes, private Vans are contracted to move mails and parcels from both Abaji and Kwali to Gwagwalada daily before 12 noon except Sundays for clearance to Abuja. Similarly, mails and parcels are cleared from Keffi to New-Karu for collection. Meanwhile, the mails at Suleja are cleared to the Hub in Abuja by the inter-city truck which happens to lie on one of the National Mail route Networks from Wuse through Suleja, Minna to Sokoto.

The three Vehicles cover a daily total distance of 537.6Km and a total Fuel cost of N11,000:00, excluding Sundays while the annual expenditure for the two contracted vehicles is N120,000. This translates to an annual cost on transportation of N3, 563,000:00. Despite reformations and restructuring of NIPOST over the years, their mail delivery service is still not effective. The probable reason being.that NIPOST is not engaging the current trend of logistic delivery chain. The main aim of this study is to analyse the delivery system of Abuja Post Office as a case study and apply the model of [2] with some modifications on the delivery system of the Post Office. It is hoped that this will bring about the necessary improvement required for NIPOST to move forward.

This paper is organized as follows: Section 2 presents review of related literature on the problem. Section 3 presents a Mathematical formulation of the problem.A lexi-search algorithm is developed in Section 4. Computational experience for the algorithm has been reported in Section 5. Finally, Section 6 presents comments and concluding remarks.Figure 1 below shows the study site while the distance matrix between the Abuja Post Office and the other Post Offices can be found in appendix A.

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Figure 1- The Study Site

2 Review of Related Literature

The VRP belongs to the class of NP-hard problems and is considered as one of the most difficult problems. Because of the extreme difficulty of the problem, exact solution methods have been often implemented on high-performance computers. Methods to solve the VRP as well as any combinatorial optimization problem are classified into two broad categories – exact and heuristic with Exact methods giving exact solution to the problem. An implicit way of solving the VRP is to list all the feasible solutions, evaluate their objective function values, and pick out the best. However it is obvious that this 'brute-force technique' is grossly inefficient and impracticable because of vast number of possible solutions to the problem, even for a problem of moderate size. In fact, computational time grows exponentially with the problem size.

It is observed that as the problem size increases obtaining exact solution to the problem is very difficult. On the other hand, heuristic methods don't guarantee the optimality of the solution, but give near exact solution very quickly. However, there are situations where exact solutions are very important. In this paper, we considered the distanceconstrained VRP (DVRP) where the total travelled distance by each vehicle in the solution is less than or equal to the maximum possible travelled distance. If the distance from city i to city j differs from that of city j to city i, we call this problem asymmetric (ADVRP), otherwise it is called as symmetric (SDVRP). We seek exact solution to our problem which is a DVRP of the second case.

The different methods used to achieve a global optimum for the VRP and its variations include branch-and-bound, cutting planes or combinations of these methods, like branch-and-cut and dynamic programming. Branch-and-bounds are the most known and used algorithms and are defined from allocation and cutting rules, which define lower bounds for the problem.

A branch-and-bound algorithm for the VRP clearly requires a lower bound, because we have to minimize the total cost. Over the past 50 years, many lower bounds have been suggested for the VRP. An excellent survey of lower bounds is given in (Baldacci and Mingozzi, 2006).

The two objective functions to DVRP - minimize total distance and minimize number of vehicles was considered in [3]. The DVRP was transferred into a multiple traveling salesman problem with time windows (m-TSPTW), where the time window constraint $[a_i, b_i]$ for any customer *i* means that it is not allowed to serve customer *i* before a_i or after b_i In other words, the vehicle has to wait until time a_i to start before dealing with customer *i*. The problem was solved using a column generation approach. The authors presented and analyzed the worst case performance for DVRP with a heuristic and provided results with up to 100 customers.

The Lexi-search derives its name from lexicograph.This approach has been used to solve various combinatorial problems efficiently, like the Assignment problem, the Travelling Salesman Problem (TSP), the job scheduling problem and so on. In all these problems the lexicographic search was found to be more efficient than the Branch bound algorithms. The algorithm is deterministic and is always guaranteed to find an optimal solution by [4] opined that it has the structure of the search algorithm that does not require huge dynamic memory during execution.

In [5], the Lexi-search algorithm was found to be one of the best exact algorithms, and genetic algorithm was found to be one of the best heuristic algorithm for the quadratic assignment problem(QAP). Hence, in [5], a hybrid algorithm that combines lexi-search algorithm with genetic algorithm was developed to find heuristic solution to the QAP. A comparative study was carried out betweenthe proposed algorithm and unified particle swarm optimization (UPSO)for some QAPLIB instances, and found that the proposed algorithm was better in [6].

Excel Spreadsheet solver was applied to model the Tangier problem in [7] as a distance constrained vehicle routing problem. It was the problem of designing routes for vehicles that should supply differentcustomers with defined locations and specific demand from a single or various depots. The main objective in this case wasminimizing the total cost of delivery or maximizing the profit while taking into consideration distance constraint that vary from a case to another. In the paper, a definition of the problem was given, presentation of a mathematical model to describe it, discussed aboutthe existing solutions to solve it and used different tools to solve a real VRP of acompany in tangier.

While [8], developed and applied the Spreadsheet solver to the solution of two real life issues. The authorintroduced VRP Spreadsheet Solver, an open source Excel based tool for solving many variants of the Vehicle Routing Problem (VRP). Case studies of two real-world applications of the solver from the healthcare and tourism sectors that demonstrate its use were presented. The solution algorithm for the solver, and computational results on benchmark instances from the literature were provided. The solver was found to be capable of solving Capacitated VRP and Distance-Constrained VRP instances with up to 200 customers within 1 h of CPU time.

VRP model using a generic multiple traveling salesperson was proposed by [9] and formulated with sub-tour elimination constraints. The model involves assigning cities to the hub, and determining the number of vehicles as well as their routing.

Formulations for the general situations involving simultaneous location and routing was presented in [10]. The given integer programming formulations for various location routing problems (multiple depots, single and multiple vehicles, depots having fixed costs) were then handled using a constraint relaxation procedure named STRAIGHT. This approach was based on solving the problems with only a limited number of the original constraints. The other constraints were gradually introduced into the problems as they were found to be violated.

A model formulated todetermined the number and size ofthe service territories was carried out by [11]. Each service territory consisted ofa service facility and an area that houses the customers to be served (some type of delivery or visits). The model was applied to United States Postal Service (*USPS*) network, sizing territories around a central sortation facility, and locating delivery units to each service territory.

The asymmetric, uncapacitated, multiple allocation p-hub median problem was first formulated by Campbell in [12] as a mixed integer linear programming. The paper defined a p-hub median as analogous to a p-median and formulated it for the multiple and single allocation p-hub median problems. Two heuristics for the single allocation p-hub median problem were evaluated. These heuristics derived a solution to the single allocation p-hub median problem from the solution to the multiple allocation p-hub median problem. Computational results were presented for problems with 10 – 40 origins/destination and up to 80 hubs.

The Swiss parcel delivery system was studied in [13]. The Swiss Postal service was restructuringdrastically andthey needed to modify their logistic system. The new structure contained transshipment points (parcel processing centers, delivery bases) and the problemwas composed of deciding on the number, capacity size and location of these transshipment points. They also worked on the allocationsofcustomers to the delivery bases and the assignment of the delivery bases/post offices to the parcel processing centers. Theysupported their decision by means of a discrete facility location model. The authors gave the details ofthe determination and derivation of cost components existing in the system.

VRP with softtime windows under service-time uncertainty was proposed by [14] as a robust optimization model to address its problem. The problem arose in the dispatchingof technicians to customer sites to repair broken equipment. In this context, significantservice-time uncertainty exists due to the high potential for misdiagnosis of the failureat the time when the client requests a repair. The authors developed a branch-and-pricemethod to solve the problem under several uncertainty descriptions and applied theirapproach on real data sets from the industry.

Two scheduling models (with and without waiting for feeding flights) concerning the arrival and departure times of flights in a typical hub airport have been presented in [15]. The research was carried in a way that can easily be applied to other problems such as express parcel deliveries, ground transportation hubs or consolidation operations at central warehouses. Since the addressed problems are nonlinear and highly combinatorial, heuristic solution methods were given for life-size problems.

The mixed integer programming model as presented by [2] was formulated to solve the capacitated *VRP* with time windows for the South African Post Office. The model was solved using a heuristic technique. The pilot location used was Emalahleni mail centre with satellite locations around it. The distances from the mail centre to each of the retail outlets was done using the latitude and longitude of each of the retail outlets and these locations fed into the heuristic. Route formulation was done and model validated.

For the Abuja Post Office problem, we will formulate it as a mixed integer programming model by modifying the model of [2].

3 The Mathematical Model

In this section, we present the mathematical model to be used in solving the Abuja Post Office problem. It is a modification of the model of [2]. We now present the model of [2] before our modification of it.

The mixed integer programming model of [2] was formulated to solve the capacitated *VRP* with time windows for the South African Post Office. For the purpose of the model, an arc represents the connection between two nodes, as well as the direction travelled. Vertex 0 was used to represent the depot and the remaining vertices were used to represent the retail outlets. But before then, we will have to define the following notations.

Denote: $i \neq j; i, j \in \{0,1,2, \dots, N\}$ where N is the total number of retail outlets
Denote: $k \in \{0,1,2, \dots, K\}$ where K is the total number of vehicles used by the South African Post Office.
The decision variables include:

$$X_{ijk} = \begin{cases} 1, \text{if vehicle } k \text{ travels from node } i \text{ to node} j \\ 0, \quad \text{otherwise} \end{cases}$$

The model parameters included:
T_i : The arrival time at node i
C_{ij} : Total cost incurred from node i to node j for the arc
t_{ij} : The travel time between node$i$ to node j
m_i : Demand at node i
q_k : The carrying capacity of vehicle k
f_i : Service time at node i
e_i : The earliest arrival time at node i
l_i : The latest arrival time at node i
r_k : Maximum route time allowed for vehicle k
The objective function was stated as,
Minimise,

$$\sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{j\neq i,k=1}^{K} C_{ij}X_{ijk} \qquad (1)$$

Subject to,

$$\sum_{k=1}^{K}\sum_{j=1}^{N} X_{ijk} \leq K \qquad \forall i = 0 \qquad (2)$$

$$\sum_{j=1}^{N} X_{ijk} = 1 \qquad \forall i = 0 \ , k \in \{1, \dots, K\} \quad (3)$$

$$\sum_{k=1}^{K}\sum_{j=0,j\neq i}^{N} X_{ijk} = 1 \qquad \forall i \in \{1, \dots, N\} \qquad (4)$$

$$\sum_{k=1}^{K}\sum_{i=0,i\neq j}^{N} X_{ijk} = 1 \qquad \forall j \in \{1, \dots, N\} \qquad (5)$$

$$\sum_{i=1}^{N} m_i \sum_{j=0,j\neq i}^{N} X_{ijk} \leq q_k \qquad \forall k \in \{1, \dots, K\} \qquad (6)$$

$$\sum_{i=1}^{N}\sum_{j=0,j\neq i}^{N} X_{ijk}(t_{ij} + f_i) \leq r_k \qquad \forall k \in \{1, \dots, K\} \qquad (7)$$

$$\sum_{k=1}^{K}\sum_{i=0,j\neq i}^{N} X_{ijk}(t_{ij} + f_i) \leq T_i \qquad \forall i \in \{1, \dots, K\} \qquad (8)$$

$$e_i \leq \left(t_{ij} + f_i\right) \leq l_i \forall i \in \{1, \dots, N\} \qquad (9)$$

$$X_{ijk} \in \{0,1\} \forall i, j \in \{1, \dots, N\} \qquad (10)$$

The objective function (1) minimises cost while we intend to minimise the total distance covered by the vehicles in Abuja Post Office. The assignment constraint (2) in [2] which assigns less than or equal to K vehicles will be similar to our model. Also, constraints (3) – (4) state that one and only one vehicle enters and leaves each retail Post Office are similar to our constraints. Constraint (5) of [2] stipulates that the total demands for the retail Post Offices should be less than or equal to the carrying capacity of the vehicle q_k for each route which we shall also adopt in our case. In the model of [2], constraint (6) ensures that, the travel time between retail Post offices and waiting time should be less than or equal to the maximum route time allowed, while constraint (7) ensures that the travel time between retail Post offices and waiting time should be less than or equal to the arrival time at node i. Lastly, constraint (8) ensures that the travel time between retail Post offices and waiting time should be less than or equal to the latest arrival time. In our case, we shall not consider constraints (6) – (9) in our first model because of the low probability of keeping timing windows. These constraints will be modified and incorporated in our subsequent models. Constraint (10) in model of [2] which is the integrality will be considered in our case too, to be sure that the assignment constraints are integers. That is not all, the model of [2] did not consider subtour elimination constraints but, we shall include subtour elimination constraints in our model. This is because, the routes so formed ought to be feasible solutions.

Based on modifications of [2] our model will be defined as follows:
Parameter definitions will be the same as for the model of [2] except for,
d_{ij} : The distance covered in moving from node i to node j for the arc
L: The maximum distance travelled
u_i : The set of retail Post Offices on that route
Our model then is as follows;
Minimize,

$$\sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{j\neq i,k=1}^{K} d_{ij}X_{ijk} \qquad (11)$$

Subject to,

$$\sum_{k=1}^{K}\sum_{j=1}^{N} X_{ijk} \leq K \qquad \forall i = 0 \qquad (12)$$

$$\sum_{j=1}^{N} X_{ijk} = 1 \qquad \forall i = 0 \ , k \in \{1, \dots, K\} \qquad (13)$$

$$\sum_{k=1}^{K}\sum_{j=0,j\neq i}^{N} X_{ijk} = 1 \qquad \forall i \in \{1, \dots, N\} \qquad (14)$$

$$\sum_{k=1}^{K}\sum_{i=0,i\neq j}^{N} X_{ijk} = 1 \qquad \forall j \in \{1, \dots, N\} \qquad (15)$$

$$d_{iN}X_{Ni} + u_i \leq L \quad i = 1,2, \dots, N \qquad (16)$$

$$U_i - U_j + nX_{ij} \leq n - 1 \text{ for } i, j = 2, .., n \qquad (17)$$

$$1 \leq U_i \leq n - 1 \quad i = 2, .., n \qquad (18)$$

$$X_{ijk} \in \{0,1\} \qquad \forall i, j \in \{1, \dots, N\} \qquad (19)$$

Constraint (11) which is the objective function minimises the total distance travelled in delivery of mails between one Post Office and another. Constraint (12) ensures that the total number of vehicles does not exceed$K$. Constraint (13) means that, only one vehicle connects any two Post Offices. Constraint (14) states that there is exactly one vehicle which enters each Post Office$i$. While, constraint (15) states that there is exactly one vehicle leaving each Post Office$j$ once and only once. Constraint (16) ensures that the total length for all the routes is equal to or less than the set distance, L equals to 200km for two or more vehicles. However, this constraint is not possible for a single vehicle since it has to go to all the Post Offices, therefore we set our L in this case to 300km. Constraints (17) and (18) are the subtour elimination constraints which guarantee that each optimal route should be feasible and (19) states that the assignment constraints should be integers.

The solution in this case entails looking for one tour over all Post Offices with minimum travelled distance. Since, the problem is NP-hard, it is very difficult to solve. We consider the transformation of DVRP to TSP.We study the natural set covering formulation of distance constrained vehicle routing.This is a special case of the set partitioning model for vehicle routing with time windows, as studied in [16]. There is a binary variable X_{ij} for every *r*-tour i of length at most *D*. Theconstraints require that every vertex be covered by at least one such *r*-tour. The LP relaxationis obtained by dropping the integrality on the variables and is as follows.

$$\min \sum_{i} X_{ij}$$

Subject to,

$$(LP) \qquad \sum_{i:v\in i} X_{ij} \geq 1 \ \forall v \in V / r$$

$$X_{ij} \geq 0$$

Although this *LP* has an exponential number of variables, it can be solved approximatelyin polynomial time.

4A lexi-search algorithm for the problem

In lexi-search algorithm, the set of all possible solutions to a problem is arranged in a hierarchy, such that each incomplete word represents the block of words with this incomplete word as the leader. For the VRP, each node is considered as a letter in an alphabet and tour set can be represented as a word with this alphabet. Thus the entire set of words in this dictionary (namely, the set of solutions) is partitioned into blocks. Bounds are computed for the values of the objective function over the blocks of words, which are then compared with the 'best solution value' found so far. If no word in the block can be better than the 'best solution value' found so far, jump over the block to the next one. If the current block, which is to be jumped over, is the last block of the present super-block, then jump out to the next super-block. Further, if the value of the current leader is already greater than or equal to the 'best solution value'; no need for checking the subsequent blocks within this super-block. However, if the bound indicates a possibility of better solutions in the block, enter into the sub block by concatenating the present leader with appropriate letter and set a bound for the new sub-block so obtained by [5].

4.1 Alphabet Table

Alphabet matrix in appendix C, $A=[a(i,j)]$, is a matrix of order n by ($n+m$ -1) formed by positions of elements of the augmented distance matrix in appendix B, $D=d_{ij}$. The *i*th row of matrix *A* consists of the positions of the elements in the *i*th row of the matrix D when they are arranged in the non-decreasing order of their values. If $a(i,p)$ stands for the *p*th element in the *i*th row of *A*, then $a(i,1)$ corresponds to the position of smallest element in *i*th row of the matrix D, as in [5]. Alphabet table "$\lfloor a(i,j)-d_{i,\alpha(i,j)}\rfloor$" is the combination of elements of matrix Aand their values as shown in appendix B.

Table 1List of Post Offices and their acronyms with Latitude And Longitude

Location ID	Name	Address	Latitude (y)	Longitude (x
0	Depot	A10.FCT	9.0667000	7.4833000
1	Customer 1	Nya,FCT	9.0561000	7.5789000
2	Customer 2	A1,FCT	9.0579000	7.4951000
3	Customer 3	Krv,FCT	9.0469000	7.7636000
4	Customer 4	Fed,FCT	9.0627000	7.4983000
5	Customer 5	A11,FCT	9.0241000	7.4783000
6	Customer 6	Mam,FCT	9.0899000	7.5197000
7	Customer 7	Nas,FCT	9.0671000	7.5098000
8	Customer 8	Old,FCT	9.0722600	7.4913000
9	Customer 9	Jik,FCT	9.1009000	7.2680000
10	Customer 10	Mog,FCT	9.0500000	7.5396000
11	Customer 11	Wz3,FCT	9.0596000	7.4719000
12	Customer 12	Bhr,FCT	9.2180000	7.4080000
13	Customer 13	Kub,FCT	9.2020000	7.3990000
14	Customer 14	Dei,FCT	9.1142000	7.2598000
15	Customer 15	Lsb,FCT	9.2760000	7.3593000
16	Customer 16	Gwa,FCT	8.9508000	7.0767000
17	Customer 17	Sul,NIG	9.1806000	7.1794000
18	Customer 18	Kuj,FCT	8.6590000	7.2705000
19	Customer 19	Kwl,FCT	8.7356000	6.9678000
20	Customer 20	Uab,FCT	8.8508000	7.0667000
21	Customer 21	Abj,FCT	8.8921000	6.8182000

5 Analysis and Results

In this study, we are using the Lexi-search algorithm which Erdogan incorporated in an Ecxel spreadsheet solver and left it as open source solver package. The package was applied by Erdogan as reviewed in related literature. Though the spreadsheet solver comes with a manual, in trying to apply it to solve the Abuja Post Office problem, I had to interact with him personally by Electronic Mail (email) in his University of Bath, United Kingdom. The Lexi-search algorithm as applied by [5] is a special case of the branch-and-bound algorithm.In this section, we apply the spreadsheet solver to analyse the NIPOST delivery chain using the distance matrix in Appendix A.

The model's result for one vehicle, two vehicles and then three vehicles are depicted graphically in Figures 2, 3 and 4which were generated with the aid of Google maps and the location of the Post Offices in Latitudes and Longitudes, Table 1. If NIPOST decides to use one vehicle, the distance covered for this TSP is 295km with the vehicle returning to the depot at 3:00 pm with a working time of five hours. The optimal route shows that the vehicle leaves the depot at 8:00am and travels to Area 11, then Federal secretariat and then National Assembly and so on as shown in Figure 2 below.

The optimal route for one vehicle is: A10 => A11=>Fsec =>Nass => Mam =>Mog =>Nya =>Krv =>Jik =>BHr => BLs =>Gwa =>Uni =>Abj =>kwl =>Kuj =>Sul => Dei =>Kub => Wz3 =>Osec => A1 => A10, distance covered is 295 Km and a working time of five hours.

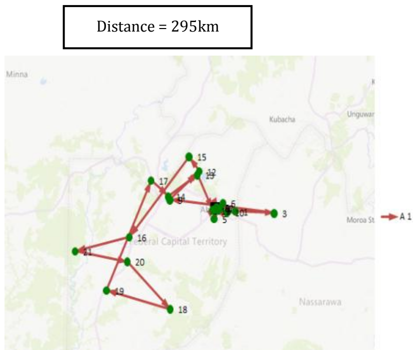

Figure 2: Generated Route for one Vehicle

While, in the case where two vehicles are used, the total distance covered is 357km but will return to the depot earlier, with the last vehicle working for four hours. Their optimal routes as shown in Figure 3 are as follows: Vehicle A1 leaves the depot and travels to Area 11 and then National Assembly and so on, and back to the depot. Next vehicle A2 starts it's journey from the depot, then moves to Area1, then Bwari Law school, Gwagwalada Market Post Office and so on, and back to the depot.

The optimal routes for two vehicles are:**A1** - A10 => A11=>Fsec =>Nass => Mam =>Mog =>Nya =>Krv =>Jik =>BHr =>Kwl =>Sul => Dei =>Kub => Wz3 => A10.

The optimal route for vehicle two, **A2** is: A10 => A1=>Bls =>Gwa =>Abj =>Uni =>Kuj =>Osec => A10.

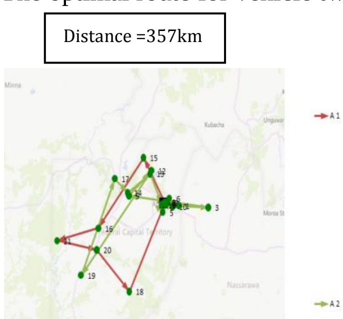

Figure 3: Generated Routes for two Vehicles

Lastly, for the three vehicles, we have the routes shown in Figure 4. The figure is self-explanatory and the optimal routes too, with a total distance of 361km. The vehicle A1 with the longest route returns to the depot at about 12:00 noon with a working time of four hours. It is worthy of note that, the route for vehicle A2 is not conspicuous in Figure 4 because the Post Offices are clustered together.

The optimal routes for three vehicles are:A**1** ; A10 => A11=>Fsec =>Nass => Mam =>Mog =>Nya =>Krv =>Jik =>BHr =>Bls =>Gwa =>Abj =>UnA => A10.

Vehicle**A2** - A10 =>OlSec => A10.

Vehicle**A3** - A10 => A1=>Kuj =>Kwl =>Sul => Dei =>Kub => Wz3 => A10.

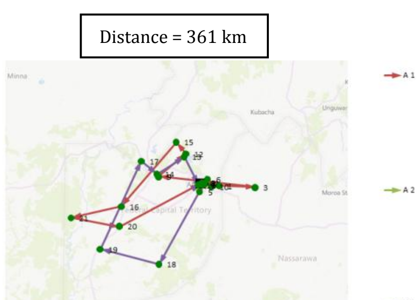

Figure 4: Generated routes for three Vehicles

Since NIPOST is currently using three vehicles, we draw our comparison using three vehicles, see Table 2 below showing the average Petrol price(N/Litre) put at N142, N163 and N213.

Table 2: Cost of average daily and annual Petrol Consumption

No. of Vehicles			1	2	3
Distance covered (Km)			295	357	361
Consumption rate (L/100km)			9.5	9.5	9.5
Average Petrol Price (N/litre)	142	Daily Cost (N)	3,951.53	4,815.93	4,869.89
		Annual Cost (N)	1,365,599.15	1,507,386	1,524,275.57
	163	Daily Cost (N)	4,568.08	5528.15	5,590.09
		Annual Cost (N)	1,429,809.04	1,730,309.39	1,749,696.61
	213	Daily Cost (N)	5,969.33	7223.90	7,304.84
		Annual Cost (N)	1,868,400.29	2,261,079.14	2,286,413.36

Assuming that the Abuja Post Office uses three vehicles for their mail delivery, the annual savings for three vehiclesis as shown in Table 3 below.

Table 3: Savings analyses for three Vehicles

Average Petrol Price(N)	Annual savings
142	N3, 563,000 – N1,524,275.57 = N2,038,724.43
163	N3, 563,000 – N1,749,696.61= N1, 813,303.39
213	N3, 563,000 – N2,286,413.36 = N1,276,586.64

6 Conclusion

In this study, we apply the Lexi-search algorithm to the distance constrained vehicle routing problem for the Abuja Post Office through the spreadsheet solver and the results so obtained show that the total distance covered increases with increase in the number of vehicles which is expected. This is because each vehicle starts and ends at the depot and so as the number of vehicles increase, the distance covered by the vehicles must also increase.Note however, that in getting our savings, we do not take care of the service time in mail clearance. If that were to be done, we may have to require more vehicles for the mails to be delivered and cleared on time. Also, we draw our comparison with the case of three vehicles because, the Abuja Post Office uses three vehicles instead of one or two and their aim is to improve delivery time. It can be seen in our analysis that, for three vehicles, the latest vehicle returns to the depot at about 12 noon as against one vehicle which returns after eight hours. It is evaluated and found that, with their current transportation schedule and based on our assumption an annual savings of N2,038,724.43, N1,832,690.61 and N1,813,303.39 at N142, N163 and N213 per Litre respectively is to be realised for three vehicles.

Appendix A: 22 × 22 Distance Matrix in Km

		A10	Nya	A1	Krv	Fsec	A11	Mam	Nas	Osec	Jik	Mog	Wz3	Bhr	Kub	Dei	Bls	Gwa	Sul	Kuj	Kwl	UniA	Abj
		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
A10	0	0	17.2	1.9	19.4	3.7	3.9	8.51	5.9	2.6	22.7	9.1	5	50.2	29.9	42	47.8	51.3	65	40.6	74.3	48.9	114.5
Nya	1	17.2	0	12.2	2.3	15	12	12.5	16.2	16.3	6.7	4.5	12.4	52.6	34.2	46	52.2	68	68.8	56.1	84.2	65.6	126.7
A1	2	1.9	12.1	0	14.4	5.6	5.5	12.2	7.6	2	16.5	11.1	6	57.6	37.2	41	35.2	51.3	65.9	35.7	73.5	48.9	114.2
Krv	3	19.4	2.3	14.4	0	12.5	12.3	13	13.1	16.4	4.1	6.8	14.7	51.6	39.2	51	57.2	66.2	73	53.8	86.5	63.9	127.4
Fsec	4	3.7	15	5.6	12.5	0	3.7	5.3	2.2	6.3	20.6	9.7	4.3	50.3	29.9	37	47.9	54.9	59.8	40.8	74.6	52.5	116.9
A11	5	3.9	12	5.5	12.3	3.7	0	6	4.8	7.5	11.8	7.7	7	53.7	33.3	37	51.3	57.3	59.7	43	77.2	55.3	117.5
Mam	6	8.5	12.5	12.2	13	5.3	6	0	4.6	12.9	17.1	7.7	9.5	49.2	28.8	33	46.8	61.4	55.2	45.7	82.9	59	122.6
Nas	7	5.9	16.2	7.6	13.1	2.2	4.8	4.6	0	9.3	20.3	8.1	6.1	52.2	30.9	34	49.9	57.1	56.4	42.7	76.8	54.7	118.4
Osec	8	2.6	16.3	2	16.4	6.3	7.5	12.9	9.3	0	18.5	11.8	6.6	53.4	32.9	36	50.9	68.6	58.8	38	72.3	66.2	127.7
Jik	9	22.7	6.4	16.5	4.1	20.6	11.8	17.1	20.3	18.5	0	15.2	18.5	61.3	40.9	44	58.9	72.7	67.3	62.3	90.6	70.3	137.5
Mog	10	9.1	4.5	11.1	6.8	9.7	7.7	7.7	8.1	11.8	15.2	0	11.1	51.7	31.3	35	49.3	61.5	57.7	47.8	123	59.1	122.7
Wz3	11	5	12.4	6	14.7	4.3	7	9.5	6.1	6.6	18.5	11.1	0	48.7	28.3	32	46.3	53.2	54.7	39.4	64.7	51.3	122.3
Bhr	12	50.2	52.6	57.6	51.6	50.3	53.7	49.2	52.3	53.4	61.3	51.7	48.7	0	20.9	43	2.4	64.8	67.5	74.2	70.6	65.1	183.7
Kub	13	29.9	34.2	37.2	39.2	29.9	33.3	28.8	30.9	32.4	40.9	31.3	28.3	20.9	0	14	22.4	44.4	47.1	53.8	53.9	44.7	163.3
Dei	14	42.4	46.2	41.3	50.7	37.2	37.1	32.6	33.8	36.2	44.7	35.1	32.1	42.8	14.1	0	26.2	48.2	50.9	57.6	57.7	48.5	167.1
Bls	15	47.9	53.2	55.2	56.2	47.9	51.3	46.8	49.9	50.9	58.9	49.3	46.3	2.4	32.1	26	0	61.6	65.1	71.8	71.9	62.7	181.3
Gwa	16	51.3	68	51.3	66.2	54.9	57.7	61.4	57.1	68.6	72.7	61.5	53.7	64.8	37.8	24	61.6	0	40.2	22.7	16.7	2.4	63.2
Sul	17	65	68.8	63.9	73.3	59.8	59.7	55.2	56.4	58.8	67.3	57.7	54.7	67.5	36.7	23	48.8	40.2	0	51.3	63	38.8	189.7
Kuj	18	40.6	56.1	35.2	52.8	40.8	43	45.7	42.7	38	62.3	47.8	39.4	74.2	56.7	43	54.3	23.7	51.3	0	27.9	22.5	158.7
Kwl	19	74.3	84.2	73.5	86.5	74.6	77.2	82.9	76.8	72.3	90.6	123	64.7	70.6	53.9	58	71.9	16.9	63	27.9	0	17.3	137.1
UniA	20	48.9	65.6	48.9	63.9	52.5	55.3	59	54.7	66.2	70.3	59.1	51.3	61.8	35.4	21	61.7	2.4	37.8	20.3	14.3	0	60.8
Abj	21	115	127	114	128	117	118	123	118	128	138	123	122	184	163	167	181	63.2	190	159	137	60.8	0

Appendix B: Augmented Distance Matrix

		A10	Nya	A1	Krv	Fsec	A11	Mam	Nas	Osec	Jik	Mog	Wz3	Bhr	Kub	Dei	Bls	Gwa	Sul	Kuj	Kwl	UniA	Abj	A10
		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	0
A10	0	0	17.2	1.9	19.4	3.7	3.9	8.51	5.9	2.6	22.7	9.1	5	50.2	29.9	42	47.8	51.3	65	40.6	74.3	48.9	114.5	0
Nya	1	17.2	0	12.2	2.3	15	12	12.5	16.2	16.3	6.7	4.5	12.4	52.6	34.2	46	52.2	68	68.8	56.1	84.2	65.6	126.7	17.2
A1	2	1.9	12.1	0	14.4	5.6	5.5	12.2	7.6	2	16.5	11.1	6	57.6	37.2	41	35.2	51.3	65.9	35.7	73.5	48.9	114.2	1.9
Krv	3	19.4	2.3	14.4	0	12.5	12.3	13	13.1	16.4	4.1	6.8	14.7	51.6	39.2	51	57.2	66.2	73	53.8	86.5	63.9	127.4	19.4
Fsec	4	3.7	15	5.6	12.5	0	3.7	5.3	2.2	6.3	20.6	9.7	4.3	50.3	29.9	37	47.9	54.9	59.8	40.8	74.6	52.5	116.9	3.7
A11	5	3.9	12	5.5	12.3	3.7	0	6	4.8	7.5	11.8	7.7	7	53.7	33.3	37	51.3	57.3	59.7	43	77.2	55.3	117.5	3.9
Mam	6	8.5	12.5	12.2	13	5.3	6	0	4.6	12.9	17.1	7.7	9.5	49.2	28.8	33	46.8	61.4	55.2	45.7	82.9	59	122.6	8.5
Nas	7	5.9	16.2	7.6	13.1	2.2	4.8	4.6	0	9.3	20.3	8.1	6.1	52.2	30.9	34	49.9	57.1	56.4	42.7	76.8	54.7	118.4	5.9
Osec	8	2.6	16.3	2	16.4	6.3	7.5	12.9	9.3	0	18.5	11.8	6.6	53.4	32.9	36	50.9	68.6	58.8	38	72.3	66.2	127.7	2.6
Jik	9	22.7	6.4	16.5	4.1	20.6	11.8	17.1	20.3	18.5	0	15.2	18.5	61.3	40.9	44	58.9	72.7	67.3	62.3	90.6	70.3	137.5	22.7
Mog	10	9.1	4.5	11.1	6.8	9.7	7.7	7.7	8.1	11.8	15.2	0	11.1	51.7	31.3	35	49.3	61.5	57.7	47.8	123	59.1	122.7	9.1
Wz3	11	5	12.4	6	14.7	4.3	7	9.5	6.1	6.6	18.5	11.1	0	48.7	28.3	32	46.3	53.2	54.7	39.4	64.7	51.3	122.3	5
Bhr	12	50.2	52.6	57.6	51.6	50.3	53.7	49.2	52.3	53.4	61.3	51.7	48.7	0	20.9	43	2.4	64.8	67.5	74.2	70.6	65.1	183.7	50.2
Kub	13	29.9	34.2	37.2	39.2	29.9	33.3	28.8	30.9	32.4	40.9	31.3	28.3	20.9	0	14	22.4	44.4	47.1	53.8	53.9	44.7	163.3	29.9
Dei	14	42.4	46.2	41.3	50.7	37.2	37.1	32.6	33.8	36.2	44.7	35.1	32.1	42.8	14.1	0	26.2	48.2	50.9	57.6	57.7	48.5	167.1	42.4
Bls	15	47.9	53.2	55.2	56.2	47.9	51.3	46.8	49.9	50.9	58.9	49.3	46.3	2.4	32.1	26	0	61.6	65.1	71.8	71.9	62.7	181.3	47.9
Gwa	16	51.3	68	51.3	66.2	54.9	57.7	61.4	57.1	68.6	72.7	61.5	53.7	64.8	37.8	24	61.6	0	40.2	22.7	16.7	2.4	63.2	51.3
Sul	17	65	68.8	63.9	73.3	59.8	59.7	55.2	56.4	58.8	67.3	57.7	54.7	67.5	36.7	23	48.8	40.2	0	51.3	63	38.8	189.7	65
Kuj	18	40.6	56.1	35.2	52.8	40.8	43	45.7	42.7	38	62.3	47.8	39.4	74.2	56.7	43	54.3	23.7	51.3	0	27.9	22.5	158.7	40.6
Kwl	19	74.3	84.2	73.5	86.5	74.6	77.2	82.9	76.8	72.3	90.6	123	64.7	70.6	53.9	58	71.9	16.9	63	27.9	0	17.3	137.1	74.3
UniA	20	48.9	65.6	48.9	63.9	52.5	55.3	59	54.7	66.2	70.3	59.1	51.3	61.8	35.4	21	61.7	2.4	37.8	20.3	14.3	0	60.8	48.9
Abj	21	115	127	114	128	117	118	123	118	128	138	123	122	184	163	167	181	63.2	190	159	137	60.8	0	114.5
A10	0	0	17.2	1.9	19.4	3.7	3.9	8.51	5.9	2.6	22.7	9.1	5	50.2	29.9	42	47.8	51.3	65	40.6	74.3	48.9	114.5	0

Appendix C – Alphabet Matrix

		A10	Nya	A1	Krv	Fsec	A11	Mam	Nas	Osec	Jik	Mog	Wz3	Bhr	Kub	Dei	Bls	Gwa	Sul	Kuj	Kwl	UniA	Abj	A10
		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	0
A10	0	0	1.9	2.6	3.7	3.9	5	5.9	8.5	9.1	17.1	19.4	22.7	29.9	40.6	42.4	47.8	48.9	65	40.6	51.3	65	74.3	114.5
Nya	1	0	2.3	4.5	6.7	12	12.2	15	16.2	17.2	17.2	4.5	12.4	52.6	34.2	46.2	52.2	68	68.8	65.6	68	68.8	84.2	126.7
A1	2	0	1.9	2	5.5	5.6	6	7.6	11.1	12.1	12.2	14.4	16.5	35.2	35.7	37.2	41.3	48.9	51.3	57.6	65.9	73.5	48.9	114.2
Krv	3	0	2.3	4.1	6.8	12.2	12.5	13	13.1	14.4	14.7	16.4	17.4	19.4	39.2	50.1	51.6	57.2	63.8	63.9	66.2	73	86.5	127.4
Fsec	4	0	2.2	3.7	3.7	3.7	4.3	5.3	5.6	6.3	9.7	12.5	15	20.6	29.9	37.2	40.8	47.9	50.3	52.5	54.9	59.8	74.8	116.9
A11	5	0	3.7	3.9	3.9	4.8	5.5	6	7	7.5	7.7	11.8	12	12.3	33.3	37.1	43	51.3	53.7	55.3	57.3	59.7	77.2	117.5
Mam	6	0	4.6	5.3	6	7.7	8.5	8.5	9.5	12.2	12.5	12.9	13	17.1	28.8	32.6	45.7	46.5	49.2	55.2	59	61.4	82.9	122.6
Nas	7	0	2.2	4.6	4.8	5.9	5.9	6.1	6.6	7.6	8.1	9.3	13.1	16.2	20.3	30.9	33.8	42.7	49.9	62.2	64.7	67.1	76.8	118.4
Osec	8	0	2	2.6	2.6	6.3	6.6	7.5	9.3	11.8	12.9	16.3	16.4	18.5	32.9	36.2	38	50.9	53.4	58.8	66.2	68.6	72.3	127.7
Jik	9	0	4.1	6.4	11.8	15.2	16.5	17.1	18.5	18.5	20.3	20.6	22.7	22.7	40.9	44.3	58.9	61.3	62.3	67.3	70.3	72.7	90.6	137.5
Mog	10	0	4.5	6.8	7.7	7.7	8.1	9.1	9.1	9.7	11.1	11.1	11.8	15.2	31.3	35.1	47.8	49.3	51.7	57.7	59.1	59.8	122.7	122.9
Wz3	11	0	4.3	5	5	6	6.1	6.6	7	9.5	11.1	12.4	14.7	18.5	28.3	32.1	39.4	46.3	47.8	48.7	51.3	57.3	64.1	122.3
Bhr	12	0	2.4	20.9	42.8	48.3	48.7	49.2	50.2	50.2	50.3	51.6	51.7	52.6	53.4	53.7	57.6	61.3	64.8	65.1	67.5	70.6	74.2	183.7
Kub	13	0	14.1	20.9	22.4	28.3	28.8	28.8	29.9	29.9	29.9	30.3	31.3	32.4	33.3	34.2	39.2	40.9	44.4	44.7	47.1	53.8	53.9	163.3
Dei	14	0	14.1	26.2	32.6	33.8	35.1	36.2	37.1	37.2	41.3	42.4	42.4	42.8	45.2	48.2	48.5	50.7	50.9	57.6	57.7	58.2	60.1	167.1
Bls	15	0	42.8	44.7	46.2	46.8	47.9	47.9	47.9	48.2	48.5	48	48.2	48.5	53.2	55.2	56.2	58.9	61.6	62.7	65.1	71.8	71.9	181.3
Gwa	16	0	2.4	16.7	22.7	23.7	37.8	40.2	42.5	44.2	45.7	48	53.7	51.3	51.3	51.3	61.5	61.6	63.2	64.8	66.2	68	68.6	72.7
Sul	17	0	22.6	36.7	38.8	40.2	48.8	55.2	56.4	58.8	59	51.3	55.2	56.4	59.7	63	63.9	65	65	67.3	67.5	68.8	73.8	189.7
Kuj	18	0	22.5	23.7	28.5	37	38	45.7	42.7	38	62.3	47.8	40.6	40.6	40.8	42.6	50	52.8	51.3	54.3	56.7	62.3	74.2	158.7
Kwl	19	0	16.9	17.3	27.9	53.9	57.7	63	64.7	72.3	74.3	74.3	64.7	70.6	53.9	57.7	71.9	16.9	83	84.2	86.5	90.6	122.8	137.1
UniA	20	0	14.2	20.3	21.3	35.4	40.3	43	44.5	47.5	48.9	48.9	51.3	61.8	35.4	21.3	52.5	55.3	59	61.7	61.8	63.9	66.2	70.3
Abj	21	0	60.8	63.2	83.5	114.2	115	114.5	116.9	117.5	118.4	122.3	122.6	122.7	127.6	127.7	127.7	137.1	158.7	163.3	167.1	181.3	183.7	189.7
A10	0	0	1.9	2.6	3.7	3.9	5	5.9	8.5	9.1	17.1	19.4	22.7	29.9	40.6	42.4	47.8	48.9	65	40.6	51.3	65	74.3	114.5

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