Transactions of the Nigerian Association of Mathematical Physics
***Volume 15, (April –June, 2021 Issue*), pp91 – 96**
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DEVELOPING A NEW TRANSMUTED RAYLEIGH PARETO DISTRIBUTION

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Abstract

A new probability distribution called the Transmuted Rayleigh Pareto (TRP) distribution was introduced. Some of its statistical properties such as the r^{th} moments, moment generating function, survival function, hazard function, reversed hazard function, quantile function, and pdf of the r^{th}order statistics were derived. The maximum likelihood estimation method was used to estimate the parameters of the new distribution.

INTRODUCTION

The lifetime distributions of random variables are widely used in data modelling. These lifetime distributions are greatly employed in reliability analysis, survival analysis, social sciences, medicine and a lot of other applications. In reliability engineering for example, it is important to model the survival time for electrical components. In medicine, one of the major applications of these distributions is to calculate the survival time of patients after surgery has been done. An interesting application in social sciences is in modelling the life time of marriages.

Aim and objectives:

The aim of this study is to introduce a new continuous probability distribution called the Transmuted Rayleigh Pareto Distribution (TRP). The specific objectives are to:

i) Define the pdf and cdf of the new distribution.

ii) Derive some of the mathematical and statistical properties of the new distribution

Problem definition.

Using lifetime distributions to model the life expectancy of real world phenomenon is the greatest challenge faced by statisticians in Big data analysis, reliability testing, survival analysis in biomedical sciences. Considering this challenge, statisticians have developed large classes of this life time models when the classical models do not follow real world scenarios. Significant amount of studies have been carried out in order to develop additional classes of continuous probability distributions. Nevertheless, there are so many instances where the real data does not follow any standard or classical problem.

LITERATURE REVIEW

The Pareto distribution is a popular probability distribution used in modeling skewed data in so many fields. It was named after a Swedish Professor of economics Wilfredo Pareto [2]. The pareto distribution was first generalized by Pickands in 1975 when making inferences on the upper tail of a probability distribution function.

The kumaraswammy distribution is a double bounded continuous probability distribution defined on the interval [0, 1]. This distribution was proposed by Indian Hydrologist Poondi Kumaraswammi. The distribution is similar to Beta distribution but much simpler to use especially in simulation studies because of its simple closed form of the probability density and cumulative distribution functions. It has been identified as a viable alternative to Beta distribution. They both have the same basic shape properties (unimodal, monotone or constant).

The Rayleigh Pareto distribution was introduced by [2]. Several properties of the distribution were studied. Some of the properties studied wererth moments, moment generating function, hazard function reversed hazard function.

The Kumaraswammi Pareto distribution, a compound distribution was studied by [1]. The distribution was shown to have a decreasing and upside down bathhub shape depending on the values of the parameter. Statistical properties of the distribution such as the moment, moment generating function, order statistics and mean deviation were also derived. The maximum likelihood estimation method was used to estimate the model parameters and a simulation studies was conducted to ascertain the precision of the maximum likelihood estimators.

The basic concept of distributional alchemy was discussed by [3]. This was defined using the concept of transmutation maps which were the functional composition of the cumulative distribution function of one distribution to the quantile function (inverse cumulative distribution function), which will become easy to introduce skewness into a distribution. The maps were shown to have many applications in statistics, risk calculations and mathematical finance. One of the weakness of their work was that it did not provide simple formulas for some of the basic properties of the basic mathematical properties of Transmuted families of distribution.

The Logistic-X family of distributions was developed by [4],The Log-Gamma-G family of distribution was developed by [5], The Gamm-G (Type I) family of distributions was derived by [6] and The ExponentiatedGeneralised (EG) family of distributions was defined by [7].

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METHODOLOGY

Introduction:

In this section, we shall study some of the properties of the Transmuted Rayleigh Pareto distribution such as the rth moment, moment generating function,quantile function, order statistics. We will also discuss the reliability analysis such as the survival function, hazard function and the reversed hazard function. The maximum likelihood estimation method was used to derive the model parameters.

3.1 Baseline Distribution:

The baselinedistribution to be used in this research is the Rayleigh Pareto distribution. The cumulative distribution function and the probability density function as defined by [2] is given by equation (1) and (2) respectively.

$$G_{RP}(x;p,b,\alpha)=1-e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}. \quad (1)$$

$$g_{KP}(x;p,b,\alpha)=\frac{\alpha}{2b^2p}\left(\frac{x}{p}\right)^{\alpha-1}e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}}. \quad (2)$$

for $0\le x<\infty$

Transmuted Family of Distributions:

The Transmuted family of distributions was proposed by [3]. Suppose $G(x)$ *and* $g(x)$ are the cumulative distribution function (cdf) and probability density function (pdf) of any baseline distribution, then the cdf and the pdf of the Transmuted family of distributions are given by equation (3) and (4) respectively.

$$F(x)=(1+\theta)G(x)-\theta\left(G(x)\right)^2 \quad (3)$$

$$f(x)=g(x)[(1+\theta)-2\theta G(x)] \quad (4)$$

Where $|\theta|\le 1$

3.3 The Proposed Distribution:

Usingthe work of [3], we propose a new distribution called the Transmuted Rayleigh Pareto Distribution (TRP). To obtain the cdf and the pdf of the new distribution, we substitute our baseline distribution in equations and (3) and (4) respectively. Therefore, the cdf and pdf of Transmuted Rayleigh Pareto Distribution (TRP) are obtained by equations (5) and (6) respectively.

$$F_{TRP}(x;p,b,\alpha,\theta)=(1+\theta)\left(1-e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right)-\theta\left(1-e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right)^2 \quad (5)$$

$$f_{TRP}(x;p,b,\alpha,\theta)=\frac{\alpha}{2b^2p}\left(\frac{x}{p}\right)^{\alpha-1}e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}}\left[(1+\theta)-2\theta\left(1-e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right)\right] \quad (6)$$

The rthMoments:

therthmoments for the transmuted Rayleigh Pareto distribution is given by

$$E(x^r)=\int_{-\infty}^{\infty}x^r\times f_{TRP}(x;p,b,\alpha,\theta)dx$$

$$=\int_0^{\infty}x^r\left(\frac{\alpha}{2b^2p}\left(\frac{x}{p}\right)^{\alpha-1}e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}}\left[(1+\theta)-2\theta\left(1-e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right)\right]\right)$$

$$=\int_0^{\infty}x^r\left(\frac{\alpha}{2b^2p}\left(\frac{x}{p}\right)^{\alpha-1}e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}}\left[(1-\theta)+2\theta\left(e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right)\right]\right)$$

$$=\int_0^{\infty}x^r(1-\theta)\frac{\alpha}{2b^2p}\left(\frac{x}{p}\right)^{\alpha-1}e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}}+\int_0^{\infty}x^r2\theta\frac{\alpha}{2b^2p}\left(\frac{x}{p}\right)^{\alpha-1}e^{-\frac{1}{b^2}\left(\frac{x}{p}\right)^{\alpha}}$$

Let

$$u=\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}\Rightarrow 2b^2u=\left(\frac{x}{p}\right)^{\alpha}$$

$$\left(2b^2u\right)^{\frac{1}{\alpha}}=\frac{x}{p}$$

$$\Rightarrow x=p\left(2b^2\right)^{\frac{1}{\alpha}}u^{\frac{1}{\alpha}}$$

Also,

$$\Rightarrow du = \frac{\alpha}{2b^2 p}\left(\frac{x}{p}\right)^{\alpha-1} dx$$

Also let

$$v = \frac{1}{b^2}\left(\frac{x}{p}\right)^{\alpha} \Rightarrow b^2 v = \left(\frac{x}{p}\right)^{\alpha}$$

$$\therefore x = \left(b^2 v\right)^{\frac{1}{\alpha}} p$$

Substituting back, we have

$$\therefore E\left(x^r\right) = \int_0^\infty \frac{\alpha}{2b^2 p}\left(\frac{x}{p}\right)^{\alpha-1} p^r \left(2b^2\right)^{\frac{r}{\alpha}} (u)^{\frac{r}{\alpha}} (1-\theta) \frac{du}{\frac{\alpha}{2b^2 p}\left(\frac{x}{p}\right)^{\alpha-1}} + \int_0^\infty 2\theta \left(b^2\right)^{\frac{r}{\alpha}} \frac{\alpha}{b^2 p}\left(\frac{x}{p}\right)^{\alpha-1\frac{r}{\alpha}} p^r \int_0^\infty v^{\left(\frac{r}{\alpha}+1\right)-1}$$

$$\times \frac{dv}{\frac{\alpha}{b^2 p}\left(\frac{x}{p}\right)^{\alpha-1}}$$

$$= p^r \left(2b^2\right)^{\frac{r}{\alpha}} (1-\theta) \int_0^\infty (u)^{\left(\frac{r}{\alpha}+1\right)-1} du + 2\theta \left(b^2\right)^{\frac{r}{\alpha}} p^r \int_0^\infty v^{\left(\frac{r}{\alpha}+1\right)-1} dv$$

$$\therefore E\left(x^r\right) = p^r \left(2b^2\right)^{\frac{r}{\alpha}} (1-\theta) \Gamma\left(\frac{r}{\alpha}+1\right) + 2\theta \left(b^2\right)^{\frac{r}{\alpha}} p^r \Gamma\left(\frac{r}{\alpha}+1\right).$$

Moment Generating Function:

The moment generating function for the Transmuted Raylegh Pareto Distribution is given by:

$$E(e^{tx}) = \int_{-\infty}^{\infty} e^{tx} \times f_{TRP}(x; p, b, \alpha, \theta) dx$$

$$= \int_0^\infty e^{tx} \left(\frac{\alpha}{2b^2 p}\left(\frac{x}{p}\right)^{\alpha-1} e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}} \left[(1-\theta) + 2\theta \left(e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}} \right) \right] \right)$$

$$\int_0^\infty e^{tx} (1-\theta) \left(\frac{\alpha}{2b^2 p}\left(\frac{x}{p}\right)^{\alpha-1} e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}} \right) + 2\theta \frac{\alpha}{2b^2 p}\left(\frac{x}{p}\right)^{\alpha-1} \left(e^{-\left(\frac{1}{b^2}\right)\left(\frac{x}{p}\right)^{\alpha}} \right)$$

Using series expansion for $e^{tx} = \sum_{j=0}^{\infty} \frac{t^j x^j}{j!}$, we have;

$$E(e^{tx}) = \int_0^\infty \sum_{j=0}^{\infty} \frac{t^j x^j}{j!} (1-\theta) \left(\frac{\alpha}{2b^2 p}\left(\frac{x}{p}\right)^{\alpha-1} e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}} \right) + \int_0^\infty \sum_{j=0}^{\infty} \frac{t^j x^j}{j!} 2\theta \frac{\alpha}{2b^2 p}\left(\frac{x}{p}\right)^{\alpha-1} \left(e^{-\left(\frac{1}{b^2}\right)\left(\frac{x}{p}\right)^{\alpha}} \right)$$

$$= \sum_{j=0}^{\infty} \frac{t^j}{j!} (1-\theta) \int_0^\infty x^j \left(\frac{\alpha}{2b^2 p}\left(\frac{x}{p}\right)^{\alpha-1} e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}} \right) + \sum_{j=0}^{\infty} \frac{t^j}{j!} 2\theta \int_0^\infty \frac{\alpha}{2b^2 p}\left(\frac{x}{p}\right)^{\alpha-1} \left(e^{-\left(\frac{1}{b^2}\right)\left(\frac{x}{p}\right)^{\alpha}} \right)$$

Let

$$u = \frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha} \Rightarrow 2b^2 u = \left(\frac{x}{p}\right)^{\alpha}$$ Also, $z = \left(\frac{1}{b^2}\right)\left(\frac{x}{p}\right)^{\alpha} \Rightarrow x = p(b^2)^{\frac{1}{\alpha}} z^{\frac{1}{\alpha}}$

$$\left(2b^2 u\right)^{\frac{1}{\alpha}} = \frac{x}{p}$$

$$\Rightarrow x = p\left(2b^2\right)^{\frac{1}{\alpha}} u^{\frac{1}{\alpha}}$$

Also,

$$\Rightarrow du = \frac{\alpha}{2b^2 p}\left(\frac{x}{p}\right)^{\alpha-1} dx$$

$$\therefore E(e^{tx}) = (1-\theta)abk(kj+k-n)\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{n=0}^{\infty}\frac{t^n\beta^{n+km}}{n!}\binom{b-1}{i}\binom{a(i+1)-1}{j}(-1)^{i+j} + 2\theta abk\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}$$

$$\frac{t^n\beta^n}{n!}(kj+km+k-n)\binom{b-1}{i}\binom{a(i+1)-1}{j}\binom{b}{l}\binom{al}{m}(-1)^{i+j+m+l}.$$

3.4.3Survival Function:

The survival function for the Transmuted Kumaraswamy Pareto distribution is given by;

$$s(x) = 1 - F_{TRP}(x; p, b, \alpha, \theta)$$

$$\therefore S(x) = 1 - \left\{(1+\theta)\left(1-e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right) - \theta\left(1-e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right)^2\right\}.$$

3.4.4 Hazard Function:

The hazard function of the Transmuted Reyleigh Pareto distribution is:

$$h(x) = \frac{f_{TRP}(x; p, b, \alpha, \theta)}{S(x)}$$

$$\therefore h(x) = \frac{\frac{\alpha}{2b^2 p}\left(\frac{x}{p}\right)^{\alpha-1} e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}}\left[(1+\theta) - 2\theta\left(1-e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right)\right]}{1-\left\{(1+\theta)\left(1-e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right) - \theta\left(1-e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right)^2\right\}}$$

3.4.5 Reversed Hazard Function:

The reversed hazard function is defined by

$$\tau(x) = \frac{f_{TRP}(x; p, b, \alpha, \theta)}{F_{TRP}(x; p, b, \alpha, \theta)}.$$

$$\therefore \tau(x) = \frac{\frac{\alpha}{2b^2 p}\left(\frac{x}{p}\right)^{\alpha-1} e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}}\left[(1+\theta) - 2\theta\left(1-e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right)\right]}{\left\{(1+\theta)\left(1-e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right) - \theta\left(1-e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right)^2\right\}}$$

3.4.6 Quantile Function:

The Quantile function for Transmuted Reyleigh Pareto distribution can be obtained by solving the equation $F(x) = u$

$$\Rightarrow (1+\theta)\left(1-e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right) - \theta\left(1-e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right)^2 = u$$

$$\theta\left(1-e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right)^2 - (1+\theta)\left(1-e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right) + u = 0$$

Let $p = \left(1-e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right)$

Therefore, $p = \frac{-b-\sqrt{b^2-4ac}}{2a}$; $a = \theta, b = -(1+\theta), c = u$

$$\left(1-e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right) = \frac{(1+\theta) - \sqrt{(1+\theta)^2 - 4\theta u}}{2\theta}$$

$$\left(e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right) = 1 - \frac{(1+\theta) - \sqrt{(1+\theta)^2 - 4\theta u}}{2\theta}$$

$$-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha} = \log\left(1 - \frac{(1+\theta) - \sqrt{(1+\theta)^2 - 4\theta u}}{2\theta}\right)$$

$$\left(\frac{x}{p}\right)^{\alpha} = 2b^2\log\left(\frac{(1+\theta) - \sqrt{(1+\theta)^2 - 4\theta u}}{2\theta}\right)$$

$$\left(\frac{x}{p}\right)=\left(2b^2\log\left(\frac{(1+\theta)-\sqrt{(1+\theta)^2-4\theta u}}{2\theta}\right)\right)^{\frac{1}{\alpha}}$$

Therefore the quantile function is given by

$$X_u=p\left[\left(2b^2\log\left(\frac{(1+\theta)-\sqrt{(1+\theta)^2-4\theta u}}{2\theta}\right)\right)^{\frac{1}{\alpha}}\right]$$

And $X_i=Q(u_i); i=1,2,...,n$ is a random sample drawn from the Transmuted Rayleigh Pareto distribution with parameters $x;p,b,\alpha,\theta$ and it is uniformly distributed on [0,1].

Also, if $u=\frac{1}{2}$, it gives the median.

3.4.7 Order Statistics:

Suppose $X_1,X_2,X_3,\dots X_n$ is a random sample of size n from TRP distribution, and $X_{(1)},X_{(2)},X_{(3)},\dots,$ are the corresponding order statistics, then the pdf of the rth order statistic is given as:

$$f_{r,n}=\frac{n!}{(r-1)!(n-r)!}f(x)F(x)^{r-1}(1-F(x))^{n-r}$$

Using binomial series expansion,

$$(1-F(x))^{n-r}=\sum_{i=0}^{n-r}\binom{n-r}{i}(-1)^i F(x)^i$$

Substituting back, we have;

$$f_{r,n}=\frac{n!}{(r-1)!(n-r)!}f(x)\sum_{i=0}^{n-r}\binom{n-r}{i}(-1)^i F(x)^{i+r-1}$$

But

$$\binom{n-r}{i}=\frac{(n-r)!}{(n-r-i)!i!}\quad f_{r,n}=\sum_{i=0}^{n-r}(-1)^i\frac{n!}{(r-1)!(n-r-i)!i!}\frac{\alpha}{2b^2p}\left(\frac{x}{p}\right)^{\alpha-1}e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}}\left[(1+\theta)-2\theta\left(1-e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right)\right]\times$$

$$\left[(1+\theta)\left(1-e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right)-\theta\left(1-e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right)^2\right]^{i+r-1}$$

The order Statistics can be used to find the minimum and maximum order statistic. For example, in reliability analysis, the minimum and maximum time to failure of light hub bulbs can be tested. This can be achieved by testing n samples of the bulb simultaneously and recording the time to failure as they occur. These observations can be ordered as$X_1,X_2,...X_n$.; X_1 denotes the minimum time to failure and X_n denotes the maximum time to failure.

To obtain the minimum and maximum order statistic, we substitute r = 1 and r = n respectively in the pdf of the rthorder Statistics.

$$f_{1,n}=\sum_{i=0}^{n-1}(-1)^i\frac{n!}{(n-1-i)!i!}\frac{\alpha}{2b^2p}\left(\frac{x}{p}\right)^{\alpha-1}e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}}\left[(1+\theta)-2\theta\left(1-e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right)\right]\times$$

$$\left[(1+\theta)\left(1-e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right)-\theta\left(1-e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right)^2\right]^{i}$$

And

$$f_{n,n}=nf(x)\left[F(x)\right]^{n-1}$$

$$f_{n,n}=n\frac{\alpha}{2b^2p}\left(\frac{x}{p}\right)^{\alpha-1}e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}}\left[(1+\theta)-2\theta\left(1-e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right)\right]\times$$

3.4.9 Maximum Likelihood Estimators:

$$\left[(1+\theta)\left(1-e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right)-\theta\left(1-e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right)^2\right]^{n-1}$$

The maximum likelihood estimation method is used in estimating the parameters of the model. Suppose $x_1,x_2,x_3,...,x_n$is a random sample from TRP distribution. The likelihood function is given by:

$$L(x_1,x_2,...x_n;x;a,b,k,\beta,\theta)=\prod_{i=1}^{n}f_{TRP}(x;a,b,k,\beta,\theta)$$

The likelihood function is given by;

$$L=\prod_{i=1}^{n}\left(\frac{\alpha}{2b^2 p}\left(\frac{x}{p}\right)^{\alpha-1} e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}}\left[(1+\theta)-2\theta\left(1-e^{-\left(\frac{1}{2b^2}\right)\left(\frac{x}{p}\right)^{\alpha}}\right)\right]\right)$$

The log likelihood function is also given by;

$$ll=n\log\alpha-n\log 2b^2-n\log p+(\alpha-1)\sum_{i=1}^{n}\log x_i-n(\alpha-1)\log p-\frac{1}{2b^2}\sum_{i=1}^{n}\frac{x^{\alpha}}{p^{\alpha}}+\sum_{i=0}^{n}\log\left[(1-\theta)+2\theta e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}}\right].$$

To get the maximum likelihood estimates of the parameters, we differentiate each of the parameters partially and set the result to zero. Therefore,

$$\frac{\partial ll}{\partial p}=-\frac{n}{p}-\frac{n(\alpha-1)}{p}+\frac{\alpha}{2b^2p^{\alpha+1}}-\sum_{i=0}^{n}\frac{2\theta x_i^{\alpha}\dfrac{\alpha}{2b^2p^{\alpha+1}}e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}}}{(1-\theta)+2\theta e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}}}$$

$$\therefore\frac{n}{p}+\frac{n(\alpha-1)}{p}-\frac{\alpha}{2b^2p^{\alpha+1}}+\sum_{i=0}^{n}\frac{2\theta x_i^{\alpha}\dfrac{\alpha}{2b^2p^{\alpha+1}}e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}}}{(1-\theta)+2\theta e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}}}=0$$

$$\frac{\partial ll}{\partial b}=-\frac{2n}{b}+\frac{1}{b^3}\sum_{i=1}^{n}\left(\frac{x^{\alpha}}{p^{\alpha}}\right)-\sum_{i=0}^{n}\frac{2\theta\dfrac{1}{b^3}\left(\dfrac{x}{p}\right)^{\alpha}e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}}}{(1-\theta)+2\theta e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}}}$$

$$\Rightarrow\frac{2n}{b}-\frac{1}{b^3}\sum_{i=1}^{n}\left(\frac{x^{\alpha}}{p^{\alpha}}\right)+\sum_{i=0}^{n}\frac{2\theta\dfrac{1}{b^3}\left(\dfrac{x}{p}\right)^{\alpha}e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}}}{(1-\theta)+2\theta e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}}}=0$$

$$\frac{\partial ll}{\partial\alpha}=\frac{n}{\alpha}+\sum_{i=0}^{n}\log x_i-n\log p-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}\sum_{i=0}^{n}\left(\frac{x}{p}\right)-\sum_{i=0}^{n}\frac{2\theta\dfrac{1}{2b^2}\left(\dfrac{x}{p}\right)^{\alpha}\log\left(\dfrac{x}{p}\right)e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}}}{(1-\theta)+2\theta e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}}}$$

$$\Rightarrow\frac{n}{\alpha}+\sum_{i=0}^{n}\log x_i-n\log p-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}\sum_{i=0}^{n}\left(\frac{x}{p}\right)-\sum_{i=0}^{n}\frac{2\theta\dfrac{1}{2b^2}\left(\dfrac{x}{p}\right)^{\alpha}\log\left(\dfrac{x}{p}\right)e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}}}{(1-\theta)+2\theta e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}}}=0$$

$$\frac{\partial ll}{\partial\theta}=\sum_{i=0}^{n}\frac{2e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}}}{(1-\theta)+2\theta e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}}}.$$

$$\Rightarrow\sum_{i=0}^{n}\frac{2e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}}}{(1-\theta)+2\theta e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\alpha}}}=0$$

These equations are intractable and can only be solved iteratively.

Conclusion:

In this study, we proposed a new class of distribution called the Transmuted Rayleigh Pareto Distribution (TRP).The properties of the proposed distribution such as the r^{th}Moments, moment generating function, quantile function, survival function, hazard function, reversed function, Order statistics and Maximum likelihood estimates were presented.

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