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MODIFIED ORDER-4 EXPLICIT LINEAR MULTISTEP TECHNIQUE FOR TIME SERIES FORECASTING

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Abstract

The purpose of this paper is to introduce a new time series smoothing technique derived from the modification of order-4 explicit linear multistep numerical technique. The proposed smoothing technique in this paper called: Modified Adams-Bashforth Order-4 (mABT Order-4) smoothing technique was tested on a dataset with high fluctuation (Nigeria external reserve, from 1981 to 2015) and compared with simple moving average and simple exponential smoothing ($\alpha = 0.8$) techniques. When the dataset was tested on the proposed mABT Order-4, it produced an R^2 of 0.9938, and RMSE of 0.1982. A forecasting model was produced using ARIMA (2, 1, 1) for the series produced by mABT Order-4. Model performance indicators such as: MAE, MSE, RMSE, and MAPE was determined to verify the suitability of the proposed technique and the results were displayed in table 3. These results indicates that the mABT Order-4 smoothing technique is adequate for the data and can perform better when applied on other data sets. It also suggests that the mABT Order-4 smoothing technique is a good time series smoothing technique that can compare favorably with other existing techniques.

Keywords: *Linear Multistep, Adams-Bashforth, Explicit, Order-4, Smoothing technique, ARIMA, Forecasting***.**

1. Introduction

Time series can be said to be a sequence of consistent observations, ordered in time (or space). In the field of engineering, especially, electrical engineering, signals are collected over time and classified as time series. A set of numerical data collected over a regular intervals can be described as time series. A time series is a set of observation over time [1].Time series data are mostly natural occurrences obtained in various scientific areas such as technology, medicine, weather and climate, economics, finance, etc. There are two types of time series data [2] namely; discrete and continuous time series data. The data used in this study is a discrete data obtained from external reserve for the period of 1981 to 2015 exhibiting high fluctuation.

A well-ordered collection of measurements taken at regular time intervals is called time series data. The objective of time series modelling is to forecast a number of future measures based on the study and analysis of past and present data [3].Time series data is essential when forecasting something that is in a constant change over time (e.g., sales figures, profit, stock prices, etc.). Anything that is observed sequentially over time is time series [4].

There are various ways to define forecasting but in a simple form, we can say it's the process of making future predictions of events using past and present information by way of analysis. In simpler terms we could say that forecasting is a way of prediction but requires some set of techniques. The word "information" as mentioned above can be referred to as time series data.

Over the time, two common time series filters or smoothing methods, namely; the simple moving average and the

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exponential moving average have been steadily deployed to filter data [5]. Forecasting methods are very complex [6]. Using adequate method for financial time series forecasting is important to detect any signs of trends of seasonality so as to increase proficiency and accuracy of the forecast. There are lots of forecasting techniques for various types of data as not all kinds of models can be used for all kinds and produce a good fit within the given data [7].

A major objective in time series technical analysis is the separation of the trend from stochastic fluctuations (white noise) [5]. Smoothing the time series data allows removing white noise and other form of variations [8]. Just like we intend in this study, the model developed is designed to smoothen discrete time series data just in the manner of the moving average techniques and the exponential smoothing techniques.

A discrete time series looks at a scenario where observations (entries) are made at (mostly) equispaced intervals. This can be represented as $[x_t : t = 1, 2, \dots, N]$ in where t indicates the time at which the datum x_t was observed.

1.1 Brief Background of the Order-4 Explicit Numerical Method

The Adams-Bashforth Order-4 method is an explicit method for finding numerical solutions for ordinary differential equations [9]. The method is appropriate for obtaining numerical solutions of time dependent ordinary and partial differential equations. This beautiful numerical technique is also a multistep method because of the efficiency it acquires as a result of not deleting but keeping (saving) and using information from previous steps, a characteristic non-multistep methods lack. The method was developed by John Couch Adams to provide numerical solutions to differential equation modelling capillary action due to Francis Bashforth [10].

$$y_{n+1} = y_n + \frac{h}{24}(55f_n - 59f_{n-1} + 37f_{n-2} - 9f_{n-3}) \qquad (1)$$

1.2 Time Series Smoothing Techniques

The concept of time series smoothing is also applicable to technology, tourism, sensors, weather forecasting, and traffic estimation where both the smoothness and accuracy of the filtered signal are important [11]. There are diverse time series smoothing techniques such as the moving average technique and exponential smoothing average technique.

The exponential moving average which is one of the moving average techniques for smoothing time series. The exponential moving average is applied in the technical analysis as both an independent method and a component of other indicators [12]. Largely because of its ease to implement, the simple moving average smoothing is can be seen as the most common time series smoothing method [13]. In finance, moving average methods are utilized to smooth stock price data and forecast the direction of trend [8].

One of the major problems of the simple moving average is the fact that always lags the last observation and making old observations drop out of the average. The simple moving average method is most useful when observation has no recognized trend nor seasonality [13]. Moving average methods are predominantly used in two main ways: Two-sided (weighted) moving average methods which are used to "smooth" a time series data in order to predict or highlight underlying trend, while the one-sided (weighted) moving average methods are deployed as simple forecasting methods for time series [14]. In the case of a weighted moving average, the weight imposed on the observational values is a bye product of the system taken [15]. The weighted moving average results are strongly dependent on the weighting (smoothing) factors used. Forecasting in exponential smoothing method is carried out by a repeated continuous calculations using the latest data, where recent values are bestowed with a relatively greater weight than older values.

Simple exponential smoothing is best applied to time series that do not exhibit trend and do not exhibit seasonality [8]. The weighted moving average results are strongly dependent on the weighting (smoothing) factors used. The suitability criteria of a smoothing technique are defined by smoothness and accuracy of the technique on a particular time series data.

2. Review of Literature

In determining the optimal value of exponential constant, a technique described as Trial and Error method was used in [16] to select the optimal smoothing constant and thus minimize the mean square error (MSE) and mean absolute deviation (MAD). The single and double exponential smoothing was applied in [17] to estimate the number of fatalities and injuries caused by road traffic accidents in Jordan within the year: 1981-2016. Exponential smoothing constants that minimize error measures associated with a large number of forecasts where considered in [18]. Techniques helpful in forecasting univariate time series data were developed in [19], the study used single exponential smoothing (SES), double exponential smoothing (DES), Holt's (Brown) and adaptive response rate exponential smoothing (ARRES) techniques. The study in [20] combined local trends and local seasonal effects to generate a parsimonious method of exponential smoothing for time series. An underlying system for obtaining predictions where they also compared simple moving average against aggregation algorithm (AA) was developed in [21]. The study produced predictions from usual statistical learning approach which was outlined along on-line protocol, then a competitive on-line learning algorithm on those predictions was used. The study in [5] introduced a new adaptive moving average technical indicator (VAMA) aimed at providing smooth results as well as providing fast decision timing. The study compared results from the technique to results of other adaptive moving averages to verify the profitability of the new indicator. The problem of forecasting prices of cryptocurrencies was analyzed and described in [12]. The study applied the simple moving average, exponential moving average and weighted moving average techniques for forecasting prices and analyzing cryptocurrency financial market. The study revealed that simple moving average technique had the least value of mean-square deviation, but reported that the exponential moving average was better more sensitive amongst all, although with greater error. The ARIMA intervention approach to investigate gross domestic product in Nigeria was done in [22]. The study applied ARIMA to make forecast of local productions and the economic, and financial expectations. An optimized custom moving average was proposed in [8] for a suitable stock time series data smoothing. Five common moving average methods were compared to the proposed method using synthetic and real stock data. The study reported that the proposed method performed better in 99.5% of the cases on the synthetic data and 91% on real data.

3. Research Methodology

The technique is best utilized for smoothing quantitative time series data in the same manner moving averages and exponential smoothing techniques are used by forecast enthusiast. It is designed to produce comparable smoothing results. The new time series smoothing technique proposed in this study was derived by modifying order-4 explicit linear multistep numerical technique also known as the Adams-Bashforth Order-4 numerical technique.

3.1 Technique Derivation

The derivation of the proposed smoothing technique can be seen below starting with the stating of the initial value problem;

$$y' = f(x,y), \qquad y(x_0) = y_0 \tag{2}$$

With f the problem has a unique solution on some open interval containing x_0.

If we integrate $y' = f(x,y)$ from the interval $x_n \; to \; x_{n+1} = x_n + h$, we have:

$$\int_{x_n}^{x_{n+1}} y'(x)dx = y(x_{n+1}) - y(x_n) = \int_{x_n}^{x_{n+1}} f\big(x, y(x)\big)\, dx \tag{3}$$

Replace $f\big(x, y(x)\big)$ with an interpolating polynomial $p(x)$ such that:

$$y_{n+1} = y_n + \int_{x_n}^{x_{n+1}} p(x)\, dx \tag{4}$$

Using cubic polynomial $p_3(x)$ such that by Newton-Gregory backward scheme, we have

$$p_3(x) = f_n + s\nabla f_n + \frac{s(s+1)}{2}\nabla^2 f_n + \frac{s(s+1)(s+2)}{6}\nabla^3 f_n \tag{5}$$

$$y_{n+1} - y_n = \int_0^1 \left(f_n + s\nabla f_n + \frac{s(s+1)}{2}\nabla^2 f_n + \frac{s(s+1)(s+2)}{6}\nabla^3 f_n\right) h\, ds \tag{6}$$

$$y_{n+1} - y_n = \left[\left(sf_n + \frac{s^2}{2}\nabla f_n + \frac{2s^3+3s^2}{12}\nabla^2 f_n + \frac{s^4+4s^3+4s^2}{24}\nabla^3 f_n\right) h\right]_{\mathbf{0}}^{\mathbf{1}} \tag{7}$$

$$y_{n+1} - y_n = h\left(f_n + \frac{1}{2}\nabla f_n + \frac{5}{12}\nabla^2 f_n + \frac{3}{8}\nabla^3 f_n\right) \tag{8}$$

By Newton-Gregory backward difference, we have:

$$\frac{1}{2}\nabla f_n = \frac{1}{2}f_n - \frac{1}{2}f_{n-1} \qquad (i)$$

$$\frac{5}{12}\nabla^2 f_n = \frac{5}{12}f_n - \frac{5}{6}f_{n-1} + \frac{5}{12}f_{n-2} \qquad (ii)$$

$$\frac{3}{8}\nabla^3 f_n = \frac{3}{8}f_n - \frac{9}{8}f_{n-1} + \frac{9}{8}f_{n-2} - \frac{3}{8}f_{n-3} \qquad (iii)$$

Substituting for (i), (ii) and (iii) in (8), gives:

$$y_{n+1} - y_n = h\left(\frac{1}{2}f_n - \frac{1}{2}f_{n-1} + \frac{5}{12}f_n - \frac{5}{6}f_{n-1} + \frac{5}{12}f_{n-2} + \frac{3}{8}f_n - \frac{9}{8}f_{n-1} + \frac{9}{8}f_{n-2} - \frac{3}{8}f_{n-3}\right) \tag{9}$$

$$y_{n+1} - y_n = \frac{h}{24}(55f_n - 59f_{n-1} + 37f_{n-2} - 9f_{n-3}) \tag{10}$$

This linear multistep techniques is called explicit because the right-hand-side does not have: $f_{n+1} = f(x_{n+1},\ y_{n+1})$, [23].

Since we are considering a time series data, we say let $n = t$.

$$y_{t+1} - y_t = \frac{h}{24}(55f_t - 59f_{t-1} + 37f_{t-2} - 9f_{t-3}) \tag{11}$$

This study is poised to deploy this technique for time series data smoothing, thus, we make $n = t$ and also derive the value of the step size h by defining it as the number of trials between successive observations.
Time intervals between successive observations be denoted as $t, t+1, t+2, t+3, .., t-N$. Such that the time between two observations is given as $h = (t+1) - (t) = 1$. Where t is weekly, monthly or yearly data.
Let y_t which we described as the initial forecast be zero ($y_t = 0$) for all forecast. Let also $y_{t+1} = Y_t$. Therefore, we have the modified smoothing technique as:

$$Y_t = \frac{1}{24}(55f_t - 59f_{t-1} + 37f_{t-2} - 9f_{t-3}) \quad (12)$$

Where;
Y_t is the forecast at time t.
$f_t \;\forall\; t = t,\; t-1,\; t-2,\; t-3$ is the time series observations.
The constants are the coefficient of data adjustment and 24 is the averaging constant.

3.2 Updating the Parameters of the Model

Let $f_t \;\forall\; t = t,\; t+1,\; t+2,\; t+3, \ldots . t-N$ be time series data and Y_{t+3} be forecasts such that $t = 1, 2, 3, 4, 5, \ldots, N$
The first forward forecast, Y_{t+3} is estimated at $t = 4$, such that (12) becomes:

$$Y_4 = \frac{1}{24}(55f_4 - 59f_3 + 37f_2 - 9f_1) \quad (13)$$

The second forward forecast is estimated at $t = 5$. Thus, we have:

$$Y_5 = \frac{1}{24}(55f_5 - 59f_4 + 37f_3 - 9f_2) \quad (14)$$

The procedure is continued until the last forecast is achieved.
For the backward (down to top) smoothing procedure, we change the t-subscript to suit the action to be implemented (see Eq. (17) and (18) below).

3.3 Procedure for Obtaining the Forecast

There are three algorithmic steps to obtaining the forecast.

1. For the forward smoothing method (from top to down), we apply (12), placing the first smoothened value at the 4th point $(n+3), n = 1, 2, 3, .. N$. Continue same for 5th, 6th and so on.

2. For the backward smoothing method (from down to top), we apply (12), placing the smoothened value at the fourth before last point $(N-3)$. Continue the upward smoothing procedure until last smoothened value is achieved.

3. Take the average of the forward and backward smoothened values to achieve the final forecast.

Example: let $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}$ be time series observations.
The algorithm for the propose method is demonstrated in the table below.
We denote the forward smoothing with Y_t^f and the backward smoothing with Y_t^b

$$forward\text{: } at\ t = 4, \quad Y_4^f = \frac{1}{24}(55x_4 - 59x_3 + 37x_2 - 9x_1) \quad (15)$$

$$forward\text{: } at\ t = 5, \quad Y_5^f = \frac{1}{24}(55x_5 - 59x_4 + 37x_3 - 9x_2) \quad (16)$$

$$backward\text{: } at\ t = 4, \quad Y_4^b = \frac{1}{24}(55x_9 - 59x_{10} + 37x_{11} - 9x_{12}) \quad (17)$$

$$backward\text{: } at\ t = 5, \quad Y_5^b = \frac{1}{24}(55x_8 - 59x_9 + 37x_{10} - 9x_{11}) \quad (18)$$

Table 1
Method Demonstration.

Serial Number	*Observation*	*Forward mABT Order-4*	*Backward mABT Order-4*	*Average = Smoothened Value*
1.	x_1		Y_{12}^b	Y_{12}^b
2.	x_2		Y_{11}^b	Y_{11}^b
3.	x_3		Y_{10}^b	Y_{10}^b
4	x_4	Y_4^f	Y_9^b	$(Y_4^f + Y_9^b)/2$
5.	x_5	Y_5^f	Y_8^b	$(Y_5^f + Y_8^b)/2$
6.	x_6	Y_6^f	Y_7^b	$(Y_6^f + Y_7^b)/2$
7.	x_7	Y_7^f	Y_6^b	$(Y_7^f + Y_6^b)/2$
8.	x_8	Y_8^f	Y_5^b	$(Y_8^f + Y_5^b)/2$
9.	x_9	Y_9^f	Y_4^b	$(Y_9^f + Y_4^b)/2$
10.	x_{10}	Y_{10}^f		Y_{10}^f
11.	x_{11}	Y_{11}^f		Y_{11}^f
12.	x_{12}	Y_{12}^f		Y_{12}^f

4. Testing the mABT Order-4 Model Efficiency

4.1 Comparison of mABT Order-4 to SMA and SES

To test the workability, efficiency and compatibility of this new technique to time series dataset, we have fitted it to a high fluctuation dataset of the Nigeria external reserve (1981 to 2015), and also compared it with the 4-step (ordere-4) simple moving average (SMA) and simple exponential smoothing techniques (SES), with a smoothing constant of $\alpha = 0.8$.

4.2 Model Performance Measurement

It is important to test the performance of time series smoothing techniques with real dataset and also recommended to compare results to that of other techniques. This helps to evaluate the overall performance of the various techniques using various common indicators such as the Root mean squared error (RMSE), Mean absolute error (MAE), Mean squared error (MSE), Mean absolute percentage error (MAPE), [24].

$$MAE = \frac{1}{n}\sum_{t=1}^{n}|e_t| \quad (19)$$

$$MSE = \frac{1}{n}\sum_{t=1}^{n} e_t^2 \quad (20)$$

$$RMSE = \sqrt{MSE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n} e_t^2} \quad (21)$$

$$MAPE = \frac{1}{n}\sum \frac{|e_t|}{f_t}.100\% \quad (22)$$

4.3 Practical Comparison

Table 2
The mABT Order-4, SMA, and SES smoothened values.

Serial	*Actual*	*Forward mABT Order-4*	*Backward mABT Order-4*	*Average = Smoothened Value*	*SMA Order-4*	*SES* $\alpha = 0.8$
1.	9.67		9.596	**9.596**	***9.67**	***9.67**
2.	9.74		9.598	**9.598**	***9.74**	**9.726**
3.	9.59		9.673	**9.673**	***9.59**	**9.617**
4.	9.08	8.623	9.237	**8.93**	**9.52**	**9.187**
5.	8.39	8.038	9.184	**8.611**	**9.2**	**8.549**
6.	7.78	7.606	7.987	**7.796**	**8.71**	**7.934**
7.	7.84	8.37	7.457	**7.914**	**8.273**	**7.859**
8.	8.01	7.931	7.973	**7.952**	**8.005**	**7.98**
9.	7.78	7.307	8.356	**7.832**	**7.853**	**7.82**
10.	7.5	7.47	7.018	**7.244**	**7.783**	**7.564**
⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮
131.	11.82	11.83	11.74	**11.78**	**11.82**	**11.83**
132.	11.78	11.76	11.88	**11.82**	**11.83**	**11.79**
133.	11.65	11.52	11.79	**11.66**	**11.78**	**11.68**
134.	11.61	11.7	11.5	**11.6**	**11.72**	**11.62**
135.	11.66	11.72	11.62	**11.67**	**11.68**	**11.65**
136.	11.57	11.38	11.68	**11.53**	**11.62**	**11.59**
137.	11.42	11.35	11.57	**11.46**	**11.57**	**11.45**
138.	11.37	11.45		**11.45**	**11.51**	**11.39**
139.	11.43	11.51		**11.51**	**11.45**	**11.42**
140.	11.38	11.23		**11.23**	**11.4**	**11.39**

4.4 Application of the Method

The proposed mABT Order-4 time series smoothing technique is suitable for quantitative time series datasets with high white noise. It can be used to provide smoothing and even forecasting if adequately utilized. The backward and forward smoothing procedure which we have applied to the mABT Order-4 makes it even better, as can be seen in table 1 and 2 (column 3 and 4). The mABT Order-4 can be used without the application of the backward procedure and the results are as good and comparable to other time series smoothing techniques, as can be seen in column 3 of table 2. Although, this can only result to the missing of the first 3 smoothing values which conventionally is allowed be filled in with the actual values, just as is in the case of simple moving average and simple exponential smoothing in column 6 and 7. The "*" in the first few entries of column 6 and 7 of table 2 shows that the values were lifted from the actual series and not produced by the technique.

Table 3
Performance measurement of 3-step mABT, SMA, and SES smoothing techniques.

Serial	*mABT Order-3*	*SMA Order-3*	*SES* ($\alpha = 0.8$)
R^2	0.9938	0.9986	0.9999
MAE	0.3110	0.2055	0.0429
MSE	0.3963	0.0872	0.0047
RMSE	0.1982	0.0436	0.0023
MAPE	3.6344	2.2191	0.4696

Table 4
ARIMA Forecast Using mABT Order-4 Smoothened Data

Time	Forecast	UCL	LCL
Q1 2016	11.57	12.67	10.46
Q2 2016	11.26	12.38	10.15
Q3 2016	11.56	13.13	9.98
Q4 2016	11.35	12.92	9.77
Q1 2017	11.51	13.32	9.70
Q2 2017	11.45	13.27	9.62
Q3 2017	11.46	13.38	9.55
Q4 2017	11.53	13.52	9.55
Q1 2018	11.45	13.46	9.43
Q2 2018	11.58	13.72	9.45
Q3 2018	11.46	13.61	9.31
Q4 2018	11.60	13.88	9.32
Q1 2019	11.51	13.80	9.21
Q2 2019	11.60	14.00	9.21
Q3 2019	11.56	13.98	9.14
Q4 2019	11.60	14.08	9.11
Q1 2020	11.62	14.15	9.09
Q2 2020	11.60	14.17	9.03
Q3 2020	11.66	14.29	9.02
Q4 2020	11.62	14.28	8.95
Q1 2021	11.68	14.41	8.96
Q2 2021	11.65	14.40	8.89
Q3 2021	11.70	14.52	8.88
Q4 2021	11.68	14.53	8.84

1. Results

The proposed modified Adams-Bashforth Order-4 (mABT Order-4) smoothing technique was used to produce the values as can be seen in column 3, 4, and 5 of table 2. The smoothened series generated by the technique can also be seen as forecast. ARIMA (2, 1, 1) was used to generate a forecasting model for the series generated by the mABT Order-4 (see (23)). Forecast were generated using ARIMA model for the mABT Order-4 smoothened series for the period of 6 years, on quarterly basis, with confidence limits. The ARIMA forecast values generated from model values produced by the new technique can be seen in table 4.
ARIMA (2, 1, 1) estimated equation:

$$\text{mABT Order} - 4 = y_t = 0.013 - 1.729x_{t-1} - 0.832x_{t-2} - 0.581\varepsilon_{t-1} \qquad (23)$$

2. Conclusion

The beauty of this research lies in the very great forecast results the proposed mABT Order-4 produced using data generated for the purpose of this study. The mABT Order-4 offers researchers and data analysts a wonderful competitive options to other smoothing techniques. The proposed technique did great with an R^2 of 0.9938 and RMSE of 0.0.1982 when applied on the data used in this study and can even do better when other data sets are considered. More performance and adequacy results from the comparison with SMA and SES can be seen in table 3. This method does not give room for missing forecasts (based on the smoothened values). The proposed mABT Order-4 is highly suitable for various quantitative time series. The application of ARIMA (2, 1, 1) on the smoothened proposed mABT Order-4 values, shows the applicability and adequacy of the new technique. It is also considered an easy smoothing and forecasting approach we believe can perform competitively against other smoothing techniques, especially on data with high fluctuation. The proposed technique is a direct method and updating parameters are easy to perform. The technique is adequate for diverse quantitative time series data and provides a practical time series smoothing option. It is also self-explanatory and less cumbersome to apply.

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