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ON THE ANALYSIS OF A TWO PATCH SCHISTOSOMIASIS MODEL.

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Abstract

In this paper, we carried out an elaborate quantitative analysis of the Reproduction Number of a two patch metapopulation model by varying key sensitive parameters. Numerical simulation was also carried using parameter values from literature.

***Keywords*:** Schistosomiasis, Metapopulation, Reproduction Number, Numerical Simulation, parameters.

1.0. INTRODUCTION

Schistosomiasis remains one of the most challenging public health issues in the tropics and subtropics [1]. Mathematical Models for Schistosomiasis transmission dynamics have been formulated since 1973 as seen in [1]-[20]. These formulated models have indeed brought great insight into dynamics of schistosomiasis but none of these models have quantitatively investigated the influence of the metapopulation paradigm on Schistosomiasis dynamics to the best of our knowledge.

In this paper, we analyzed the Reproduction number and also numerically simulated a non-linear coupled deterministic mathematical model which described the disease dynamics for Schistosomiasis in a 2- patch population setting. The cumulative incidence over a period of time is obtained by simulating the forces of infections of the model (for both human and snail populations in the patches) using parameter values from literature. Control parameters built to the force of infections (which seeks to prevent the interactions between humans and Cercariae; between Snail and Miracidia on the other hand) are varied within a defined range and its impact on the metapopulation schistosomiasis model is reported in both the Analysis of the reproduction number and Numerical simulation.

The metapopulation model we seek to analyze is given by:

$$
\begin{aligned}
\frac{dS_{h_1}}{dt} &= \Lambda_{h_1} - \lambda_{h_1} S_{h_1} + \gamma_{h_1} I_{h_1} - \mu_h S_{h_1} - \eta_{12} S_{h_1} + \eta_{21} S_{h_2},\\
\frac{dE_{h_1}}{dt} &= \lambda_{h_1} S_{h_1} - (\kappa_{h_1} + \mu_h + a_{12}) E_{h_1} + a_{21} E_{h_2},\\
\frac{dI_{h_1}}{dt} &= \kappa_{h_1} E_{h_1} - (\gamma_{h_1} + \delta_{h_1} + \mu_h + b_{12}) I_{h_1} + b_{21} I_{h_2},\\
\frac{dM_1}{dt} &= \theta_{M_1} I_{h_1} - \mu_M M_1,\\
\frac{dU_1}{dt} &= \Lambda_{s_1} - \lambda_{s_1} U_1 - \mu_s U_1,\\
\frac{dL_1}{dt} &= \lambda_{s_1} U_1 - (\kappa_{s_1} + \mu_s) L_1,\\
\frac{dI_{s_1}}{dt} &= \kappa_{s_1} L_1 - (\delta_{s_1} + \mu_s) I_{s_1},\\
\frac{dC_1}{dt} &= \theta_{C_1} I_{s_1} - \mu_C C_1,\\
\frac{dS_{h_2}}{dt} &= \Lambda_{h_2} - \lambda_{h_2} S_{h_2} + \gamma_{h_2} I_{h_2} - \mu_h S_{h_2} - \eta_{21} S_{h_2} + \eta_{12} S_{h_1},\\
\frac{dE_{h_2}}{dt} &= \lambda_{h_2} S_{h_2} - (\kappa_{h_2} + \mu_h + a_{21}) E_{h_2} + a_{12} E_{h_1},\\
\frac{dI_{h_2}}{dt} &= \kappa_{h_2} E_{h_2} - (\gamma_{h_2} + \delta_{h_2} + \mu_h + b_{21}) I_{h_2} + b_{12} I_{h_1},\\
\frac{dM_2}{dt} &= \theta_{M_2} I_{h_2} - \mu_M M_2,\\
\frac{dU_2}{dt} &= \Lambda_{s_2} - \lambda_{s_2} U_2 - \mu_s U_2,\\
\frac{dL_2}{dt} &= \lambda_{s_2} U_2 - (\kappa_{s_2} + \mu_s) L_2,\\
\frac{dI_{s_2}}{dt} &= \kappa_{s_2} L_2 - (\delta_{s_2} + \mu_s) I_{s_2},\\
\frac{dC_2}{dt} &= \theta_{C_2} I_{s_2} - \mu_C
\end{aligned}
\tag{1.1}
$$

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with $\lambda_{h_i} = \beta_{h_i}\frac{(I-\phi\xi_i)C_i}{C_0+\varepsilon C_i}$ and $\lambda_{s_i} = \beta_{s_i}\frac{(I-\pi\nu_i)M_i}{M_0+\varepsilon M_i}$ (I = 1, 2) being the forces of infection for human and snail subpopulations. Where the State variables and Parameters of the model are defined in Tables (1.1) and (1.2)

Table 1.1: Model Variables and Description

Variables	Description
$S_{h_i}(t)$	Susceptible individuals in patch i
$E_{h_i}(t)$	Latently tainted individuals in patch i
$I_{h_i}(t)$	Infected individuals in patch i
$M_i(t)$	Miracidia concentration in patch i
$U_i(t)$	Uninfected snails in patch i
$L_i(t)$	Latently-infected snails in patch i
$I_{s_i}(t)$	Tainted snails not yet releasing cercariae in patch i
$C_i(t)$	Free swimming Cercariae ready to enter human skin patch i.

Table 1.2: Model Parameters and Description

Parameter	Description
$\mu_k(k = h, M, s, C)$	Natural death rate for the kth sub population.
$\Lambda_{k_i}(k = h, s)$	Recruitment rate for the kth sub population in patch i.
$\beta_{k_i}(k = h, s)$	Cercariae and Miracidia infectious rate respectively for the kthsub population in patch i.
C_0	Saturation constant for Cercariae.
M_0	Saturation constant for Miracidia.
ε	Limitation of growth velocity of Cercariae and Miracidia.
$\kappa_{k_i}(k = h, s)$	Progression rate from latent class to infectious classes in the kth sub population in patch i.
$\delta_{k_i}(k = h, s)$	Disease and parasite induced death respectively for humans and snails the kth sub population in patch i.
γ_{h_i}	Recovery rate for humans in patch i
θ_{M_i}	Rate at which egg produced by adult Schistosome hatch and develop to free swimming Miracidia in patch i.
θ_{C_i}	Rate at which patent infected snails release cercariae in patch i.
η_{ij}	Movement rate of susceptible individuals from patch i to j.
a_{ij}	Rate at which latently infected individuals move from patch i to j.
b_{ij}	Movement rate of infected individuals from patch i to j.
ϕ	Efficacy of control in the human population.
ξ_i	Availability of control in the human population in patch i.
π	Efficacy of control in the aquatic (Snail) environment.
ν_i	Availability of control in the aquatic environment in patch i.

Using the method of next generation matrix operator proposed by [21] and using notations similar to the ones used in [21], we obtained the Reproduction Number as follows:

$\mathcal{R}_0^2 = \frac{1}{2}[h_1 + \sqrt{h_1^2 - 4h_2}]$,

where:

$h_1 = \frac{H_1 + H_2}{H_3}$

$H_1 = A_1A_2(P_7P_8\kappa_{h_1} + a_{12}b_{21}\kappa_{h_2})P_9P_{10}P_{11}\beta_{h_1}\beta_{s_1}\theta_{M_1}\theta_{C_1}\Lambda_{s_1}\kappa_{s_1}$

$H_2 = A_3A_4P_4P_5P_{12}(a_{21}b_{12}\kappa_{h_1} + P_2P_3\kappa_{h_2})\beta_{h_2}\beta_{s_2}\theta_{M_2}\theta_{C_2}\Lambda_{s_2}\kappa_{s_2}$

$H_3 = (P_2P_7 - a_{12}a_{21})(P_3P_8 - b12b_{21})P_4P_5P_9P_{10}P_{13}^2C_0^2M_0^2\mu_C\mu_M\mu_s$,

$h_2 = \frac{H_4}{H_5}$

$H_4 = A_1A_2A_3A_4\beta_{h_1}\beta_{s_1}\beta_{h_2}\beta_{s_2}\theta_{M_1}\theta_{C_1}\theta_{M_2}\theta_{C_2}\Lambda_{s_1}\Lambda_{s_2}\kappa_{h_1}\kappa_{h_2}\kappa_{s_1}\kappa_{s_2}$

$H_5 = (P_2P_7 - a_{12}a_{21})(P_3P_8 - b12b_{21})P_4P_5P_9P_{10}P_{13}^2C_0^2M_0^2\mu_C^2\mu_M^2\mu_s^2$ $A_1 = (1 - \pi\nu_1)$, $A_2 = (1 - \phi\xi_1)$, $A_3 = (1 - \pi\nu_2)$,

$A_4 = (1 - \phi\xi_2)$,
$P_7 = (\kappa_{h_2} + \mu_h + a_{21})$,
$P_8 = (\gamma_{h_2} + \delta_{h_2} + \mu_h + b_{21})$, $P_9 = (\kappa_{s_2} + \mu_s)$, $P_{10} = (\delta_{s_2} + \mu_s)$,
$P_{13} = (P_1P_6 - \eta_{12}\eta_{21}) = (\eta_{12}\mu_h + \eta_{21}\mu_h + \mu_h^2) > 0$,
$P_{14} = (P_2P_7 - a_{12}a_{21}) > 0$

2.0. ANALYZING THE REPRODUCTION NUMBER.

The threshold quantity, $\mathcal{R}_0$, is analyzed with respect to its sensitivity to some key parameters that describes the impact of Metapopulation and some control parameters on the disease dynamics of Schistosomiasis. This is done by investigating $\mathcal{R}_0$ sensitivity to certain key parameters, namely: availability of control measures in human population in patch 1 (ξ_1), availability of control measures in human population in patch 2 (ξ_2), efficacy of control measures in the human population (ϕ), availability of control measures in aquatic environment in patch 1 (ν_1), availability of control measures in aquatic environment in patch 2 (ν_2), efficacy of control measures in aquatic environment(π) . Hence, we examine 2D contour plots (Figures (2.1) – (2.5)), 2D region plots (Figures (2.6) – (2.10)) and 2D density plots (Figure (2.11) – (2.12)) of $\mathcal{R}_0$ as functions of any two of the following parameters at a time, ξ_1, ξ_2, ϕ, ν_1, ν_2 and π where the two chosen parameters are varied while other parameter values in Table 4.1 not varied are upheld. Note that the values enclosed in the squares, along the curves or contours in Figures (2.1) to (2.5), represent the computed reproduction number using various combination of parameter values. The values of the parameters use for generating these plots are gotten from Table 4.1. The plots are produced using the *ContourPlot* command in Wolfram Mathematica 8.0.

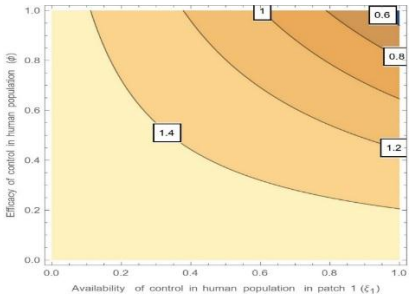

Figure 2.1. Contour plot of $\mathcal{R}_0$ as a function of availability of control in human population in patch 1 (ξ_1) and efficacy of control measures in the human population (ϕ)

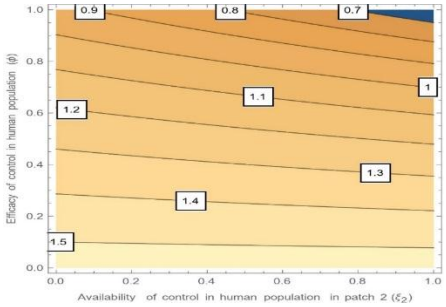

Figure 2.2. Contour plot of $\mathcal{R}_0$ as a function of availability of control in human population in patch 2 (ξ_2) and efficacy of control measures in the human population (ϕ)

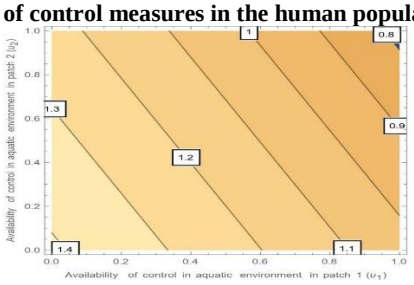

Figure 2.3. Contour plot of $\mathcal{R}_0$ as a function of availability o f control in aquatic environment in patch 1 (ν_1) and availability of control in aquatic environment in patch 2 (ν_2)

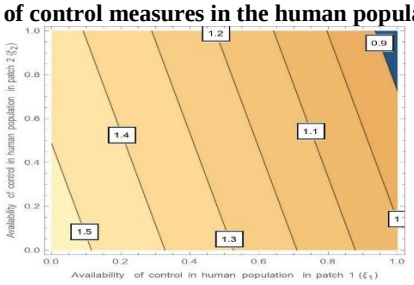

Figure 2.4. Contour plot of $\mathcal{R}_0$ as a function of availability of control in human population in patch 1 (ξ_1) and availability of control in human population in patch 2 (ξ_2)

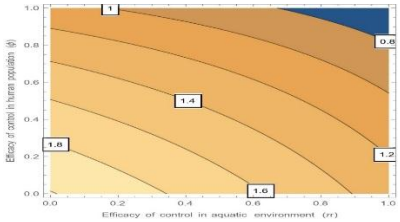

Figure 2.5. Contour plot of $\mathcal{R}_0$ as a function of efficacy of control in the aquatic environment(π) and efficacy of control measures in human population (ϕ)

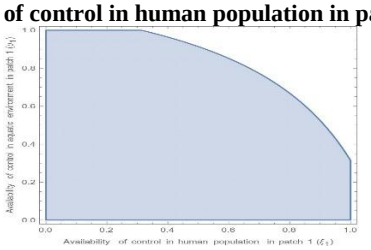

Figure 2.6. Region plot of $\mathcal{R}_0$ as a function of availability of control measures in human population in patch 1(ξ_1) and availability of control measures in the aquatic environment in patch 1 (ν_1)

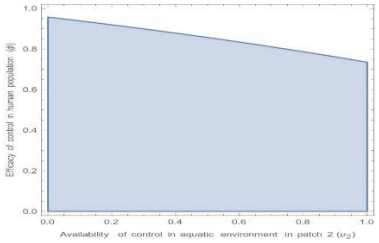

Figure 2.7. Region plot of $\mathcal{R}_0$ as a function of availability of control measures in the aquatic environment in patch 2 (ν_2) and efficacy of control measures in human population(ϕ).

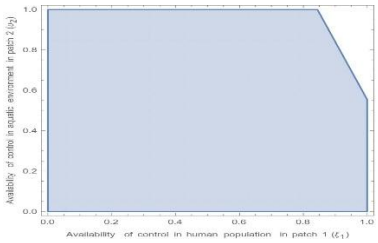

Figure 2.8. Region plot of $\mathcal{R}_0$ as a function of availability of control measures in human population in patch 1 (ξ_1) and availability of control measures in the aquatic environment in patch 2 (ν_2).

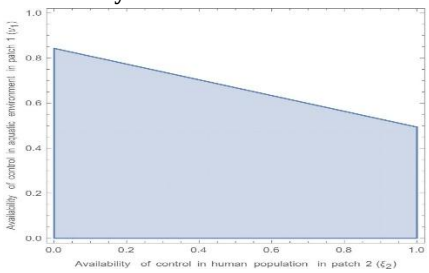

Figure 2.9. Region plot of $\mathcal{R}_0$ as a function of availability of control measures in human population in patch 2 (ξ_2) and availability of control measures in the aquatic environment in patch 1 (ν_1).

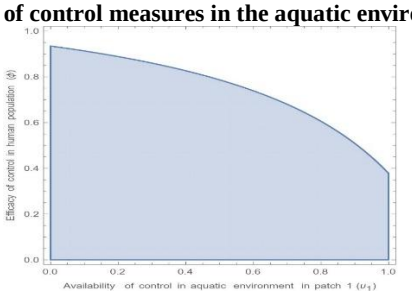

Figure 2.10. Region plot of $\mathcal{R}_0$ as a function of availability of control measures in the aquatic environment in patch 1 (ν_1) and efficacy of control measures in the human population (ϕ).

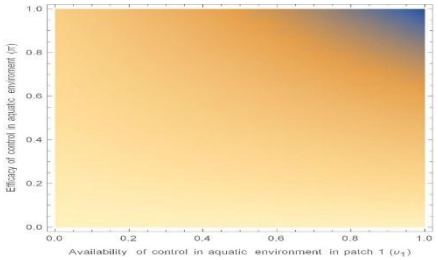

Figure 2.11. Density plot of $\mathcal{R}_0$ as a function of availability of measures of control in the aquatic environment in patch 1 (ν_1) and efficacy of measures of control in the aquatic environment (π).

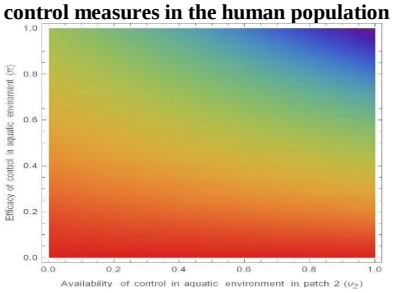

Figure 2.12. Density plot of $\mathcal{R}_0$ as a function of availability of control measures in the aquatic environment in patch 2 (ν_2) and efficacy of control measures in the aquatic environment (π).

3.0. DISCUSSION ON CONTOUR, REGION AND DENSITY PLOTS.

The plot in Figure (2.1) implies that if efficacy of the control measures in the human population can be almost **70**% along with making the control measures available to about **92**% of those prune to Schistosomiasis in patch 1, then eradication of the disease from the population (both patches) is feasible. Also making the control measures available to about **80**% of those at risk of Schistosomiasis in patch 1 along with efficacy of the control measures in the human population of almost **79**% is required to eliminate Schistosomiasis both patches. Also from Figure (2.1), **eradicating Schistosomiasis** from the population (both patches) is **feasible by embarking on *mass administration* of the control measures to those prune to Schistosomiasis in patch 1**.

Figure (2.2) shows that if efficacy of the control measures in the human population can be almost **70**% and the control measures can be made available to about **96**% of those at risk in patch 2 is required to eliminate Schistosomiasis. Also from Figure [2.2], making the control measures available to about **50**% of those at risk in patch 2 along with efficacy of the control measures in the human population of almost **79**% is required to eliminate Schistosomiasis. Another possible combination obtained from Figure (2.2) is that disease eradication is possible if the control measures can be made available to about **14**% of those at risk in patch 2 along with ensuring that efficacy of the control measures in the human population is almost **87**%.

Figure (2.3) suggest that providing control measures in the aquatic environment in patch 1 of nearly **57**% of the time along with making the control measures in the aquatic environment in patch 2 available nearly **100**% of the time is required to eliminate Schistosomiasis. Figure [2.3] also suggest that providing control measures in the aquatic environment in patch 1 of nearly **67**% of the time along with making the control measures in the aquatic environment in patch 2 available **80**% of the time is required to eliminate Schistosomiasis. Also from Figure (2.3), we see that making the control measures in the aquatic environment in patch 1 available nearly **99**% of the time along with making the control measures in the aquatic environment in patch 2 available **14**% of the time is required to eliminate Schistosomiasis.

From Figure (2.4), we can infer that Schistosomiasis eradication is possible if the control measures in the human population can be available to nearly **80**% of the individuals vulnerable to Schistosomiasis in patch 1 along with making the control measures available to about **96**% of the human population vulnerable to Schistosomiasis in patch 2. Also Figure (2.4) suggest that making the control measures available to nearly **92**% of the human population at risk of Schistosomiasis in patch 1 along with making the control measures available to about **50**% of the human population vulnerable to Schistosomiasis in patch 2 is needed to eradicate the disease from the entire population. This Figure also suggest that making the control measures available to nearly **98**% of the human population prune to Schistosomiasis in patch 1 along with making the control measures available to about **22**% of the human population at risk of Schistosomiasis in patch 2 can lead to the eradication the disease from the entire population.

Figure (2.5) suggests that if efficacy of the control measures in the aquatic environment can be as high as **70**% together with the efficacy of the control measures in the human population being about **79**% then disease eradication is possible from the entire population. Figure (2.5) also suggest that for disease eradication from the entire population, efficacy of control measures in the aquatic environment has to be as high as about **82**% together with the efficacy of the control measures in the human population as high as about **70**%. However, Figure [2.5] suggest that if emphasis is placed on the efficacy of the control measure in the human population by making it as high as nearly **98**% with efficacy of the control measures in the aquatic environment of about **23**% is just sufficient to eliminate Schistosomiasis.

From Figure (2.6), it can be inferred that Schistosomiasis can be eradicated if the control measures in the human population in patch 1 is made available to about **80**% of those at risk alongside making the control measures in the aquatic environment in patch 1 available about **68**% of the time. Also, for disease eradication from the entire population, Figure [2.6] suggests that the control measures in the human population in patch 1 has to be available to about **91**% of the individuals at risk with making the control measures in the aquatic environment availability about **50**% of the time in patch 1. Figure (2.6) also infers that enforcing strict compliance of about **99**% usage of the control measures in the human population by those at risk of Schistosomiasis in patch 1 as well as making the control measures in the aquatic environment in patch 1 available about **34**% of the time is just sufficient for the eradication of Schistosomiasis.

From Figure (2.7), we see that efficacy of about **86**% of the control measures in the human population and making the control measures in the aquatic environment available at about **51**% of the time in patch 2 is required to eradicate the disease from the entire population. Figure (2.7) suggest that if efficacy of the control measures in the human population can be sustained at about **73**% and the control measures in the aquatic environment is available all the time (**100**%) in patch 2 then Schistosomiasis can be eradicated from the entire population. Very high efficacy (about **95**%) of control measures in the human population is required to eradicate Schistosomiasis.

The following conclusions on possibilities of eradicating Schistosomiasis from the entire population can be inferred from Figure (2.8): control measures in the human population should be available to **83**% of those at risk of Schistosomiasis in patch 1 and the control measures in the aquatic environment made available all the time(**100**%) in patch 2; control measures in the human population should be made available to all individuals at risk in patch 1 and the control measures in the aquatic environment available at about **56**% of the time in patch 2; and the control measures has to be largely available in the aquatic environment in patch 2 **but extreme caution must be exercise to safeguard other aquatic lives**.

From Figure (2.9), we can infer that disease eradication is possible from the entire population if the control measures in the human population can be made available to about **95**% of those humans at risk in patch 2 and the control measures in the aquatic environment is made available about **50**% of time in patch 1. Figure (2.9) also suggests that Schistosomiasis can be eradicated if the control measures in the human population can be made available to about **99**% of those prune to the disease in patch 2 and the control measures in the aquatic environment should be available for use about **50**% of the time in patch 1. The disease can also be eradicated if the control measures in the human population can be made available to about **63**% of the humans at rick in patch 2 and the control measures in the aquatic environment needed for disease control should be available for use about **63**% of the time in patch 1 . Also the disease can be eradicated if the control measures in the human population can be made available to about **50**% of the humans at risk in patch 2 and the control measures in the aquatic environment should be available for use about **66**% of the time in patch 1. Disease eradication is also possible if about **83**% of the control measures in the aquatic environment needed is available for use in patch 1.

From Figure (2.10), Schistosomiasis can be eliminated if the efficacy of the measures of control in the human population can be as high as about **93**%. Also, efficacy of the measures of control in the human population of about **70**% and making the measures of control in the aquatic environment availability about **66**% of the time in patch 2 can eradicate the disease.

Figure (2.11) suggests that disease elimination is possible if the efficacy of the control measures in the aquatic environment can be as high as about **100**% and the control measures in the aquatic environment needed in patch 1 is available for use about **35**% of the time. Also disease eradication can be achieved if the efficacy of the control measures in the aquatic

environment can be as high as about $\mathbf{70}\%$ and about $\mathbf{66}\%$ of the control measures in the aquatic environment needed in patch 2 is available. Figure (2.11) also suggest that Schistosomiasis eradication is possible if the efficacy of the control measures in the aquatic environment can be as high as about $\mathbf{83}\%$ and about $\mathbf{50}\%$ of the control measures in the aquatic environment needed in patch 2 is available.
From Figure (2.12), Schistosomiasis eradication can be achieved if the efficacy of the measures of control in the aquatic environment can be as high as about $\mathbf{75}\%$ together with about $\mathbf{99}\%$ of the control measures in the aquatic environment needed in patch 1 is made available. Also disease eradication from the entire population can be attained if the efficacy of the control measures in the aquatic environment can be as high as about $\mathbf{83}\%$ with about $\mathbf{80}\%$ of the control measures in the aquatic environment needed in patch 2 is available.

4.0. NUMERICAL SIMULATION RESULTS

In this section, the system is numerically simulated so as to investigate the impact of varying certain important parameters. The parameter values in Table 4.1 are used for the simulation. Demographic and epidemiological parameters relevant and connected to Nigeria are used in this simulations. In 2015, Nigeria population was estimated to be 184,635,279 (Countrymeter, 2016), hence $\Lambda_h/\mu_h = 184{,}635{,}279$ at the DFE. The mean mortality rate per year was calculated to be $\mu_h = 0.02041$ or $\mu_h = 0.000056$ per day (UNAIDS-WHO, 2004). Two communities (patches) relatively close to each other around water bodies which the residence depend on for means of livelihood and other human activities are used for this simulation. Community (patch) 1 is assumed to have an estimated population size of $50{,}000$ individuals with Schistosomiasis prevalence of $\mathbf{43.4}\%$ while community (patch) 2 is assumed to have an estimated population size of $30{,}000$ individuals with Schistosomiasis prevalence as high as $\mathbf{80.4}\%$. Community (patch) 1 is assumed to have access to public health facilities, vector control measures, resources, modern amenities, lots of wealth creation opportunities and serious awareness/enlightenment/sensitization program in place while community (patch) 2 is contrast. Hence the prevalence of Schistosomiasis in community (patch) 1 is low when compare to community (patch) 2. The parameter values for movement between the patches is assumed within this premise; with more individuals tending to move to community (patch) 1 than community (patch) 2. The entire simulation (as well as the associated plots) performed in this section have been done using *MATLAB R2012a*.

Table 4.1. **Baseline values for the parameters of the metapopulation Schistosomiasis model.**

Parameter	**Baseline values**	**References**
μ_h	0.000056 day^{-1}	[24]
μ_M	2.526 day^{-1}	[25]
μ_s	0.003 day^{-1}	[25]
μ_C	1.000 day^{-1}	[25]
Λ_{h_1}	2.8 day^{-1}	Calculated
Λ_{h_2}	1.68 day^{-1}	Calculated
Λ_{s_1}	180 day^{-1}	[25]
Λ_{s_2}	200 day^{-1}	[5]
β_{h_1}	0.000406 day^{-1}	Assumed
β_{h_2}	0.0092 day day^{-1}	Assumed
β_{s_1}	0.00091 $(L\ day)^{-1}$	Assumed
β_{s_2}	0.000091 $(L\ day)^{-1}$	[25]
C_0	900,000	[5]
M_0	1,000,000	[5]
ε	0.2	[5]
κ_{h_1}	0.017857 day^{-1}	[17]
κ_{h_2}	0.0017857 day^{-1}	Assumed
κ_{s_1}	0.036 day^{-1}	[25]
κ_{s_2}	0.036 day^{-1}	[25]
δ_{h_1}	0.0039 day^{-1}	[17]
δ_{h_2}	0.0041 day^{-1}	Estimate

δ_{s_1}	0.0004012 day^{-1}	[5]
δ_{s_2}	0.0006 day^{-1}	Assumed
γ_{h_1}	0.8 day^{-1}	[17]
γ_{h_2}	0.5 day^{-1}	Assumed
θ_{M_1}	500 day^{-1}	[25]
θ_{M_2}	700 day^{-1}	[25]
θ_{C_1}	2.6 day^{-1}	[5]
θ_{C_2}	2.8 day^{-1}	[5]
η_{12}	0.4 day^{-1}	Assumed
η_{21}	0.8 day^{-1}	Assumed
a_{12}	0.4 day^{-1}	Assumed
a_{21}	0.8 day^{-1}	Assumed
b_{12}	0.08 day^{-1}	Assumed
b_{12}	0.1 day^{-1}	Assumed
ϕ	0.7	Assumed
ξ_1	0.8	Assumed
ξ_2	0.5	Assumed
π	0.7	Assumed
ν_1	0.5	Assumed
ν_2	0.8	Assumed

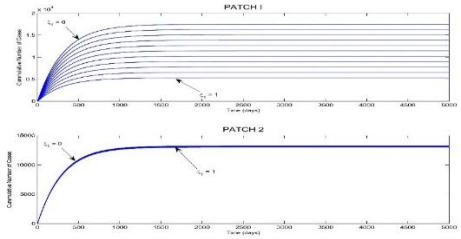

Figure 4.1. Aggregate number of new Schistosomiasis cases in both patches with varied availability of measures of control in the human population in patch 1 (ξ_1).

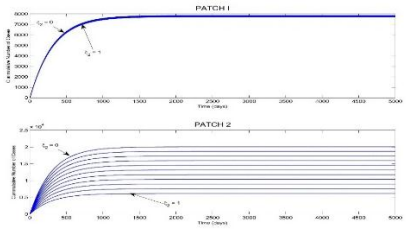

Figure 4.2. Cumulative number of new Schistosomiasis cases with varied availability of control measures in the human population in patch 2 (ξ_2).

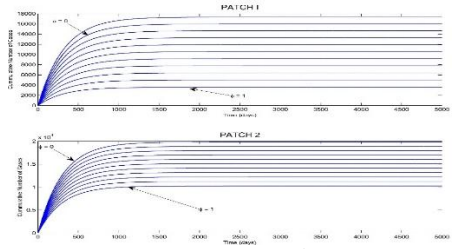

Figure 4.3. Aggregate number of new schistosomiasis cases with varied efficacy of measures of control in the human population (ϕ).

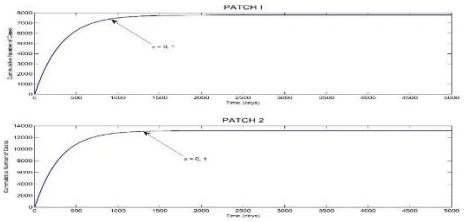

Figure 4.4. Aggregate number of new Schistosomiasis cases with varied **efficacy of measures of control in the aquatic environment** (π).

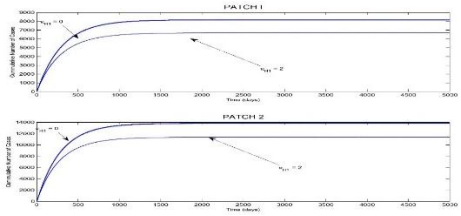

Figure 4.5. Aggregate number of new Schistosomiasis cases with varied **progression rate of latently infected to infected humans in patch 1** (κ_{h_1}).

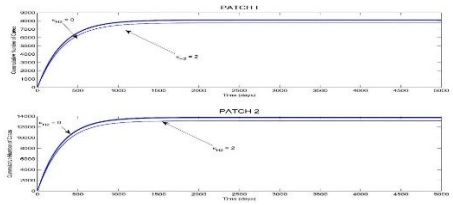

Figure 4.6. Aggregate number of new Schistosomiasis cases with varied **progression rate of latently infected to infected humans in patch 2** (κ_{h_2}).

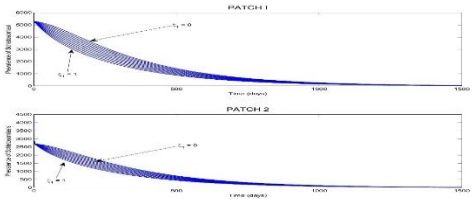

Figure 4.7. Prevalence of Schistosomiasis cases with varied **availability of control measures in the human population in patch 1** (ξ_1).

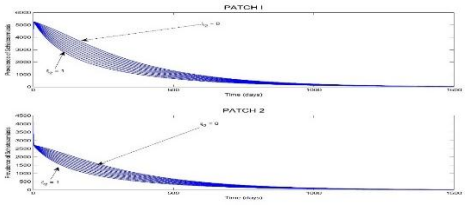

Figure 4.8. Prevalence of Schistosomiasis cases with varied **availability of control measures in the human population in patch 2** (ξ_2).

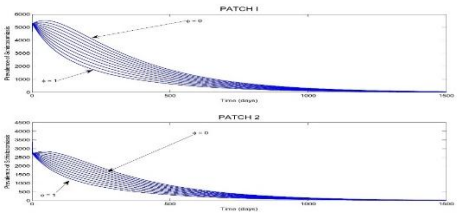

Figure 4.9. Prevalence of Schistosomiasis cases with varied **efficacy of control measures in the human population** (ϕ).

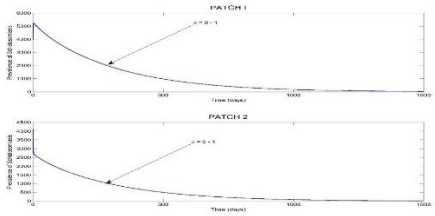

Figure 4.10. Prevalence of Schistosomiasis cases with varied **efficacy of control measures in the aquatic environment** (π).

5.0. DISCUSSION ON NUMERICAL SIMULATIONS.

Figure (4.1) shows the aggregate number of new Schistosomiasis cases (Schistosomiasis incidence) in both patches as the proportion of availability of measures of control in the human population in patch 1 is varied from 0 to 1. It can be seen that the incidence of Schistosomiasis in patch 1 dropped significantly as the fraction of availability of control measures in the human population in patch 1 increases (i.e., $\xi_1 \to 1$) while the impact of varying this parameter is not significant in patch 2. The simulation result implied that the incidence of Schistosomiasis in patch 1 could be reduced as the proportion of availability of control measures in the human population in patch 1 is increased. Increasing the proportion of the control measures available in the human population could result in the prevention of about 12,149 new Schistosomiasis cases over a period of 5,000 days in patch 1. Whereas, the disease burden in patch 2 remains at about 13,230 new Schistosomiasis cases (with no reduction as the proportion of the control measures in the human population available in patch 1 is increased) over a period of 5,000 days.

Figure (4.2) shows the aggregate number of new Schistosomiasis cases (Schistosomiasis incidence) in both patches as the proportion of availability of control measures in the human population in patch 2 is varied from 0 to 1. It can be seen that the incidence of Schistosomiasis in patch 2 dropped significantly as the fraction of availability of control measures in the human population in patch 2 increases (i.e., $\xi_2 \to 1$) while the impact of varying this parameter is not significant in patch 1. The simulation result implied that the incidence of Schistosomiasis in patch 2 could be reduced as the proportion of availability of control measures in the human population in patch 2 is increased. Increasing the proportion of control measures available in the human population in patch 2 could result in the prevention of about 13,920 new Schistosomiasis cases over a period of 5,000 days in patch 2. Whereas, the disease burden in patch 1 remains at about 7,868 new Schistosomiasis cases (with no reduction as the proportion of the control measures in the human population available in patch 2 is increased) over a period of 5,000 days.

Figure (4.3) shows the aggregate number of new Schistosomiasis cases (Schistosomiasis incidence) in both patches as the proportion of efficacy of control measures in the human population is varied from 0 to 1. It can be seen that the incidence of Schistosomiasis in both patches dropped significantly as the fraction of efficacy of control measures in the human population increases (i.e., $\phi \to 1$). The simulation result implied that the incidence of Schistosomiasis in both patches could be reduced as the efficacy of the measures of control in the human population is increased. Increasing the proportion of the efficacy of the measures of control in the human population could result in the prevention of about 13,743 new Schistosomiasis cases in patch 1 and prevention of about 9,550 new Schistosomiasis cases in patch 2 over a period of 5,000 days.

Figure (4.4) shows the aggregate number of new Schistosomiasis cases (Schistosomiasis incidence) in both patches as the proportion of efficacy of control measures in the aquatic environment is varied from 0 to 1. It can be seen that the impact of varying the efficacy of the control measures in the aquatic environment on the incidence of Schistosomiasis in both patches is not significant as the fraction of efficacy of control measures in the aquatic environment increases (i.e., $\pi \to 1$). The simulation result implied that the incidence of Schistosomiasis in both patches is not significantly influenced as the proportion of the measures of control in the aquatic environment is increased owing to the fact that the measures of control

in the aquatic environment is meant to prevent the Schistosome eggs deposited by humans (through urinating or defecating) into the aquatic environment from developing into Miracidia that is capable of entering a fresh water snail. Schistosomiasis is contacted only when humans come in contact with Cercariae infected waters. The disease burden in patches 1 and 2 remained respectively at about 7,868 and 13,180 new Schistosomiasis cases (with no reduction as the proportion of the efficacy of the control measures in the human population is increased) over a period of 5,000 days.

Also varying the proportion of the availability of control measures in the aquatic environment in patches 1 and 2 (ν_1 and ν_2) will not influence the incidence of Schistosomiasis in both patches significantly since the interest is in new Schistosomiasis cases and these parameters are in the Snail incidence.

Figure (4.5) shows the aggregate number of new Schistosomiasis cases (Schistosomiasis incidence) in both patches as the proportion of progression rate of latently infected humans to infected humans in patch 1 is varied from 0 to 2. It can be seen that the incidence of Schistosomiasis increased significantly as the fraction of progression rate of latently infected humans to infected humans in patch 1 is increased (i.e., $\kappa_{h_1} \to 2$). The simulation results implied that the incidence of Schistosomiasis in both patches could be increased as the proportion of progression rate of latently infected humans to infected humans in patch 1 is increased. As $\kappa_{h_1} \to 2$, new Schistosomiasis cases increased by about 1,497 in patch 1 and 2,490 in patch 2 over a 5,000 days period.

Figure (4.6) shows the aggregate number of new Schistosomiasis cases (Schistosomiasis incidence) in both patches as the proportion of progression rate of latently infected humans to infected humans in patch 2 is varied from 0 to 2. It can be seen that the incidence of Schistosomiasis increased significantly as the fraction of progression rate of latently infected humans to infected humans in patch 2 is increased (i.e., $\kappa_{h_2} \to 2$). The simulation results implied that the incidence of Schistosomiasis in both patches could be increased as the proportion of progression rate from latently infected humans to infected humans in patch 2 is increased. As $\kappa_{h_2} \to 2$, new Schistosomiasis cases increased by about 373 in patch 1 and 700 in patch 2 over a 5,000 days period.

Figure (4.7) shows the prevalence of Schistosomiasis in both patches as the proportion of availability of control measures in the human population in patch 1 (ξ_1) is varied from 0 to 1. It can be seen that the prevalence of Schistosomiasis decreased significantly as the fraction of availability of control measures in the human population in patch 1 (ξ_1) is increased (i.e., $\xi_1 \to 1$). The simulation results implied that the prevalence of Schistosomiasis in both patches could be decreased as the proportion of availability of control measures in the human population in patch 1 is increased. As $\xi_1 \to 1$, the prevalence of Schistosomiasis decreased by about 583 in patch 1 and by 472 in patch 2 within a few days of intervention.

Figure (4.8) shows the prevalence of Schistosomiasis in both patches as the proportion of availability of control measures in the human population in patch 2 (ξ_2) is varied from 0 to 1. It can be seen that the prevalence of Schistosomiasis decreased significantly as the fraction of availability of control measures in the human population in patch 2 (ξ_2) is increased (i.e., $\xi_2 \to 1$). The simulation result implied that the prevalence of Schistosomiasis in both patches could be decreased as the proportion of availability of control measures in the human population in patch 2 is increased. As $\xi_2 \to 1$, the prevalence of Schistosomiasis decreased by about 1352 in patch 1 and by 580 in patch 2 within a few days of intervention.

Figure (4.9) shows the prevalence of Schistosomiasis in both patches as the proportion of efficacy of control measures in the human population (ϕ) is varied from 0 to 1. It can be seen that the prevalence of Schistosomiasis decreased significantly as the fraction of efficacy of control measures in the human population (ϕ) is increased (i.e., $\phi \to 1$). The simulation result implied that the prevalence of Schistosomiasis in both patches could be decreased as the efficacy of control measures in the human population is increased. As $\phi \to 1$, the prevalence of Schistosomiasis decreased by about 1757 in patch 1 and by 1240 in patch 2 within a few days of intervention.

Figure (4.10) shows the prevalence of Schistosomiasis in both patches as the proportion of efficacy of control measures in the aquatic environment (π) is varied from 0 to 1. It can be seen that the prevalence of Schistosomiasis is not significantly influenced as the efficacy of control measures in the aquatic environment (π) is increased (i.e., $\pi \to 1$).

6.0. CONCLUSION

Mathematically, through the use of these contour, region and density plots above, we have been able to show that Schistosomiasis can be eradicated from a two patch model (patches that are relatively close and are in the same geographical region and individuals are free to move within and between patches) if the government, non governmental agency and all stakeholders in the health sector are committed to making various control measures available to individuals who are at risk of Schistosomiasis especially those that live or have means of livelihood around water bodies. Implying that mass administration of highly efficacious control measures which ranges from prophylaxis, vaccine, protective wears to limiting the activities of individuals around water bodies suspected to be highly infected with Cercariae by way of legislation in just a single patch can help eliminate Schistosomiasis from the entire population. Also from these plots, we see that another feasible way of eradicating the disease from the entire population is in laying emphasis on making available at all times highly efficacious control measures in the aquatic environment in either of the patches. However, care must be taken in administering these controls or may be completely discouraged so as not to affect other aquatic lives. We must emphasize here that not all diseases can be eradicated by just mass administration of control measures but we have shown that in this case, mass administration of control measures

can help eradicate Schistosomiasis from a two patch model. The key to eradicating Schistosomiasis in this Metapopulation model is mass administration of highly efficacious control measures in both human and aquatic environments.
Also, the numerical study of the system (1.1) showed that, the key to eradicating Schistosomiasis in a metapopulation model where control measures are available is mass administration of highly efficacious control measures.

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