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STATISTICAL ESTIMATION OF SOME METEOROLOGICAL VARIABLES USING THE BETA KERNEL FUNCTION

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Abstract

The study of meteorological variables is imperative due to their usefulness in our daily activities. Climate change which is now a global issue has received considerable large volume of research as its effect's cuts across all aspects of life. It is generally known that the effect of climatic changes is a great threat to agricultural activities in the Africa continent and the world at large; hence the human race is likely to encounter problem if corrective measures are not put in place soonest. Statistical analysis of meteorological data for visualization, exploratory and estimation purposes is very fundamental because the underlying structures of the variables forms the bedrock in the process of decisions making and implementation stages by meteorologists. This study investigates the relationship between temperature and relative humidity using the beta kernel function with the asymptotic mean integrated squared error as the criterion function. The results of the study using real data examples reveals that temperature and relative humidity greatly determine climatic fluctuations which can adversely affect the environment.

***Keywords*:** Kernel, Bandwidth, Climate Change, AMISE, Temperature, Humidity.

1. INTRODUCTION

Human activities are dependent on all the weather variables. Agricultural activities rely largely on the relationship between some climatic variables for quality or maximum yield. Due to the importance of agricultural products to mankind, efforts are being made globally to curb the activities that could result in climate change[1].A study of the effects of climate change was recently conducted on a region known as the Near East North Africa (NENA) where the agricultural activities and livelihoods of people within the countries in the region were critically examined with emphasis' on small-scale farming by Food and Agriculture Organization of the United Nations in Cairo, Egypt[2].

Apart from the direct effects of climate change on agricultural products, the mobility of the products and human resources to required markets can also be hampered by climatic fluctuations. The interactions between temperature and humidity were investigated in relation with cloud cover using aircraft variables to verify if temperature and humidity changes are correlated and the results revealed negative correlations between the variables [3]. Globally, the effects of climate change can affect access to food which directly affects the Sustainable Development Goals (SDGs) of poverty and hunger eradication. The United States transport large volume of food particularly grains through the waterway and should there be changes in the weather with direct effect on the waterway, there might be difficulty in transportation of such products to the designated market since there may be no alternatives routes for transporting such commodities. If there are transportation problems, the volume of products to be exported to the international market will be affected hence; the global prices for food will also be affected drastically [4].

Climatic fluctuations are not just a threat to the growth and development of the economic and agricultural sector of nations but its effects can be felt in the entire human population. The human race now faces new climatic variables at an

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exponential rate in their environment unlike the former variables in which adaptability has been developed [5]. The spread of diseases in animal and humans can also be attributed to changes in climatic variables; hence adequate measures should be taken to address this menace. One of the important climatic variables is rainfall and insufficient amount of rainfall could result in degeneration and deforestation of land as a resource leading to desertification. Due to the adverse effects of climatic fluctuations on human race and its environment, it is regarded as the severest challenge confronting the world now and its effects is worse than the activities of terrorist globally [6, 7].

Climatic variables should be properly investigated so that inherent structures can be identified and decisions can be made on reducing its effects on the environment. Orderliness of climatic variables is fundamental to curbing its activities; hence statistical analysis of some climatic data is the focus of this work. Data analysis and estimation has broad spectral in applicability virtually in all fields of studies. Analysis of data could be for visualization, exploratory and estimation purposes for communication of the findings about the observations. The visualization of data involves representation of the data graphically in order to identify trends and patterns that are present in the data. In exploratory analysis, the data are subjected to critical examination with the aid of some statistical tools while the data estimation uses the actual data values in predicting or forecasting a future occurrence.

Accurate predictions of climatic variables with the application of statistical tools could prevent disasters that are connected to climate change. Predictions or forecasting using statistical tools involves density estimation which is the construction of probability estimates using the data either from a known probability distribution or unknown probability distribution. If the observations are assumed to be members of a known probability distribution, then the estimation technique is parametric density estimation. In this estimation method, prior knowledge of the data is required and it is only the parameter of the distribution that will be estimated. On the other hand, nonparametric approach does not make any assumption about the distribution of the set of observations but the observations are given the opportunity to "speak for themselves". Nonparametric estimation gives a better approach to statistical analysis of data due to its ability to capture the true structures of the underlying distribution. One of the features of nonparametric estimation is that they are useful exploratory and visualization tools in statistical data analysis.This advantages account for nonparametric models as a better choice of robust and accurate statistical tools[8].Nonparametric density estimation is of wide applicability in multivariate analysis of data for visualization and exploratory purposes especially for observations whose prior information may not be known [9].Due to constant variations in weather variables, the nonparametric approach will be employed to avoid the assumption of imposition of distributional property on the observations.

This paper focus on density estimation with emphasis on weather variables using the beta polynomial kernel functions for accurate meteorological information. The analysis will be based on bivariate kernels only since it considers only two weather variables. The general form of the kernel estimator is presented, and the beta polynomial kernel functions discussed. The smoothing parameters and kernel estimates of the bivariate product kernel using real data examples were obtained and the kernel performances were evaluated with the asymptotic mean integrated squared error.

2. THE KERNEL DENSITY ESTIMATOR

The kernel density estimator is a probability density function with applications in exploratory data analysis and data visualization [10, 11]. It is also of indirect application in discriminant analysis, goodness-of-fit testing, hazard rate estimation and other statistical related estimation techniques [12]. The frequent usage of the kernel estimators in nonparametric density estimation is attributed to its simplicity and computational efficiency. As a standardized weighting function, the univariate kernel density estimator in its compact form is given as [10, 11]

$$\hat{f}(\mathrm{x}) = \frac{1}{nh_{\mathrm{x}}}\sum_{i=1}^{n} K\left(\frac{\mathrm{x} - X_i}{h_{\mathrm{x}}}\right), \qquad (1)$$

where $K(\cdot)$ is a kernel function,n is sample size, $h_{\mathrm{x}} > 0$ is bandwidth(also called smoothing parameter), x is range of the observations and X_i is the set of observations.Usually the kernel function is a non-negative function that must satisfy the following conditions

$$\int K(\mathrm{x})d\mathrm{x} = 1, \int \mathrm{x}K(\mathrm{x})d\mathrm{x} = 0 \quad \text{and} \int \mathrm{x}^2 K(\mathrm{x})d\mathrm{x} = \mu_2(K) \neq 0. \qquad (2)$$

The implication of the first condition in Equation (2) in kernel density estimation is that every kernel function must integrate to unity that is one; hence most kernel functions are probability density functions. The other conditions imply that the average of the kernel is zero while the variance of the kernel $\mu_2(K)$ is not zero.

Density estimation using the kernel estimator involves the smoothing parameter which plays a significant role in the estimation process; hence it is regarded as a resolution factor when viewing observations. The evaluation of the kernel estimator is strictly dependent on the smoothing parameter; therefore, its appropriate choices are very imperative. Smoothing parameter selection is the major problem confronting the implementation of kernel density estimation. There are plethora smoothing parameter selectors but no universally acceptable selector for all situations; hence, smoothing parameter selections is a grey area in kernel density estimation. The problem of smoothing parameter selection in univariate kernel is not as difficult as the multivariate kernel estimation with different forms of parameterizations [13].The general multivariate form of the kernel estimator in Equation (1) with a single bandwidth is of the form

$$\hat{f}(\mathrm{x}) = \frac{1}{nh_{\mathrm{x}}^{d}}\sum_{i=1}^{n} K\left(\frac{\mathrm{x}-\mathrm{X}_i}{h_{\mathrm{x}}}\right), \qquad (3)$$

where d is dimension of the kernel function. The multi-dimensional kernel alsosatisfies the conditions in Equation (2) and with the assumption of its contours being spherically symmetric. This multivariate kernel estimator is advantageous because the formulas for the asymptotic mean integrated squared error and the optimal smoothing parameter value can be obtained unlike other complex forms of parameterizations without explicit optimal bandwidth formula.

The two-dimensional kernel estimators can be easily obtained from the general form using the product techniques that applies different smoothing parameter for each dimension[14]. The bivariate product kernel uses the product of two univariate kernel but with different level of smoothing in each axis. In bivariate kernel density estimation, x , y are random variables having joint density function $f(\mathrm{x}\,,\mathrm{y})$, with $\mathrm{X}_i\,,\mathrm{Y}_i,\;\; i = 1, 2, \ldots, n$as set of observations, where $n$is sample size. The bivariate product kernel density estimator is of the form

$$\hat{f}(\mathrm{x}\,,\mathrm{y}) = \frac{1}{nh_{\mathrm{x}}h_{\mathrm{y}}}\sum_{i=1}^{n} K\left(\frac{\mathrm{x}-\mathrm{X}_i}{h_{\mathrm{x}}},\frac{\mathrm{y}-\mathrm{Y}_i}{h_{\mathrm{y}}}\right) = \frac{1}{nh_{\mathrm{x}}h_{\mathrm{y}}}\sum_{i=1}^{n} K\left(\frac{\mathrm{x}-\mathrm{X}_i}{h_{\mathrm{x}}}\right)K\left(\frac{\mathrm{y}-\mathrm{Y}_i}{h_{\mathrm{y}}}\right), \qquad (4)$$

where $h_{\mathrm{x}} > 0$ and $h_{\mathrm{y}} > 0$ are smoothing parameters in X and Ydirections, x and yare ranges of the variables in the different axes and $K(\mathrm{x}\,,\mathrm{y})$is a bivariate kernel function, which is the product of two univariate kernels. The bivariate product kernel is mostlybeneficial if there are variations in the scales of the observations in the respective axes or directions.The bivariate kernel bridges the gap between the univariate and other higher dimensional kernel estimators.An advantage of the bivariate kernel estimators is that their estimates are simple to understand and interpret, either as surface plots or contour plots and also a useful tool for data exploratory analysis and data visualization [15]. The multivariate product kernel is of the form

$$\hat{f}(\mathrm{x}) = n^{-1}\left(\prod_{j=1}^{d} h_j\right)^{-1}\sum_{i=1}^{n} K\left(\frac{\mathrm{x}_1-\mathrm{X}_{i1}}{h_1},\frac{\mathrm{x}_2-\mathrm{X}_{i2}}{h_2},\ldots,\frac{\mathrm{x}_d-\mathrm{X}_{id}}{h_d}\right)$$

$$= n^{-1}\left(\prod_{j=1}^{d} h_j\right)^{-1}\sum_{i=1}^{n} K\left(\frac{\mathrm{x}_j-\mathrm{X}_{ij}}{h_j}\right), \qquad (4)$$

where h_j are the smoothing parameter for the different axes and d is the dimension of the kernel.

3. THE BETA POLYNOMIAL KERNEL FUNCTIONS

There is plethora of kernel estimators, however; the beta polynomial kernel whose degree of differentiability is dependent on the power of the polynomial is our focus because its limiting case is the popular Gaussian kernel that has wide applications. The beta polynomial kernels are probability density function with their general form given by

$$K_p(t) = \frac{(2p+1)!}{2^{2p+1}(p!)^2}(1-t^2)^p, \qquad (5)$$

where $p = 0, 1,\; 2, \ldots, \infty$ is regarded as its power and$t$ takes value within the interval $-1 \le t \le 1$. Since the beta kernel functions are probability density functions, they are usually evaluated within this interval$[-1,\;1]$. The values of $p$that ranges from 0 to 3 will produce Uniform, Epanechnikov, Biweight and Triweight kernels with the uniform kernel being the simplest kernel, while at its limiting case the resulting kernel is the Gaussian kernel also known as the normal kernel. The limiting case simply means when the value of$p$ tends to infinity [16].The corresponding kernel functions for$p = 1,\; 2,$ and 3known as the Epanechnikov,Biweight and Triweightkernel are as follows.

$$K_1(t) = \frac{3}{4}(1-t^2) \qquad (6)$$

$$K_2(t) = \frac{15}{16}(1-t^2)^2 \qquad (7)$$

$$K_3(t) = \frac{35}{32}(1-t^2)^3 \qquad (8)$$

As the value of p tends to infinity, the resulting kernel is the popular Gaussian kernel given by

$$K_{\emptyset}(t) = \frac{1}{\sqrt{2\pi}}\,exp\left(-\frac{t^2}{2}\right). \qquad (9)$$

The Epanechnikov, Biweight, and Triweight kernel functions are of wider applications especially, the Epanechnikovkernel is used in computation of the efficiency of other kernel functions of the family

In the product method, several univariate kernels are multiplied to obtain the corresponding multivariate kernel and are of the form

$$K_p(t) = A^2 \prod_{i=1}^{2} (1 - t_i^2)^p, \tag{10}$$

where $A = \frac{(2p+1)!}{2^{2p+1}(p!)^2}$is the normalization constant of the kernelfunction.The bivariate beta kernel functions can be simply express as

$$K_p(t) = A^2(1 - t_1^2)^p(1 - t_2^2)^p \tag{11}$$

The corresponding bivariate Epanechnikov, Biweight and Triweight kernel are

$$K_1(t) = \left(\frac{3}{4}\right)^2 (1 - t_1^2)(1 - t_2^2) \tag{12}$$

$$K_2(t) = \left(\frac{15}{16}\right)^2 (1 - t_1^2)^2(1 - t_2^2)^2 \tag{13}$$

$$K_3(t) = \left(\frac{35}{32}\right)^2 (1 - t_1^2)^3(1 - t_2^2)^3 \tag{14}$$

Again, at the limiting case, the bivariate Gaussian kernel is of the form

$$K_\emptyset(t) = \frac{1}{2\pi} \exp\left(-\frac{t_1^2 + t_2^2}{2}\right). \tag{15}$$

The Gaussian kernel is very fundamental in density estimation because it produces smooth density estimates and the mathematical computations can be explicitly expressed.It is also continuously differentiable and possesses derivatives of all order which supported its wide uses in density estimation unlike other members of the beta family.

4. PERFORMANCE EVALUATION OF THE KERNEL ESTIMATOR.

The performance of the kernel estimator is dependent on the smoothing parameter and not the kernel function; therefore, appropriate choices are imperative.

One of the optimality criteria function in kernel density estimation with regards to dimension is the mean integrated squared error with its components is given by

$$MISE\left(\hat{f}(\mathrm{x})\right) = \int Var\left(\hat{f}(\mathrm{x})\right) d\mathrm{x} + \int Bias^2\left(\hat{f}(\mathrm{x})\right) d\mathrm{x}. \tag{16}$$

There is trade-off between the components the MISE, the bias can be reduced while the variance increases and vice versa, if the magnitude of the smoothing parameter varies. As a result of the importance of the smoothing in performance evaluation of the kernel estimator, appropriate criterion function that regulates the contributions of both components must be employed [17]. The approximate form of Equation (16) is the asymptotic mean integrated squared errorwhich produces the integrated variance and the integrated squared bias given by

$$AMISE\left(\hat{f}(\mathrm{x})\right) = \frac{R(K)}{nh_\mathrm{x}} + \frac{h_\mathrm{x}^4}{4}\mu_2(\mathrm{K})^2 \quad R(f''(\mathrm{x})), \tag{17}$$

where $R(K)$is roughness of kernel, $\mu_2(\mathrm{K})^2$ is variance of kernel while $R(f''(\mathrm{x})) = \int f''(\mathrm{x})^2 d\mathrm{x}$ is the roughness of the unknown density function. The smoothing parameter with the minimum AMISE in Equation (17) in terms of dimension is

$$h_{\mathrm{x-AMISE}} = \left[\frac{R(K)}{\mu_2(K)^2 R(f''(\mathrm{x}))}\right]^{\left(\frac{1}{4+d}\right)} \times n^{-1/(4+d)}. \tag{18}$$

where d is dimension of the kernel function. The multivariate asymptotic mean integrated squared error with the product kernel estimatoris

$$\begin{aligned} \mathrm{AMISE}\left(\hat{f}(\mathrm{x})\right) &= \frac{R(K)^d}{nh_1h_2,\dots,h_d} + \frac{1}{4}h_j^4\mu_2(K)^2 \int tr^2\left(\nabla^2 f(\mathrm{x})\right)d\mathrm{x} \\ &= \frac{R(K)^d}{nh_1h_2,\dots,h_d} + \frac{1}{4}h_j^4\mu_2(K)^2 R\left(\nabla^2 f(\mathrm{x})\right) \end{aligned} \tag{19}$$

where $R(K) = \int K^2(\mathrm{x})d\mathrm{x}$isroughness of kernel, $\mu_2(K)^2$ is variance of kernel, $R\left(\nabla^2 f(\mathrm{x})\right) = \int tr^2\left(\nabla^2 f(\mathrm{x})\right)d\mathrm{x}$ is roughness of the function, tr is trace of a matrix, n is sample size,$h_1, h_2, \dots, h_d$ are smoothing parameters to be determined, d is dimension of kernel and $\nabla^2 f(\mathrm{x})$ is Hessian matrix (matrix of second partial derivatives) of the function. The corresponding smoothing parameter with the minimum AMISE value is

$$H_{AMISE} = \left[\frac{dR(K)^d}{\mu_2(K)^2 \quad R\left(\nabla^2 f(\mathrm{x})\right)}\right]^{\left(\frac{1}{d+4}\right)} \times n^{-\left(\frac{1}{d+4}\right)} \qquad l \qquad (20)$$

Equation (18) and Equation (20) depend on the second derivative of the unknown function, that is $R(f''(\mathrm{x}))$ and $R\left(\nabla^2 f(\mathrm{x})\right)$ respectively,thereforewill be difficult to evaluate without the knowledge of the true density function $f(\mathrm{x})$. Several researches and suggestions have been made about the value of $R(f''(\mathrm{x}))$ and $R\left(\nabla^2 f(\mathrm{x})\right)$ for different kernel functions. We obtain the value of the roughness of the function, $R\left(\nabla^2 f(\mathrm{x})\right)$ with reference to the Epanechnikov, Biweight, and Gaussian functions.

5. RESULTS AND DISCUSSIONS

The statistical properties of the weather variable were obtained with their informative graphical displays also presented in bivariate form with the aid of Mathematica ver.12 software [18]. This analysis aims at exploring the general distributional properties of the weather data and obtaining the necessary information for decision making and possibly future predictions.The beta kernel functions employed in this analysis are Epanechnikov, Biweight, and Gaussiankernel functions. The smoothing parameter values that minimize the AMISE of the data were obtained with reference to their respective kernels.

The weather data investigated is the ERA-Interim average daily meteorologicaldata that is made up of365 observations on two variables namely; temperature in degree Celsius and relative humidity in percentages[19, 20]. The two variables were the daily records of temperature and relative humidity for the year 2017.The analysis of these data addresses the relationship between temperature and relative humidity and their direct impact on the environment. The data were standardized to ensure that there are no variations in their ranges [21, 22, 23, 24, 25]. Standardization of data helps in multivariate analysis particularly in kernel density estimation where visualization of data structure is the primary goal. It should be noted that standardization of data during analysis does not affect the inherent features of the data set. The bivariate kernel estimates of the weather variables using the bivariate product kernel are in Figure 1, Figure 2 and Figure 3 using Epanechnikov, Biweight, and Gaussian kernel functions. The smoothing parameter and the asymptotic mean integrated squared error (AMISE) values are in Table 1. The Epanechnikov being the optimal kernel produce the smallest value of the AMISE amongst the three kernel functions examined. Again, the smoothing parameter increases as seen in Table 1 while the Normal kernel produced the largest value of the AMISE. Performance in kernel estimation means the closeness of the kernel density to its target density and can be evaluated using the smoothing parameter. The results in Table 1 show that the Epanechnikov kernel outperformed the Biweight and Gaussian kernels due to its production of the smallest AMISE value.

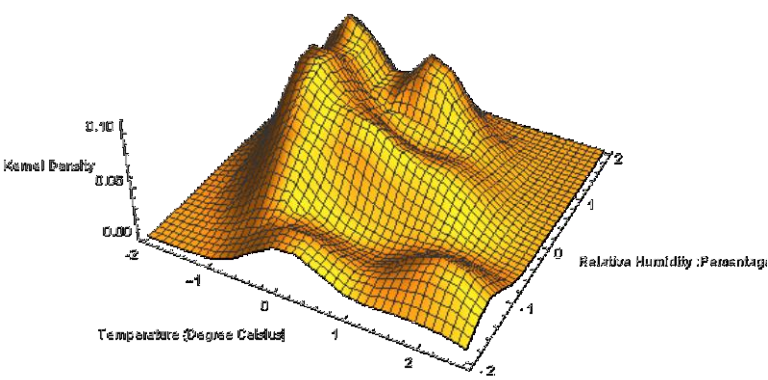

Figure 1a: Surface plot of Epanechnikov bivariate kerneldensity estimate of WeatherData

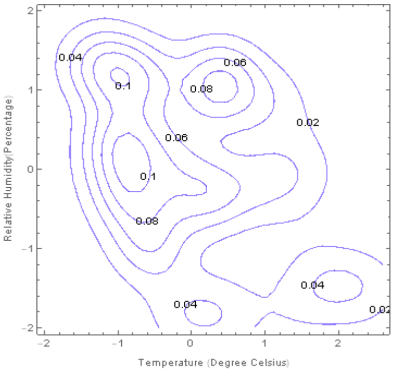

Figure 1b:Contour plot of Epanechnikov bivariate kerneldensity estimate of Weather Data

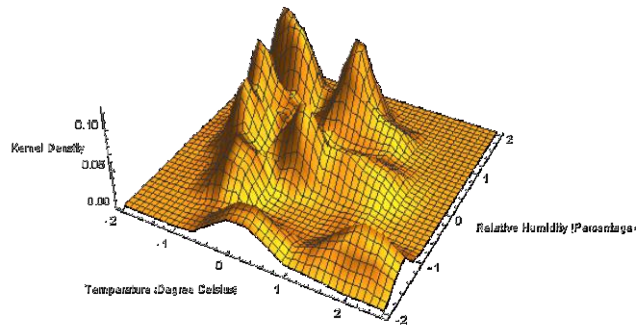

Figure 2a:Surface plot of Biweight bivariate kerneldensity estimate of WeatherData

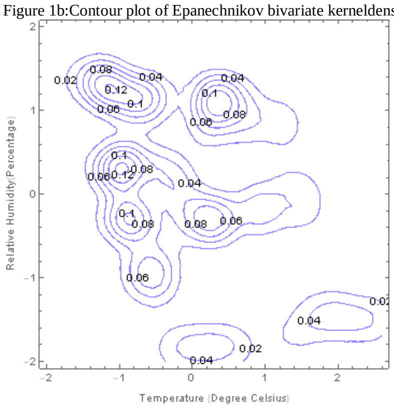

Figure 2b:Contour plot of Biweight bivariate kerneldensity estimate of Weather Data

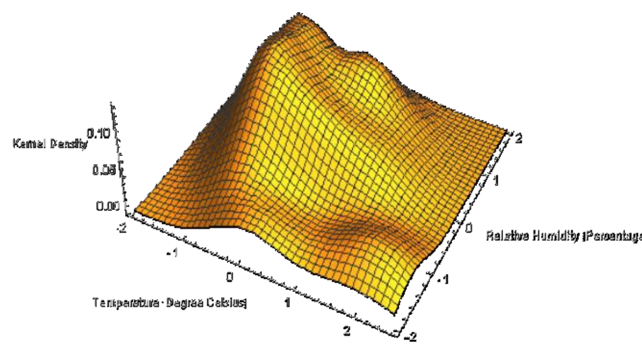

Figure 3a:Surface plot of Normal bivariate kerneldensity estimate of Weather Data

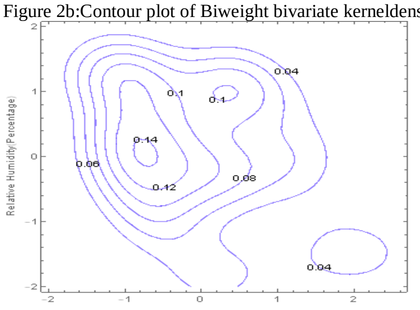

Figure 3b:Contour plot of Normal bivariate kerneldensity estimate of WeatherData

Table1: Kernel Functions, Smoothing Parameters, Variance, $Bias^2$ and AMISE of the Data.

Kernel Function	h_x	h_y	**Variance**	$Bias^2$	**AMISE**
Epanechnikov	0.486475	0.503703	0.00046124	0.00016110	0.00062234
Biweight	0.451965	0.451834	0.00111163	0.00013969	0.00125132
Gaussian	0.449998	0.457424	0.00105918	0.00105931	0.00211849

The bivariate kernel estimates of the weather variables clearly show that the data are multimodal which indicates that the variables are inversely related.The Gaussian kernel produces smooth kernel estimates as seen in Figure 3 but despite the noise in the Epanechnikov and Biweight estimates as noticed in Figure 1 and Figure 2, the clear effect of temperature and relative humidity is statistically presented. The effects of the interaction of temperature and relative humidity of the variables investigated are with high probability value as seen in the kernel estimates. The probability value ranges between 0 and 0.14 which means any action that can be affected by these variables in the environment if not prevented will result in disaster to mankind.

This inverse relationship has serious effect on man and its environment because an increase in relative humidity results in more water in the atmosphere that could circulate malodorous molecules from bacteria related sources. Therefore, maintenance of the environment should be our priority so that the effect of increase humidity and heat due to dirtiness can be avoided. The multimodality of the kernel estimates which implies inverse relationship could also result in cardiovascular diseases that could lead to death.

6. CONCLUSIONS

Temperature and relative humidity are inversely related and this inverse relationship as investigated using real life data shows its devastatingeffect on the environment if neglected without urgent control measure. The probability of their interaction is high using the statistical tools of nonparametric density estimation. Activities that could result in depletion of the Ozone layer should be avoided and environmentalists should as a matter of urgency sensitize the citizenry on the importance of urgent need of maintenance of the environment. Government at all levels in every nation should come up withstringentpolicies that regulate the activities of individuals and industries towards the maintenance of the environment to reduce the effects of climatic fluctuations.

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