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FOURIER SERIES MODELS OF SOME METEOROLOGICAL VARIABLES

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Abstract

Climate is the long-term average of weather, typically averaged over a period of 30 years. Some of the meteorological variables measured include but not limited to visibility, humidity, sunshine hours, temperature, rainfall, evaporation, solar radiation, atmospheric pressure, precipitation and wind. The purpose of this study is to develop the Fourier series models of mean monthly visibility, relative humidity, fractions of sunshine hours, maximum temperature, rainfall, evaporation and solar radiation. Fourier series approximations up to second harmonics were computed for each of the variables using the method of harmonic analysis. The results show that the models are able to represent and predict the mean monthly measures of the respective meteorological variables within a stable climatic period. These models are compact and therefore recommended.

***Keywords*: climate, meteorology, Fourier series, Harmonic analysis**

1. Introduction

Weather is the condition of the atmosphere at any particular time and place[1]. Climate is defined as the averaged weather over a long period[1]. The standard averaging period is 30 years, but other periods may be used depending on the purpose. Climate also includes statistics other than the average, such as the magnitudes of day-to-day, month-to-month or year-to-year variations. Weather and climate play a major role in our lives. Weather, for example, often dictates the type of clothing we wear, while climate influences the type of clothing we buy. Climate determines when to plant crops as well as what type of crops can be planted. Weather determines if these same crops will grow to maturity. Although weather and climate affect our lives in many ways, perhaps their most immediate effect is on our comfort. *Meteorology* is the study of the atmosphere and its phenomena. Meteorological variables or weather elements measured include but not limited to visibility, humidity, sunshine hours, temperature, rainfall, evaporation, solar radiation, atmospheric pressure, precipitation and wind [1].

The purpose of this study is to develop the Fourier series models of mean monthly visibility, relative humidity, fractions of sunshine hours, maximum temperature, rainfall, evaporation and solar radiation.

Fourier series is a periodic function composed of harmonically related sinusoids combined by a weighted summation. With appropriate weights, one period of the summation can be used to approximate an arbitrary function in that interval or the entire function if it is periodic. For texts on Fourier analysis, the reader is referred to ([2],[3], [4] and [5]). Some meteorological variables are periodic functions in the sense that their average functional values repeat after a period of one year. Fourier series can be used to represent the functions generated by meteorological data. We now use Fourier series to model mean monthly visibility, relative humidity, fractions of sunshine hours, maximum temperature, rainfall, evaporation and solar radiation.

The plan of this paper is as follows. Model formulation is done in section 2. Numerical computation and results are presented in section 3. Discussion of results and conclusive remarks are passed in sections 4 and 5 respectively.

2. Model Formulation

The Fourier series representation of a function with an arbitrary period with finite length $(-l, l)$ is given by

$$f(t) = \frac{a_0}{2} + \sum_{i=1}^{\infty}[a_n cos\frac{n\pi t}{l} + b_n sin\frac{n\pi t}{l}] \quad (1)$$

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where

$$a_0 = \frac{1}{l}\int_{-l}^{l} f(t)dt,$$

$$a_n = \frac{1}{l}\int_{-l}^{l} f(t)cos\frac{n\pi t}{l}dt$$

and

$$b_n = \frac{1}{l}\int_{-l}^{l} f(t)sin\frac{n\pi t}{l}dt.$$

However, where functional values are generated at discrete points of t, we adopt the method of Harmonic analysis to estimate the Fourier coefficients as follows.

$$a_0 = 2\ mean\ f(t),$$

$$a_n = 2\ mean\left[f(t)cos\frac{n\pi t}{l}\right],$$

$$b_n = 2\ mean\left[f(t)sin\frac{n\pi t}{l}\right].$$

Therefore,

$$f(t) = mean\ f(s) + \sum_{i=1}^{\infty}\left[2\ mean\left[f(s)cos\frac{n\pi s}{l}\right]cos\frac{n\pi t}{l} + 2\ mean\left[f(s)sin\frac{n\pi s}{l}\right]sin\frac{n\pi t}{l}\right] \quad (2)$$

3. Numerical Computation and Results

The data in Table 1 are used for the numerical computation.

Table 1: Mean monthly meteorological data used for numerical computation

Month	V(km)	n/N	R.H.(%)	$T_{max}(0_C)$	Rainfall (mm)	Solar radiation ($mJm^{-2}day^{-1}$)	Evaporation (mm)
Jan	11.62	0.75	23.30	32.03	19.06	10.61	114.80
Feb	10.23	0.68	19.80	34.13	45.75	11.40	119.85
March	18.54	0.66	19.90	38.68	141.43	11.10	120.32
April	18.54	0.62	31.20	40.72	154.9	11.14	95.15
May	18.54	0.63	46.00	39.15	209.81	11.34	100.33
June	18.46	0.60	57.40	36.30	222.39	8.81	81.93
July	16.15	0.55	71.00	32.88	460.22	8.33	80.53
Aug	7.85	0.54	78.90	31.24	361.7	7.70	65.84
Sept	9.85	0.62	74.60	32.75	261.44	4.71	72.21
Oct	14.85	0.70	54.30	36.17	293.98	5.45	82.76
Nov	11.23	0.75	28.70	35.69	142.71	8.71	87.12
Dec	13.31	0.79	25.50	33.07	22.94	6.15	92.29

Source: ([6] and [7])

To obtain the Fourier series representations of the variables in Table 1, the formula in (1) is used. Therefore, the Fourier series representation of mean monthly visibility is given by

$$f_v(t) = 14.0975 - 1.0398cos\frac{\pi t}{6} - 1.6292cos\frac{2\pi t}{6} + \cdots + 3.5519sin\frac{\pi t}{6} - 2.1752sin\frac{2\pi t}{6} + \cdots \quad (3)$$

The Fourier series representation of mean monthly relative humidity is given by

$$f_{RH}(t) = 45.05 - 25.64cos\frac{\pi t}{6} + 3.33cos\frac{2\pi t}{6} + \cdots - 12.35sin\frac{\pi t}{6} + 3.69sin\frac{2\pi t}{6} + \cdots \quad (4)$$

The Fourier series representation of mean monthly fractions of sunshine hours is given by

$$f_{n/N}(t) = 0.6575 + 0.0943cos\frac{\pi t}{6} - 0.0075cos\frac{2\pi t}{6} + \cdots - 0.0290sin\frac{\pi t}{6} - 0.0390sin\frac{2\pi t}{6} + \cdots \quad (5)$$

The Fourier series representation of mean monthly maximum temperature is given by

$$f_{Tmax}(t) = 35.23 + 0.0151cos\frac{\pi t}{6} - 2.9575cos\frac{2\pi t}{6} + \cdots + 2.6237sin\frac{\pi t}{6} - 1.0695sin\frac{2\pi t}{6} + \cdots \quad (6)$$

The Fourier series representation of mean monthly rainfall is given by

$$f_{Rain}(t) = 203.03 - 171.84cos\frac{\pi t}{6} - 11.82cos\frac{2\pi t}{6} + \cdots - 54.96sin\frac{\pi t}{6} + 45.10sin\frac{2\pi t}{6} + \cdots \quad (7)$$

The Fourier series representation of mean monthly evaporation is given by

$$f_{Eva}(t) = 92.75 + 17.90cos\frac{\pi t}{6} + 1.22cos\frac{2\pi t}{6} + \cdots + 14.56sin\frac{\pi t}{6} + 2.40sin\frac{2\pi t}{6} + \cdots \quad (8)$$

The Fourier series representation of mean monthly solar radiation is given by

$$f_{Solar}(t) = 8.7875 + 0.8434cos\frac{\pi t}{6} + 0.2417cos\frac{2\pi t}{6} + \cdots + 2.7803sin\frac{\pi t}{6} - 0.0144sin\frac{2\pi t}{6} + \cdots \quad (9)$$

The plots of equations 3 through 9 are shown in Figures 1 through 7.899

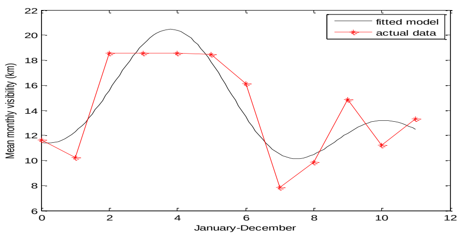

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Figure 1: Fourier series plot of mean monthly visibility

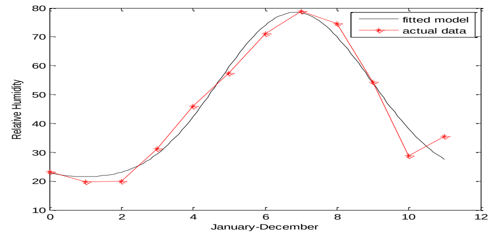

Figure 2: Fourier series plot of mean monthly relative humidity

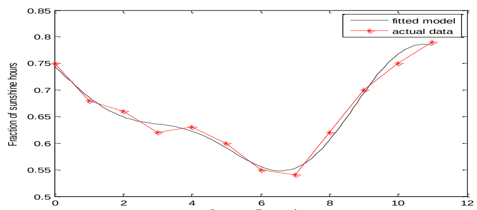

Figure 3: Fourier series plot of mean monthly fractions of sunshine hours

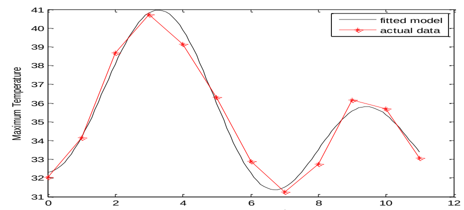

Figure 4: Fourier series plot of mean monthly temperature

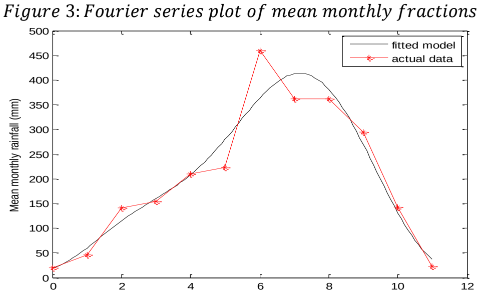

Figure 5: Fourier series plot of mean monthly rainfall

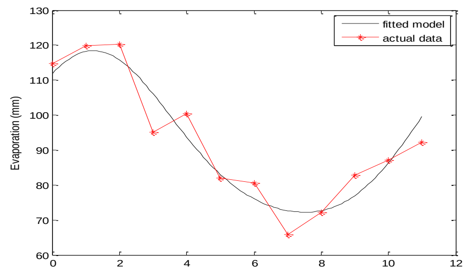

Figure 6: Fourier series plot of mean monthly evaporation

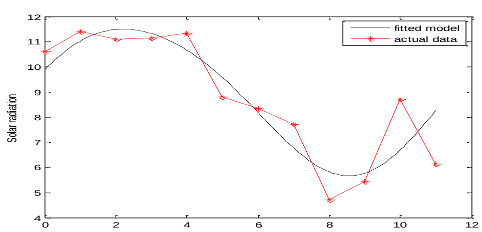

Figure 7: Fourier series plot of mean monthly solar radiation

4. Discussion

This section discusses the results of our study. The main results are equations 3 through 9 and Figures 1 through 7. Equations 3 through 9 are Fourier series representations of the mean monthly visibility, relative humidity, fractions of sunshine hours, maximum temperature, rainfall, evaporation and solar radiation. Figures 1 through 7 compare the actual data with the estimated values. These results show that the Fourier series representations adequately fit mean relative humidity, fractions of sunshine hours, maximum temperature and rainfall; and fairly represent the mean monthly visibility, evaporation and solar radiation.

Conclusion

This paper presents Fourier series models of the mean monthly visibility, relative humidity, fractions of sunshine hours, maximum temperature, rainfall, evaporation and solar radiation. The main results can be seen in section 3. Whilst some models adequately fit the mean monthly relative humidity, fractions of sunshine hours, maximum temperature and rainfall, others fairly represent the mean monthly visibility, evaporation and solar radiation. Therefore, Fourier series models are effective for representing meteorological variables.

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