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**Erratum:** The Diffusive Behaviours Associated with Some of the Ocean Wave Processes {Transactions of the Nig. Assoc. of Mathematical Physics Volume 9, (March and May, 2019), pp183 – 186}

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Because of Several errors in this paper, the entire paper is reproduced below as it ought to be.

# THE DIFFUSIVE BEHAVIOURS ASSOCIATED WITH SOME OF THE OCEAN WAVE PROCESSES

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***Abstract***

***This study describes the dynamics of the physical manifestations that characterize ocean wave events. The differential equation governing these processes and their respective solution techniques are in general quite familiar.***
***The similarity transformations techniques may also be familiar but the related parameters and variables derived here are quite new. They are indeed able to exhibit some of the physical characteristics associated with linear and nonlinear behaviour of ocean wave parameters such as the modulated frequency wave group velocity, the spectrum assumed narrow in this case.***
***Finally, the study appears to have brought to focus the remarkable effects of nonlinearity in the evolution of diffusive phenomena. These effects are indeed, drastic as shown in fig 1.***

## 1. Introduction

This paper concerns the nonlinear diffusive phenomena and evolutional differential equations that describe their physical manifestations. A number of physical processes are diffusive in behaviour and hence, are related to these. The motions induced by water waves in ocean environment [1], energy related group velocity [2], heat wave, dispersive movement of atmospheric cloud, high frequent cyelectromagnetic waves in plasma [1]; all these are highly diffusive in behaviour. Hence, these processes are governed by the identical evolution equations and in most cases, nonlinear. These equations are generally familiar. However, in this study, attempt will be made to integrate this equation not by the method of familiar integral transformation technique [3,4].

Existing classical method (integral, Fourier) effectively solve linear evolution equation only in general. Similarity technique [5] on the other hand, provides an effective way out of complexities associated with both linear and nonlinear diffusive behaviour in a process and their inherent constraints [2,6].

Consequently, attempt will be made to determine suitable similarity parameters and variables that solve bothlinearand nonlinear evolution process with diffusive behaviour in the far field and subsequent applications will follow. We will appeal to ocean observed data. [4,7] eventually.

## 2. A Unidirectional Propagating Process

A process propagating horizontally in x-direction is considered, y-axis is vertical andperpendicular to x-axis. t> 0 denotes time duration. The evolution of the process denoted by V(x,t)is governed by the nonlinear diffusion equation (7)

$$\frac{\partial V}{\partial t} + v\frac{\partial V}{\partial x} = k\frac{\partial^2 V}{\partial x^2} \qquad (1)$$

Here, the parameter k is the diffusion coefficient assumed to be constant in this study. The constraints on the V(x,t) will follow as this study progresses.

## 3. Similarity Transformation Technique for a Linear Process

We consider the linear problem stated as follows:

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$\frac{\partial V}{\partial t} = k\frac{\partial^2 V}{\partial x^2}, t > 0, x > 0$ (2)
$V(0., t) = 0, t > 0$ (2i)
V(x, t) is bounded as x →∞, t > 0 (2ii)
V (x, 0) = $V_0$ (a constant) (2iii)
Let V(x, t) = f(η) and η(x, t) = $x/h(t)$ (3)

$$\eta_x = \frac{1}{h(t)}, \eta_t(x,t) = -\left(\frac{x}{h(t)}\right)\frac{h'(t)}{h(t)} = -\eta(x,t)\frac{h'(t)}{h(t)}$$

$$V_x'(x,t) = f'(\eta)\eta_x = \frac{1}{h(t)}f'(\eta), V_{xx}'(x,t) = \frac{1}{h(t)}f''(\eta)\eta_x = \frac{1}{h^2(t)}f''(\eta)$$

$$V_t(x,t) = f'(\eta)\eta_t = -\eta(x,t)\frac{h'(t)}{h(t)}f'(\eta)$$

Substitute the above in (2)

$$\frac{k}{h^2}f''(\eta) = \frac{-h'(t)}{h(t)}\eta(x,t)f'(\eta), f''(\eta) = -\frac{h'h}{k}\eta f'(\eta)$$

Physical oceanography consideration gives

$$hh' = 2k = \frac{1}{2}\frac{d}{dt}h^2(t); thus, \frac{h^2(t)}{2} = 2kt + B. Buth(0) = 0$$

Thus, B = 0; $h(t) = 2\sqrt{kt}$ (4)
Consequently
$f''(\eta) = -2nf'(\eta)$ (5)
The similarity transformation for this problem is now as follows from equation (3)
$V(x,t) = f(\eta) and \eta(x,t) = \frac{x}{2\sqrt{kt}}$ (6)
And transformed equation (5) is a simple variable separable in that form
$V(0,t) = f(0) = 0, V(x,0) = f(V(x,t) = f(\eta) and f(\infty) = V_0 and from equaiton$ (5)

$$\frac{f''(\eta)}{f'(\eta)} = -2\eta, lnf'(\eta) = -\eta^2(x,t), f'(\eta) = k_0 e^{-\eta^2}$$

$f(\eta) = k_0\int_0^\eta e^{-\sigma^2}\,d\sigma$ (7)

$$f(\infty) = V_0 = k_0\int_0^\eta e^{-\sigma^2}\,d\sigma = V(x,0) = k_0\frac{\sqrt{\pi}}{2} asn \to 0$$

By using the definition of gamma function $\Gamma(\alpha) = \int_0^\eta R^{\alpha-1}e^{-R}dR$ and considering $\Gamma\left(\frac{1}{2}\right);\ k_0 = \frac{2V_0}{\sqrt{\pi}}$
Thus, V(x, t) = $\frac{2V_0}{\sqrt{\pi}}\int_0^\eta e^{-\sigma^2}\,d\sigma$ (8)
Where the standard error – function erf is given by
$\text{erf}(\eta) = \frac{2}{\sqrt{\pi}}\int_0^\eta e^{-\sigma^2}\,d\sigma$,thus
$V(x,t) = V_0\text{erf}(\eta)$ (9)

**4. The Non – Linear Diffusive Processes[6, 2, 5]**

These kind of diffusive phenomena are described by equation (1). The boundary condition is simply that V(x,t) → 0 as x →±∞ but the initial data is same $V_0$. Sustained investigation suggests that the similarity transformation in equation (6) is not suitable in this case. However, further investigation suggests the following transformation procedure:
$V(x,t) = V_0 t^q f(\eta), \eta(x,t) = xt^p$ (10)
qand p being similarity parameters and thus η(x, t), the similarity variable are to be determined firstly,
From equation (10), $\eta_x(x,t) = t^p, \eta_{xx}(x,t) = 0, \eta_t(x,t) = {p\eta(x,t)}/{t}$

$$V_t = V_0[qt^{q-1}f(\eta) + t^q f'(\eta)\eta_t] = V_0[qt^{q-1}f(\eta) + t^q f'(\eta)\eta_t]$$
$$= V_0 t^{q-1}[qf(\eta) + p\eta f'(\eta)], V_x = V_0 t^q f'(\eta)\eta_x = V_0 t^{p+q}f'(\eta)$$
$$V_{xx} = V_0 t^{p+q}f''(\eta)\eta_x = V_0 t^{p+2q}f''(\eta), VV_{xx} = V_0^2 t^{2p+q}f(\eta)f'(\eta)$$

Substituting in equation (1), then

$$t^{q-1}V_0[qf(\eta) + \eta pf'(\eta)] + V_0^2 t^{p+2q}f(\eta)f'(\eta) - kt^{q+2p}V_0 f''(\eta)] = 0$$
$$t^{q-1}[f(\eta) + \eta f'(\eta)] + V_0 t^{3q}f(\eta) - kt^{3q}f''(\eta)] = 0$$

Provided p = q

$$= t^{-2q-1}[f(\eta) + \eta f'(\eta)] + \frac{V_0}{2}\frac{d}{d\eta} f^2(\eta) - kf''(\eta)] = 0$$

$if\ t^{-2q-1} = 1, then - 2q - 1 = 0$ and hence, $q = \frac{-1}{2}$

The form of equation (1) which doesn't explicitly contain x and t is thus

$$-\frac{1}{2}[f(\eta) + \eta f'(\eta)] + \frac{V_0}{2}\frac{d}{d\eta} f^2(\eta) - kf''(\eta)] = 0 \quad (11)$$

The similarity transformation designed for (1) is now

$$V(x,t) = \frac{V_0}{\sqrt{t}} f(\eta), \eta(x,t) = \frac{x}{\sqrt{t}} \quad (12)$$

$$\int \eta f'(\eta) d\eta = \eta f(\eta) - \int f(\eta) + B, B = 0\ since f(\infty) = 0$$

Integrating (11), we have

$$2kf'(\eta) + \eta f(\eta) = V_0 f^2(\eta) \quad (13)$$

$$f'(\eta) + \frac{\eta f}{2k}(\eta) = \frac{V_0}{2k} f^2(\eta) \quad (14)$$

The constant of integration is zero since $f(\eta)$ and the derivative are zero as $x \to \infty$

Equation (13) is essentially Bernoulli's ordinary differential equation. Its appearances in this study is interestingly an unexpected result. For the solution, however it may be put in the form,

$$f^{-2}(\eta) f'(\eta) + \frac{2}{2k} f^{-1}(\eta) = \frac{V_0}{2k}$$

$$Let Z = f^{-1}(\eta) = Z(\eta), Z' = -f^{-2}(\eta) f'(\eta), f^{-2}(\eta) f'(\eta) = -Z$$

And

$$Z' - \frac{\eta z}{2k} = -\frac{V_0}{2k} \quad (15)$$

Equation (15) takes the form

$$\frac{d}{d\eta}[zexp\left[-\frac{\eta^2}{4k}\right]] = -\frac{V_0}{2k}\exp\left[-\frac{\eta^2}{4k}\right], \quad thus Z(\eta) = -\exp\left[-\frac{\eta^2}{4k}\right]\frac{V_0}{2k} R(\eta)$$

$$R(\eta) = \int_{-\eta}^{\eta} \exp[-\sigma^2]\, d\sigma = 2\int_0^{\eta} exp\,[-\sigma^2] d\sigma$$

For $\exp[-\sigma^2]$ is an even function of σ but $z(\eta) = f^{-1}(\eta)$, thus

$$V_0 f(\eta) = -\frac{k}{R(\eta)} \exp(-\eta), t > 0 \quad (16)$$

$$V(x,t) = \frac{V_0}{\sqrt{t}} f(\eta) = \frac{-kexp(-\eta^2)}{\sqrt{t}R(\eta)} \quad (17)$$

Equation (17) can be utilized to describe the far field behaviour of an ocean kinematic process. Further,

$\eta \to \infty, t > 0, R(\eta) \to \sqrt{\pi k}$ and $V(x,t) \to 0$ as expected.

In the subsequent calculations, the following ocean datae are used. [6,2]. $V_0 = 13$m $sec^{-1}$ and thus is the data for the modulation group velocity in the wave spectrum assumed narrows banded. Modulation wavelength L = 10.2m; water depth from the free – surface h = 10m; diffusion coefficient k = 31$m^2 sec^2$. The above are the characteristic value from an observed oceanographic datae[2,6].

**5. Conclusions**

Eqns. (16) and (17) illustrate some of the interesting diffusive behaviors which describes the kinematic variables in the concept of physical oceanography. In a slowly varying ocean medium, some of the important wave physical parameters involved are group velocity, wave number and frequency. These are often kinematically diffusive in their evolutions [4, 5].

Group velocity is denoted by V(x,t) in this consideration. It is the speed with which wave energy is transmitted. Adequate understanding of its evolution is essential in all coastal and inshore ocean activities. Its main feature is this. It is kinematic in its evolution [2] and also diffusive even in the far field [7]. This observation is justified analytically in this study.

.Finally, this study suggests the unexpected catastrophic behaviour associated with the diffusive phenomenon at the origin of its profile (fig 1) in the nonlinear case. This event would be what a marine physicist or engineer would be eager observe.

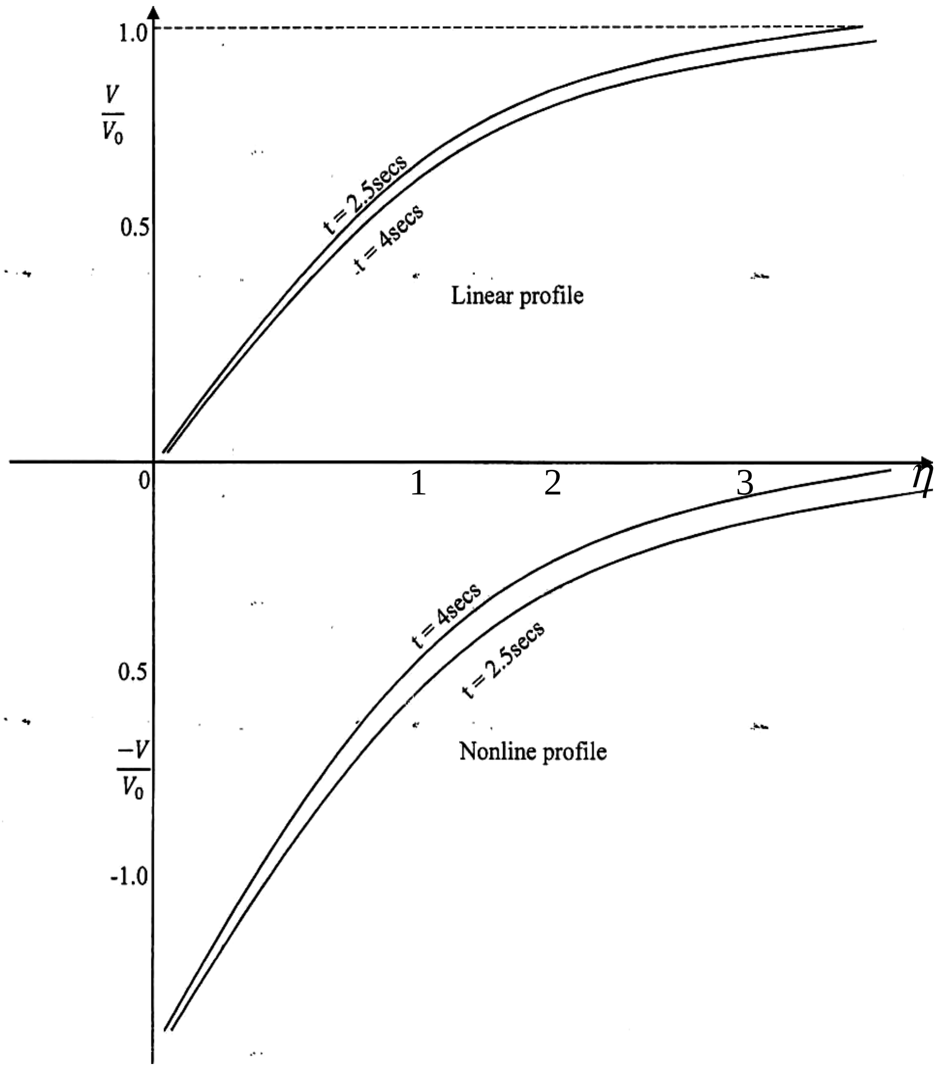


Fig I: Linear Profile (above), Non-linear profile (below)

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