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THE EFFECTS OF REYNOLDS NUMBER ON VARYING PRESSURE FROMTHE OCEAN WATER WAVES

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Abstract

The paper considers the effects of Reynolds number on the wind generated Ocean water waves where varying pressure is taken into account. We give information on wave frequency approximation by Newton iterative formula on the deep water height for waves moving from deep Ocean areas to coastal regions due to nonlinearities present. The wave amplitude modulation for the progressive waves which moves with group energy and the work done by the wave train have been detailed.
It is established that the amplitude modulation for the group energy is related to the Kinetic energy, and that the total energy with which the ocean water waves advance is a sum of both Kinetic and Potential Energy.
We give equation for the breaking waves and effects of gravity waves on the bottom layers as well as upper surface of the Ocean water. The effects of wind stress on the ocean is discussed which is dependent on the density of the air, drag coefficient and squared wind speed.

Keywords: wind generated waves, ocean water, Reynolds number, Newton iterative methods , wind stress.
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1. Introduction

Internal waves are created when water waves of two different densities travel in the same direction. This situation is due to wave dispersion. It is the distinct property of water waves, which dictates waveforms of different frequencies travelling at different speeds. Wave's measurements can be made by using various types of recorders that may be kept at the sea surface or over and below it[1]. The ocean currents are the continuous predictable directional movement of seawater driven by gravity, wind (Coriolis effects), and water density.
We define the group velocity as velocity with which wave energy is transferred by a propagating wave train. Water waves are said to be oscillatory or nearly oscillatory if the motion described by the water particle is a circular orbit that is closed or nearly closed to each wave period. The mechanical energy carried by linear waves is a factor that plays a significant role in the transformations, including breaking that the wave trains undergo during travel in deep, shallow and intermediate waters, where $\frac{h}{\lambda} < 20$ and $\frac{h}{\lambda} < \frac{1}{2}$ respectively, h is the water depth, λ the wavelength.

In deep water for example, the characteristic behavior of the wind blowing in particular direction and time are:
- (i) Constant wind blowing for a long time enough, in fetch limited conditions;
- (ii) Constant wind of minimal transient period blowing over sea area; and
- (iii) fully developed sea.

Therefore, the features of Ocean waves are mainly due to wind waves, the upper surface turbulence and the upper Ocean mixed layer depth. The mixed layer depth may be influenced by the wind stress, heating and cooling, advection, waves breaking, Langmuir circulation, and internal waves surface forcing [3].Wind generated waves have irregular wave heights and periods due to the nature of wind. Thus interest may arise in computing the statistical properties of the surface – that is, average wave height, wave periods and directions which vary slowly in time and space relative to typical wave heights and periods.
When water particles advance with wave and do not return to their original position, it is called a wave translation, as for example, the typical solitary wave. The surface elevation of the ocean water is a sum of harmonic waves generated at different times and locations and each are statistically independent of their origin[1].
Usually, waves are governed by swells and seas. We give such distinction here between swells and seas. Seas are short periodic waves created by winds, whereas, swells are waves that have moved out of the generating area [4]. Swells are more of regular gravity waves with well developed long crests and relatively long periods. Thus, a fully developed sea is when waves stop growing since the growth wind so generated waves is not indefinite. As an example, in [5] the effect of a higher order nonlinear term on the shallow water finite amplitude wave was presented. Equations for the energy waves incorporating into formulation the higher order nonlinear terms as ratio

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of wave height to wavelength scale were obtained which will be useful in discussing wave amplitude approximation by Newton iterative formula on the deep water height for waves moving from deep ocean areas to coastal regions with nonlinearities present.

Transfer of wind energy to the waves is described with a resonance mechanism and a feedback mechanism. It follows that Energy is generated that moves in the form of waves when the wind blows across the water for a period of time. Wind growth consists of linear growth and exponential growth where the wind speed is always taken at $10meters$ elevation U_{10} above the sea level. The energy so impacted to the water by the moving wind is dissipated in the form of wave breaking. Dissipation as it relates to wave energy is the sum of three different contributions -: white capping, bottom friction and depth-induced breaking. White capping is controlled by the steepness of the waves. Waves have tendency of travelling several hundreds or thousands of kilometers without losing much of energy. In practice, when the ratio of wave height over water depth exceeds a certain limit, waves start to break, thereby dissipating energy rapidly. However, while this assertion holds, some energy may be lost or dissipated internally within the fluid in the form of interaction with the air above, by turbulence upon breaking, and percolation and friction with the Seabed. Short period components have tendencies of losing more energy easily than the long period components [6].

Since bottom friction causes energy dissipation, the time rate of energy density loss at wave number k is given by $S_{bf}(k) = -<\tau_0 U_{bk}>$.

Here, τ_0 is the bottom shear stress and U_{bk} is the orbital velocity of the wave component with wave number k.

Therefore, wave breaking [7], has the following attributes:

(i) controls energy dissipation and defines spectrum in the gravity range;

(ii) generates shorter (including parasitic) surface waves , and thereby influencing shape of spectrum in the high frequency range;

(iii) Enhances the form drag due to the airflow separations from breaking crests and momentum flux to short -waves generated by breaking crests.

It follows from reasoning adduced above that the periods of swell waves tend to be much longer than the seas. Theory of gravity capillary wave curvature spectrum can be found in [6].

It is of importance to know the elevation of the free surface ocean water, the speed of the free surface oscillations, the mean square displacement of the sea surface elevation and this is proportional to the mean potential energy of the elevation.

In oceanographic calculations, the free water surface elevation is equal to the typical wave amplitude at the sea ward edge of the shallow water. Thus, when the initial amplitude is zero, the oscillations vanish at infinity.

In shallow water, triad wave-wave interactions do transfer energy from lower frequencies to higher frequencies giving rise to higher harmonics.

The remaining sections in the paper have been categorized as follows:

Section 2 describes the Euler-Bernoulli equation for Ocean water waves. The velocity (celerity) of water wave is described for the shallow and deep water. The effects of a higher order nonlinear term on the shallow water wave with finite amplitude in the sense of [5] is also described which form the basis of theoretical approximation. Approximation of wave frequency using Newton iterative formula for the wave front approaching coastal region is presented. The gains in using Newton method for approximating the wave frequency is that, in the coastal regions, the flow of water is assumed rotational associated with regular water waves partnering with nonlinearity dominating. The mean depth varies with location with mean current co-habiting with waves.

Section 3 gives the group velocity for the progressive Ocean water waves. It is established that the amplitude modulation for the group energy is related to the Kinetic energy, and that the total energy with which the ocean water waves advance is a sum of both Kinetic and Potential Energy.

The effect of Reynolds number on the Ocean water wave expressed in terms of ratio of inertia resistance velocity to the velocity of the moving body water is presented in Section 4. It is shown that high Reynolds number implies inverse relationship between momentum of the fluid and prevailing viscosity such that fluid moving from state of laminar through intermediate to turbulence forms the bulk of discussion. We give equation in the sense of [8] for the Stoke’s drift induced by the wind stress on the Ocean water.

2. The Varying Pressure from the Ocean Waves.

Generally, it is the principle in water engineering mechanics that the pressure in the water is equal to the atmospheric pressure plus the hydrostatic pressure (weight of water above) and a dynamic part due to wave motion. However, we shall include the effects of temperature profiles and vertical wind velocity and wave spectrum for the wavelengths. This is governed by the wave breaking that influences energy losses for the gravity waves and energy gains for shorter surface waves and surface drag.

Firstly, we give the Euler-Bernoulli equation in the form:

$$\frac{p}{\rho} + \frac{\partial \phi}{\partial t} + \frac{1}{2}\left(u^2 + v^2\right) + gz = \frac{p_{atm}}{\rho} \qquad (2.1)$$

The terms p and ρ respectively, denotes the pressure and density whilst u, and v are respectively horizontal and vertical velocity components, that is, $u = \frac{\partial \phi}{\partial x}, v = \frac{\partial \phi}{\partial z}$.

For small amplitude, in the linearized equation, the term $\frac{1}{2}\left(u^2 + v^2\right)$, is reasonably small enough, hence negligible.

In its simplest form the surface pressure equation is given by

$$p(x,z,t) = -\rho \frac{\partial \phi}{\partial t} - \rho g z + p_{atm} \tag{2.2}$$

Whereas, the dynamic pressure for the regular small amplitude is in the form:

$$p(x,z,t) = -\rho \frac{\partial \phi}{\partial t}(x,z,t) = \frac{\rho a g \cosh(k(z+h))}{\cosh(kh)} \sin(\omega t - kx) \tag{2.3}$$

By writing that:

$p_d - \rho g z + p_\phi =$ Total pressure in the water,

Where,

$p_d =$ dynamic pressure,

$-\rho g z$ = hydrostatic pressure,

p_ϕ =atmospheric pressure,

One obtains the representation of potential function for the Laplace equation:

$$\frac{\partial^2 \phi}{\partial t^2} + \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0, -h \le z \le \eta, \tag{2.4}$$

with boundary condition:

$$\frac{\partial \phi}{\partial z}(x,z,-h,t) = 0, \tag{2.5}$$

$$\frac{\partial \eta}{\partial t} = \omega(x,z,0,t), \tag{2.6}$$

$$\frac{\partial \phi}{\partial t}(x,z,0,t) = -g\eta. \tag{2.7}$$

Following [9], the free water surface is in the form:

$$\eta(x,t) = \sum_{n=1}^{N} a_n \sin(\omega_n t - k_n x + \alpha_n) \tag{2.8}$$

$$\phi(x,y,z,t) = \sum_{n=1}^{N} \frac{a_n g}{\omega} \frac{\cosh(k_n(z+h))}{\cosh(k_n h)} \cos(\omega_n t - k_n x + \alpha_n) \tag{2.9}$$

Where,

$a_n =$ wave amplitude,

$k_n =$ wave number,

$\omega_n^2 = g k_n \tanh(k_n k)$,the wave speed in shallow water.

We therefore relate the wave celerity to wavelength and water depth h by the equation

$$C = \sqrt{\frac{g\lambda}{2\pi} \tanh\left(\frac{2\pi\lambda}{\lambda}\right)} \tag{2.10}$$

Note that the wave number $k = \frac{2\pi}{\lambda}$, and wave angular frequency $\omega = \frac{2\pi}{T}$ hold verbatim.

Practically in deep water, $\tanh(kh) \cong 1$, hence the wave celerity is in the form:

$$C_0 = \frac{gT}{2\pi} \tag{2.11}$$

However, in shallow water where the depth of water is $\frac{2\pi h}{\lambda} < \frac{1}{4}$ or $\frac{h}{\lambda} < \frac{1}{25}$, the wave celerity is only in magnitude of

$$C = \sqrt{gh}. \tag{2.12}$$

In discussing the effects of nonlinearities present in the work, it is assumed that the flow velocity $u(x,t)$ is uniform, then equation for the surface elevation and depression [5] are expressed as $z = \eta(x,t)$ from which one obtains in the form similar to equation (2.6)

$$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x} u((h_0 + \eta)) = 0 \tag{2.13}$$

Where,

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{\partial \eta}{\partial x} = 0 \tag{2.14}$$

Following [10] as in the sense of [5], $u(x,t)$ could be rewritten in the form:

$u(x,t) = 2\sqrt{g(h_0 + \eta)} \ - 2\sqrt{gh_0}$. Using this in equations (2.13) and (2.14) via Taylor series expansion, we see that

$$\frac{\partial \eta}{\partial t} + c_0 \left(1 + \frac{3}{2}\frac{\eta}{h_0} - \frac{3}{8}\frac{\eta^2}{h_0^2}\right)\frac{\partial \eta}{\partial x} = O\left(\frac{\eta^3}{h_0^3}\right) \tag{2.15}$$

To obtain a KDV equation [5] further initiated the form of equation

$$\frac{\partial\eta}{\partial t}+c_0\left(1+\frac{3}{2}\frac{\eta}{h_0}-\frac{3}{8}\frac{\eta^2}{h_0^2}\right)\frac{\partial\eta}{\partial x}=O\left(\frac{\eta^3}{h_0^3}\right)+\beta\frac{\partial^3\eta}{\partial x^3}=0, \quad (2.16)$$

with higher order nonlinearity effect, where from, $c_0=\sqrt{gh_0}$, $\beta=\frac{1}{6}c_0h_0$. Note that the dispersion effect of order $\left(\frac{h_0}{L_0}\right)^2$ was used in equation (2.15), where, $L_0\,(=\lambda)$ is the horizontal wavelength scale with $\frac{2\pi}{L_0}$ taken as the peak wave number.

To solve equation (2.16), the approach of change of variable was again initiated by [5] in the form $\varsigma=x-u_0t;\ u_0^2=g(h_0+\eta_0);\ \eta=\eta(\varsigma)$. This would yield the resulting expression for integration of equation(2.18) as given below

$$-u_0\eta+c_0\left(\eta+\frac{3}{4h_0}\eta^2-\frac{1}{8h_0^2}\eta^3\right)+\beta\frac{d^2\eta}{d^2\varsigma}=X_0 \quad (2.17)$$

Particularly, in the sense of [1] the expression for the pressure below the sea surface will now be explicitly derived as follows:
Using the equation $\frac{\partial\phi}{\partial t}+\frac{p}{\rho}+g\eta=0$, this gives

$p=-\rho gz-\rho\frac{\partial\phi}{\partial t}$. But $\eta=-\frac{1}{g}\left(\frac{\partial\phi}{\partial t}\right)_{z=0}$.

But it is known that for a water depth d meters

$$\phi=\frac{\pi d\cosh(k(h+z))}{kT\sinh(kh)}\sin(kx-\omega t) \quad (2.18)$$

Because the horizontal velocity component for the water wave is

$u=\frac{\partial\phi}{\partial x}=\frac{\pi d\cosh(k(h+z))}{T\sinh(kh)}\cos(kx-\omega t)$, we are now able to express the pressure equation in the form

$$p=-\rho gz+\frac{\rho g\eta\cosh(k(h+z))}{\cosh(kh)} \quad (2.19)$$

The term $\frac{\cosh(k(h+z))}{\cosh(kh)}$ appearing in equation (2.19) is the response factor.

Varying the values of h in equation (2.19) will lead to various effects of pressures on the ocean water wave.

In what follows in the presentation, the Newton iterative method (or any of its variants)[11]can be used to approximate the dispersion relation $\omega^2=gk\tanh(kh)$ in the calculation of wavelength in Coastal regions for any monochromatic irrotational and linear waves with constant water depth and non mean currents. It is given in the form:

$$\omega_{n+1}=\omega_n-\frac{f(\omega_n)}{f^{/}(\omega_n)},\quad (n=0,1,2,\ldots,.) \quad (2.18)$$

Here, in equation (2.14), ω_0 is an approximate starting value for ω, the wave number, and h is the depth of water. Let us note that more advanced formulae of Newton methods are to be found in [12, 13,14,and 15].The drawback here, is that under the Coastal regions, the flow of water is rotational that is associated with irregular waves pattern with nonlinearity dominating. The mean water depth also varies with location and with the existence of the prevailing mean currents co-habiting with waves.

3. The Group Energy Waves.

Plain wave means a wave with infinitely long crests(maxima) and constant elevation along lines orthogonal to the travel direction. From the equation of free surface elevation profile as discussed in section 2, for a progressive wave,the equation of free water surface

$$\eta(x,t)=a\sin(kx-\omega t), \quad (3.1)$$

whose progressive profile in the form can be found:

$$\phi(x,z,t)=\frac{ga}{\omega}\frac{\cosh k(z+h)}{\cosh kh}\cos(kx-\omega t) \quad (3.2)$$

We give the potential energy per unit wave length as

$$P/E=\frac{1}{2}\rho g\int_0^\lambda \eta^2dx \quad (3.3)$$

Because the $\|\eta\|\le|a|$, that is , the free water surface elevation is less than and equal to wave amplitude, we can conveniently write that

$$\eta(x,t)=\frac{1}{4}\rho ga^2\lambda, \quad (3.4)$$

for $\int_0^{\lambda} \sin^2(\frac{2\pi x}{\lambda})dx = \int_0^{\lambda} \cos^2\left(\frac{2\pi x}{\lambda}\right)dx = \frac{\lambda}{2}$.

Using standard method, the kinetic energy (K/E) per unit wavelength may be written as

$\frac{1}{2}\rho\int_0^{\lambda}\int_{-h}^{h}\left\{\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial z}\right)^2\right\}dxdz = -\frac{1}{2}\rho\int_s \phi\frac{\partial\phi}{\partial x}ds$,(cf vector field theory, e.g.).

That is

$$\text{K/E} = -\frac{1}{2}\rho g a^2\int_0^{\lambda}\cos^2(kx-\omega t)dx = \frac{g\rho a^2\lambda}{4} \quad (3.5)$$

The Total Energy is the sum of both Potential and Kinetic energies and is given by

$$\text{T/E}= \frac{g\rho a^2\lambda}{2} \quad (3.6)$$

Now further analysis on superimposition of two waves leads to writing surface elevation profile in the form:

$$\eta(x,t) = a\sin(kx-\omega t)a\sin(kx+\omega t).\ \sin\frac{1}{2}\{(k+k_1)x-(\omega+\omega_1)t\} \quad (3.7)$$

Thus for $k \approx k_1$, this pushes $(k-k_1)x$ to varying with x much more slowly than does $(k+k_1)x$.

The term $2a\cos\frac{1}{2}\{(k-k_1)x-(\omega-\omega_1)t\}$ is referred to as amplitude modulation.

Then equation (3.1) to a first order approximation would yield the change in pressure δp in the form:

$\delta p = -\rho v = -\rho\frac{\partial\phi}{\partial x}$ (Variable part in the pressure)

The rate at which the wave is doing work is given by

$$v\delta p = -\rho\left(\frac{\partial\phi}{\partial x}\right)^2. \quad (3.8)$$

Using standard techniques, it can be derived that, the work done in unit time or energy carried across unit width of a section is in the form:

$$W = \int_{-h}^{0}\delta p\frac{\partial\phi}{\partial x}dz = \rho\int_{-h}^{0}\left(\frac{\partial\phi}{\partial x}\right)^2 dz = \frac{g^2\rho a^2}{\omega}k\frac{\sin^2(kx-\omega t)}{\cosh^2 kh}\int_{-h}^{0}\cosh^2 k(z+h)dz \quad (3.9)$$

That is,

$$W = \frac{g^2\rho a^2 k}{\omega}\frac{\sin^2(kx-\omega t)}{\cosh^2 kh}\left(\frac{\sinh 2kh}{4k}-\frac{h}{2}\right) \quad (3.10)$$

where,

$$\omega^2 = gk\tanh kh, \Rightarrow k = \frac{\omega^2}{g\tanh kh} \quad (3.11)$$

Therefore, equation (3.10) is a standard formula in Water Engineering mechanics that the work done across a section by the wave train is

$$\bar{W} = \frac{1}{4}\frac{g\rho a^2\omega}{k}(1+2kh\,\mathrm{cosech}\,2kh) = \frac{1}{2}g\rho a^2 U \cdot \quad (3.13)$$

The energy is transmitted at a speed equal to group velocity. In deep water, the fluid particles move in circles with constant speed. Its radius of the circle at the surface is equal to the amplitude of the wave. Thus the water particle makes one complete turn per wave period. The importance of the above presentation on Water wave train energy stems from the below discussions.

As is expected, the amount of energy in wave thus depends on its height and wavelength as well as the distance over which it breaks [4]. But we also should take note of the variance density spectrum being the Fourier transform of the auto-covariance function of the sea surface elevation

$$\hat{E}(f) = \int_{-\infty}^{\infty} C(r)e^{--2\pi i f r}d\tau \quad (3.14)$$

Where,

$C(r) = \langle\eta(t)\eta(t+\tau)\rangle$ is the auto-covariance function, $\eta(t)$ and $\eta(t+\tau)$ are the two random processes of sea surface elevation for τ being time lag. We can obtain the moment energy density spectrum from equation (3.14) defined as

$E(f) = \int_0^{2\pi}E(f,\theta)d\theta$, for where the energy is transmitted per unit unit meter in the direction θ.

It should be noted that, the surface waves do not only affect the momentum flux, heat fluxes, they also act on the mass fluxes, the side of the atmosphere. In deep water such as the Ocean water, surface waves actually do affect kinetic fluxes through process of breaking waves, upper Ocean mixing and momentum flux [7] [4], [6].Therefore, in times of high wind speeds, the drag coefficient decreases with the wind speed. Surface waves do not only affect the bottom atmospheric layer but their effects on the upper-Ocean mixing are important as well. In general, as a follow up in our presentation, we give remarks on different situations in which the surface water waves do affect the upper –Ocean mixing. These are: Wave breaking, Coriolis –Stokes force and Langmuir circulation models. During breaking waves, the lost energy flux are transferred mainly to near – surface turbulence with the turbulent kinetic energy at the boundary.

4. Synchronalization with the Reynolds Number.

The Reynolds number is a measure of ratio of inertial forces to the viscous forces. It measures how turbulent the flow is in the sea or Ocean water. Low Reynolds number flows are Laminar, while higher Reynolds number flows are turbulent**.** The Mach number is the ratio of the fluid velocity U, to the speed of sound in that fluid, c. Thus, Mach number, measures the flow compressibility mostly experienced by a kinematic object moving through the air, cloud or water. A notable example is the Airplane moving through a medium in the air under gravitational force. However, we shall not go into details on this here.

Following existing facts from literature, it holds that

$$\mathrm{Re} = \frac{inertial\ resistance}{viscous\ resistance} = \frac{\rho v \ell}{\upsilon} \qquad \textbf{(4.0)}$$

Where ,

$\mathrm{Re} =$ Reynolds number;

$\rho =$ density of the fluid;

$v =$ relative speed of the fluid;

$\ell =$ characteristic length of the system;

$\upsilon =$ dynamic viscosity of the fluid.

The Mach number therefore, is the ratio of velocity of the kinematic object in the fluid to the speed of sound wave in the fluid. This must not be confused with the Bond number, which is the ratio of influence of gravity on the surface tension in the formation of a wave bubble or droplet to surface temperature. It is given by the equation

$$B_0 = \frac{\rho g D^2}{\sigma}$$

Where,

$B_0 =$ **bond number,**

$\rho =$ density of fluid,

$g =$ acceleration due to gravity,

$D =$ bubble diameter,

$\sigma =$ surface temperature.

Following Babanin (2006), for the amplitude a -based Reynolds number we write that

$$\mathrm{Re} = \frac{aV}{\upsilon} = \frac{a^2 \omega}{\upsilon} \qquad (4.1)$$

Where, υ is the kinematic viscosity of the Ocean water.

Recalling that the Ocean water wave speed $\omega^2 = gk$, then we rewrite that

$$\mathrm{Re} = \frac{a^2 \sqrt{gk}}{\upsilon} = \sqrt{2g\pi}\, \frac{a^2}{\upsilon\sqrt{\lambda}}.$$

The Reynolds number vanishes with a given wavelength [16], as a function of depth

$$\mathrm{Re} = \frac{\omega}{v} a_0^2 \exp(-2kz) = \frac{\omega}{v} a_0^2 \exp\left(-2\frac{\omega^2}{g} z\right) \qquad (4.2)$$

Where,

$$\mathrm{Re}_\lambda(z) \approx a(z)^2 \approx \exp(-2kz). \qquad (4.3)$$

The effect of Reynolds number vanishes at the surface as $z \to 0$ when $\exp(-2kz) \to 1$.

Therefore, the Reynolds number at a given wavelength $\lambda(\omega)$ with location z is calculated in the form:

$$z_{cr} = -\frac{1}{2k} \ln\left(\frac{\mathrm{Re}_{cr}\, v}{a_0^2}\right) = \frac{g}{2\omega^2} \ln\left(\frac{a_0^2 \omega}{\mathrm{Re}_{cr}\, v}\right) \qquad (4.4)$$

Therefore, high Reynolds number implies inverse relationship existing between momentum of the fluid and the prevailing viscosity with the result of such flow being unsteady, churning, roiling, and turbulent.

For the irrotational flow for the velocity potential, the vorticity $\nabla \times V = 0$ has every point under the wave given by

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial z}.$$

Its non-dimensional vorticity is

$$\varpi = \frac{\omega}{2ak\delta} \qquad (4.5)$$

For small amplitude, the solution for the velocity in water is

$$\omega_{an} = -2ak\delta e^{\left\{\left(\frac{\delta}{2\nu}\right)^{\frac{1}{2}} h - \frac{2\delta}{R_\omega} t\right\}} e^{i\left\{kx + \left(\frac{g}{2\nu}\right)^{\frac{1}{2}} h + \delta t\right\}} \quad (4.6)$$

Where, ν is the kinematic velocity of the fluid, h is the depth of water and R_ω is the wave Reynolds number expressed as $R_\omega = \frac{\delta}{\nu k^2}$.

As is well known in water mechanics, the vorticity dampens to zero exponentially with oscillatory behavior as water depth increases. However due to the facts earlier stated above, the inviscid rotational Gerstner wave in the sense of Lamb in 1930 as reported in [17]was obtained in the form:

$$\omega = -\frac{2a^2k^2\delta e^{2kz_0}}{1 - a^2k^2e^{2kz_0}} \quad (4.7)$$

where defined that:

$$x = \frac{\theta}{k} + ae^{kz_0}\sin\theta; \quad (4.8)$$

$$h = h_0 - ae^{kz_0}\cos\theta$$

The point here is that, vorticity is damped in the Gerstner wave whereas, it does not produce oscillations.
Similarly, the Froude number which is the ratio of inertial forces to the gravitational forces is given by the equation

$$Fr = \frac{\nu}{\sqrt{g\ell}}. \quad (4.9)$$

Using the above preambles, we give the influence of breaking waves on energy flux losses by the equation of wave on the turbulence kinetic energy (TKE) in the sense of [18] in the form:

$$q_{wb,o} = m_0\rho_w u_{*w}^3 \quad (4.10)$$

For u_{*w} being the friction velocity in water and, m_0 a certain coefficient which may be taken as 100,e.g.,. The breaking wave induced stress that is transferred from the surface wave breaking to the Ocean currents [18] is actually defined to be the quantity

$$\int_{-H}^{z} \frac{A(z)dz}{\eta\rho u_*^2} \approx e^{bz}, \quad (4.11)$$

Where, b is a coefficient depending on the wind, A is a momentum density and η is the ratio of breaking stress to the wind stress.
The Stokes drift from $2D$ wave spectrum is calculated by the equation

$$u_s = \frac{16\pi^3}{g}\int_0^\infty\int_{-\pi}^{\pi} \omega^3 s_{\omega\theta}(\omega,\theta) e^{\left(\frac{8\pi^2\omega^2 z}{g}\right)} d\theta d\omega \quad (4.12)$$

Where,
$\omega =$ wave frequency,
$\theta =$ wave direction,
$s_{\omega\theta} =$ directional frequency spectrum.
The decay rate for finite wave amplitude water waves by the linearized Navier Stokes equation is expressed in the form:

$$\frac{\partial a}{\partial t} = -2\nu k^2 a \quad (4.13)$$

With decaying solution $a \approx e^{-2\nu k^2 t}$, where $a =$ wave amplitude, $\nu =$ kinematic viscosity of the fluid and, $k =$ the wave number. The Level set equation for the free surface is

$$\frac{\partial\phi}{\partial t} + U|\nabla\phi| = 0, \quad (4.14)$$

For, $U =$ speed of the interface in the outward normal direction, is the sum of interface propagation speed. That is, the speed due to curvature and velocity normal to the interface, i.e., $U.\hat{n}$, where, $\hat{n} = \frac{\nabla\phi}{|\nabla\phi|}$ is the unit normal to the interface.

The equation for calculating wind stress is given by

$$\tau = \rho_a c_d U_{10}^2 \quad (4.15)$$

Where,
$\rho_a =$ air density,
$U_{10} =$ wind speed at $10 - m$ above sea surface.
From this, we give the wave drag coefficient under neutral condition as

$$c_{d_N} = \left(\frac{\varsigma}{\ln\left(\frac{10}{z_0}\right)}\right) \quad (4.17)$$

z_0 is the sea surface roughness length , ς is 0.4 being the von Karman constant. We recall that under very high wind speeds, the drag coefficient levels off and the turbulence in the bottom atmospheric layer may be altered via the buoyancy force under sea spray influence. But it must be noted [19, 20,3,21] that τ depends on three variables namely, density of the air ρ_a, the drag coefficient, C_D, and the squared wind speed U_{10}^2. The U_{10} is so named because, it is measured at 10 meters above the air –sea level as a standard rule. The C_D thus depends on dynamic stability at the air-sea interface. This means air-sea temperature difference. It also depends on relative humidity at the air-sea interface. The C_D also depends on sea surface current speed ocean wave speed.

As a notation, the wind speed ismeasured at $10m$ above sea level in the form:

$$\vec{V} - \vec{V}C - \vec{V}W \qquad (4.18)$$

In equation (4.18), $\vec{V}$ = wind speed at $10m; VC$ = surface current speed; VW = primary wave speed.

5. Conclusion

The paper presented the features of Ocean water waves to include the wind waves, the near surface turbulence and the upper Ocean mixed layer. The effects of varying pressure on the surface Ocean water waves was discussed and related this by extension to the effect of Reynolds number when the water wave progresses from laminar through intermediate to turbulence due to wind generated wave from a fetch. We also give the group energy for the wave transform and work done by the wave energy. As a demonstration, it is showed that Reynolds number is very important in the treatments of wind stress equations and related this to Stokes drift in $2D$ in the sense of [8].We hope to present numerical examples in future paper.

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