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A NONLINEAR ANALYSIS OF THE DEPLETION OF A FERTILE TOPSOIL DUE TO INCREASING VEGETATION COVER AND RAINFALL PRESSURE

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Abstract

This paper describes dual impact of a continues mineral uptake by a growing vegetation biomass and an increasing rainfall stress on a fertile topsoil. The study identifies four mutually existing but interdependent interacting ecological components comprising of, in the first category, vapour, cloud droplets and raindrops phases, the humid phase (hot air), the fertile topsoil and the component of vegetation biomass. It is further assumed that while the density of vegetation cover exhibits a logistic type growth, the other phases undergo ecological type growth and nonlinear interactions. The stability theory of differential equations and numerical simulation were used to establish the stability behaviours of the proposed model. It is shown that the depletion of fertile topsoil could be due to the continues mineral uptake by vegetation cover and the increasing stress of rain drop

1. Introduction

The process necessitating the gradual change, overtime, in the value of a resource, in the present study – soil, is basically referred to as degradation. Land degradation, therefore, refers to a temporary or permanent decline in the productive capacity of the land, or a falloff from its potential for environmental management. Land resources broadly includes soil, landscape, vegetation, water and microclimatic components of an ecosystem[1].Soil is basically a non-renewable resource that is very prone to degradation because it is intrinsically and indispensably involved with all of life's processes and procedures [2]. Major soil degradation processes include water (raindrop) and wind erosion, depletion in the pool of the soil organic carbon (SOC), loss in biodiversity, loss of soil fertility and elemental imbalance, acidification and salinization [1, 2, 3]. The four basic types of soil degradation and their extents and effect can be looked up in [2]. The consequences of soil erosion are widespread and dire [1 – 7]. The direst economic consequence of degradation is its attendant devastation of crop cultivation intensive [2, 3].Some types of land degradation, for example severe gullying and advanced salinization (known to severely compromise the long-term biological and environmental potential of the land [5]), displacement of soil material (erosion),are irreversible [1]. The economic importance of wind and rain in accelerating the processes of soil erosion can be looked up in [8, 9, 10]. Water, for instance was compared in [9] to be about 800 times heavier than air and with about the same weight as lose topsoil. Making it easier to displace loose substances. It was further figured out that the energy of moving water is bound to increase proportionally as the mass and speed of droplets grow in size therefore, invariably making, especially, larger raindrops or heavy rain one of the important natural factors that affects the fertility of soil making it less or non-productive. Green plants (trees, grasses and other vegetative cover) soil interaction could also exert unsustainable SOC, nutrient and moisture content depletion is exacerbating degradation [1, 2]. Most types of soil degradation, however, can be prevented or reversed by adding nutrients to nutrient-depleted soil, rebuilding topsoil through soil amendments, re-establishing vegetation, or buffering soil acidity [1, 2, 3]. Relative to their natural conditions, land resources exist in one of three states – productive, undergoing degradation or degraded. About half of the estimated global cost effect of soil erosion is argued to be borne by developing countries [11].The practicality or otherwise of rehabilitating or managing degraded soil depends on the costs relative to the value of output or environmental benefits expected [1]. A considerable number of studies exist on resource depletion [8 – 10, 12 – 19].

2. Mathematical model

Our model considers the following six interacting components. The first three state variables of the model system (1), a vapour, cloud droplets and raindrops phases, which are assumed to follow a consecutive natural forming process, exhibit a mutual existence in the atmospheric space of the region being considered. Here, the natural formation process of the vapour phase is assumed to be constant, at a rate Q_v, as well as through augmentation by vaporisation of humidity and liquid on green vegetation

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through evaporation rate, ν, and transpiration rate α. It is assumed to be depleted by natural factors at a rate ϕ leading to the growth of cloud droplets, which is also considered to undergo natural depletion at a rate ψ, with an assumed fraction, π_1, aiding the formation of rain drops which is further modified by a raindrops formation proportionality rate, π_2. The density of raindrops is assumed to deplete naturally at a rate γ_0 as well as interaction by vegetation and soil surface at rates γ_1 and γ_2 respectively. Further, it is assumed that atmospheric air could be heated up naturally by sunlight and other natural factors at a constant rate Q_A. Again, it is assumed that this heated air is either naturally lost at some rate α_0, through cooling, or trapped in the atmosphere to form vapour at a α. The mass of fertile topsoil is considered to have a natural growth rate Q_S and gets depleted due to natural factors, like SOC loss, at a constant rate η_0 or by η_1 due to stress of heavy rain on the soil surface proportional to *R*. A fraction, θ, of the lost SOC is assumed to be available to the density of vegetation biomass for absorption. However, only to a fraction, μ, is actually absorbed as a part, δ, is assumed to naturally get washed away. Finally, it is assumed that the density of vegetation biomass grows intrinsically at a rate λ_0 and gets depleted through intraspecific competition at a rate λ_1, it is assumed that the growth in fertility of the mass of fertile topsoil could be augmented at a proportionality rate k.

Following from the foregoing, the dynamics of the depletion of the fertility of fertile topsoil is governed by the following system of nonlinear differential equations.

$$\begin{aligned}
\frac{dV}{dt} &= Q_V - \phi V + \upsilon\alpha AR + \nu\gamma_0 AG,\\
\frac{dC}{dt} &= \pi_1\phi V - \psi C,\\
\frac{dR}{dt} &= \pi_2\psi C - \gamma_0 R - \gamma_1 GR - \gamma_2 RS,\\
\frac{dA}{dt} &= Q_A - \alpha_0 A - \alpha AR,\\
\frac{dS}{dt} &= Q_S + k\mu GU - \eta_0 S - \eta_1 RS,\\
\frac{dU}{dt} &= \theta\eta_0 S - \delta U - \mu GU,\\
\frac{dG}{dt} &= \lambda_0 - \frac{\lambda_1}{K}G^2.
\end{aligned} \tag{1}$$

where $V(0)\geq 0, C(0)\geq 0, R(0)\geq 0, A(0)\geq 0, S(0)\geq 0, U(0)\geq 0, G(0)\geq 0.$

3. The region of attraction

The region of attraction is stated below

Lemma 1. *The set,* Ω, *stated below, where* $q_v = \min\{\gamma_0, \phi(1-\pi_1), \psi(1-\pi_2)\}$ *and* $q_s = \min\{\delta, \eta_0(1-\theta)\}$, *is the region of attraction for the model system (1) and attracts all solutions initiating in the positive octant.*

$$\Omega = \left\{(V,C,R,A,S,U,G): 0 \leq V + C + R \leq \frac{Q_V}{q_v}; 0 \leq A \leq \frac{A_A}{\alpha_0}; 0 \leq S + U \leq \frac{Q_S}{q_s}; 0 \leq G \leq \frac{\lambda_0 K}{\lambda_1}\right\}.$$

Proof

Applying the comparison theorem in [20] on model system (1), we obtain from the fourth and last equations that

$\frac{dA}{dt} \leq Q_A - \alpha_0 A$ and $0 \leq G \leq K.$

Thus, from the first, second and third equations, we obtain

$$\begin{aligned}
\frac{dV}{dt} + \frac{dC}{dt} + \frac{dR}{dt} &\leq Q_V + \nu\gamma_0 K\frac{Q_A}{\alpha_0} - \phi(1-\pi_1)V - \psi(1-\pi_2)C - \gamma_0 R + \upsilon\alpha\frac{Q_A}{\alpha_0}R\\
&\leq \frac{\alpha_0 Q_V + \nu\gamma_0 K Q_A}{\alpha_0} - q_V(V + C + R),
\end{aligned}$$

Therefore, $0 \leq V + C + R \leq \frac{\alpha_0 Q_V + \nu\gamma_0 K Q_A}{q_V \alpha_0}$, where $q_v = \min\{\gamma_0, \phi(1-\pi_1), \psi(1-\pi_2)\}$.

Similarly, from the fifth and sixth equations, we can obtain

$$\begin{aligned}
\frac{dS}{dt} + \frac{dU}{dt} &\leq Q_S - \eta_0(1-\theta)S - \delta U - \mu(1-k)KU\\
&\leq Q_S - \eta_0(1-\theta)S - \delta U.
\end{aligned}$$

This gives

$0 \le \frac{dS}{dt} + \frac{dU}{dt} \le Q_S - q_S(S+U),$
where $q_s = \min\{\delta, \eta_0(1-\theta)\}.$
Hence, the lemma follows.

4. Equilibrium analysis

The point $E^*(V^*, C^*, R^*, A^*, S^*, U^*, G^*)$, where $V^*, C^*, R^*, A^*, S^*, U^*$ and G^* are positive solutions of the algebraic equations (2) below, is a nonnegative equilibrium of the model system (1).

$$\begin{aligned}
&\lambda_0 K - \lambda_1 G = 0,\\
&Q_A - \alpha_0 A - \alpha AR = 0,\\
&V = \frac{\lambda_1 Q_V(\alpha_0 + \alpha R) + Q_A(\upsilon\alpha\lambda_1 R + \nu\pi\lambda_0 K)}{\phi\lambda_1(\alpha_0 + \alpha R)} = f(R),\\
&\pi_1\phi f(R) - \psi C = 0,\\
&\phi\pi_1\pi_2\lambda_1 f(R) - \gamma_0\lambda_1 R - \gamma_1\lambda_0 KR - \gamma_2\lambda_1 RS = 0,\\
&\lambda_1 Q_S + \mu k\lambda_0 KU - \lambda_1(\eta_0 + \eta_1 R)S = 0,\\
&\theta\eta_0 S - \delta U - \mu GU.
\end{aligned} \tag{2}$$

To establish the existence of the equilibrium E^* we proceed as follows.
It can be noted from equation (2) that

$$\phi\pi_1\pi_2\lambda_1 f(R) = [\gamma_1\lambda_0 K + \lambda_1(\gamma_0 + \gamma_2 S)]R \tag{3}$$

and

$$Q_S(\delta\lambda_1 + \mu\lambda_0 K) + [k\mu\eta_0\theta\lambda_0 - (\delta\lambda_1 + \mu\lambda_0 K)(\eta_0 + \eta_1 R)]S = 0. \tag{4}$$

We further note from equation (3) that

a. For $S = 0,$ one positive value of $R,$ say $\overline{R},$ then will be obtained as a root of the quadratic equation $\alpha(\gamma_0\lambda_1 + \gamma_1\lambda_0 K)R^2 + a_1 R - \pi_1\pi_2(\alpha_0\lambda_1 Q_V + \nu\pi\lambda_0 Q_A K) = 0,$ where $a_1 = \alpha_0(\gamma_0\lambda_1 + \gamma_1\lambda_0 K) - \alpha\pi_1\pi_2(Q_V + \upsilon Q_A).$ In particular, $\overline{R} < Q_V/\gamma_0$.

b. As $S \to \infty, R$ attains a constant value

c. $\frac{dS}{dR} = \frac{\pi_1\pi_2[\alpha Q_A R(\upsilon\alpha_0\lambda_1 - \nu\pi\lambda_0 K) - \phi\lambda_1(\alpha_0 + \alpha R)^2 f(R)]}{\gamma_2\lambda_1 R^2(\alpha_0 + \alpha R)^2} < 0,$ in the first quadrant provided that $\upsilon\nu\pi\lambda_0 RK + \phi\lambda_1(\alpha_0 + \alpha R)^2 f(R) > \upsilon\alpha\alpha_0\lambda_1 Q_A R.$

We equally note from (4) that

a. For $R = 0,$ then $S = \frac{Q_S(\delta\lambda_1 + \mu\lambda_0 K)}{\eta_0(\delta\lambda_1 + \mu\lambda_0 K) - \mu k\eta_0\theta\lambda_0\lambda_1 K} > 0,$ is ensuing positive value of S provided that $\delta\lambda_1 + \mu\lambda_0 K(1 - k\theta) > 0.$

b. As $R \to \infty, S$ attains a constant value

c. $\frac{dR}{dS} = -\frac{Q_S}{\eta_1 S^2} < 0.$

Thus, using the values of G^*, R^* and S^* we obtain the values corresponding to V^*, C^*, A^* and U^* from (2)

5. Stability analysis

In this section, we analyse the stability behaviour of the nonnegative equilibrium $E^*(V^*, C^*, R^*, A^*, S^*, U^*, G^*)$, where V^*, C^*, R^*, A^*, S^*, U^* and G^* are positive solutions of the algebraic equations (2). To study the stability behaviour of the equilibrium, we propose the following theorems.

Theorem 1: *The equilibrium E^* if it exists, is locally asymptotically stable inside the region of attraction Ω.*

Proof

To prove the LAS of E^*, it is sufficient to obtain the variational matrix of the model system (1). The variational matrix of the model system (1) is given as

$$J = \begin{pmatrix}
-\phi V^* & 0 & \upsilon\alpha A^* & \upsilon\alpha R^* + \nu\pi G^* & 0 & 0 & \nu\pi A^*\\
\pi_1\phi & -\psi & 0 & 0 & 0 & 0 & 0\\
0 & \pi_2\psi & -(\gamma_0 + \gamma_1 G^* + \gamma_2 S^*) & 0 & -\gamma_2 R^* & 0 & -\gamma_1 R^*\\
0 & 0 & -\alpha A^* & -\alpha_0 - \alpha R^* & -\alpha A^* & 0 & 0\\
0 & 0 & -\eta_1 S^* & 0 & -\eta_0 - \eta_1 R^* & \mu k G^* & \mu k U^*\\
0 & 0 & 0 & 0 & \eta_0\theta & -\delta - \mu G^* & -\mu U^*\\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{\lambda_1}{K}G^*
\end{pmatrix},$$

It can easily be verified that all the eigenvalues of the matrix above are negative. Thus, the equilibrium point E^* is locally asymptotically stable.

Theorem 2:*Let the following inequalities hold*

$$\frac{\alpha^2(\eta_0+\eta_1 R^*)}{\gamma_0+\gamma_1 G^*+\gamma_2 S^*}\max\left\{5\upsilon^2, \frac{15}{\eta_0+\eta_1 R^*}, \frac{3\eta_1 S^* A^*(\gamma_0+\gamma_1 G^*+\gamma_2 S^*)}{\gamma_2 R^*(\alpha_0+\alpha R^*)}\right\} < d_3 < \min\left\{\frac{\delta+\mu G^*}{2\mu}, \frac{\eta_1 G^* S^*(\eta_0+\eta_1 R^*)}{4k}, \frac{\gamma_0+\gamma_1 G^*+\gamma_2 S^*}{5\pi_1^2\pi_2^2}, \frac{\gamma_2 k G^*(\mu U)^2(\gamma_0+\gamma_1 G^*+\gamma_2 S^*)}{5\gamma_1^2 R^2}\right\} \quad (5)$$

$$\max\left\{\left(\frac{A^*}{V^*}\right)^2, \frac{20\gamma_2\eta_1 S^* R^*}{\gamma_0+\gamma_1 G^*+\gamma_2 S^*}, \frac{2\mu k\theta\eta_0 G^*}{\delta+\mu G^*}\right\} < \eta_0+\eta_1 R^* < \min\left\{\frac{1}{3}\left(\frac{\alpha_0+\alpha R^*}{\upsilon\alpha R^*+\nu\pi G^*}\right)^2 \gamma_2 k G^* R^*\left(\frac{\mu U^*}{2\nu\pi}\right)\right. \quad (6)$$

then the equilibrium E^* *is nonlinearly stable inside the region of attraction* Ω.

Proof

To establish the theorem, we consider the following positive definite function about E^*

$$F = \frac{1}{2}\left(d_1V_1^2 + d_2C_1^2 + d_3R_1^2 + d_4A_1^2 + d_5S_1^2 + d_6U_1^2 + d_7G_1^2\right), \quad (7)$$

where $V_1, C_1, R_1, A_1, S_1, U_1$ and G_1 are small perturbations about E^* as follows

$V = V_1 + V^*, C = C_1 + C^*, R = R_1 + R^*, A = A_1 + A^*, S = S_1 + S^*, U = U_1 + U^*$ and $G = G_1 + G^*$.

Differentiating F with respect to t around the linearised system of the model system (1), gives

$$\begin{aligned}\frac{dF}{dt} &= -d_1\phi V^* V_1^2 - d_2\psi C_1^2 - d_3(\gamma_0+\gamma_1 G^*+\gamma_2 S^*)R_1^2 - d_4(\alpha_0+\alpha R^*)A_1^2 - d_5(\eta_0+\eta_1 R^*)\\
&- d_6(\delta+\mu G^*)U_1^2 - \frac{d_7\lambda_1}{K}G^* G_1^2 + d_1\upsilon\alpha A^* R_1 V_1 + d_1\nu\pi G_1 V_1 + d_2\phi\pi_1 C_1 V_1 + d_6(\delta+\mu G^*)A_1 V\\
&+ d_3\pi_2\psi C_1 R_1 - d_3\gamma_1 R^* G_1 R_1 - d_4\alpha A^* A_1 R_1 - d_4\alpha A^* A_1 S_1 - (d_3\gamma_2 R^* + d_5\eta_1 S^*)R_1 S_1\\
&+ (d_5\mu k G^* + d_6\eta_0\theta)S_1 U_1 + d_5\mu k U^* G_1 S_1 - d_6\mu U^* G_1 U_1.\end{aligned}$$

Then dF/dt will be negative definite subject to the following inequalities

$$\begin{aligned}
&d_1(\upsilon\alpha A^*)^2 < \frac{1}{5}d_3\phi V^*(\gamma_0+\gamma_1 G^*+\gamma_2 S^*),\ d_1(\upsilon\alpha R^*+\nu\pi G^*)^2 < \frac{1}{3}d_4\phi V^*(\alpha_0+\alpha R^*),\\
&d_1(\nu\pi A^*)^2 < \frac{1}{4}d_7\frac{\phi\lambda_1 V^* G^*}{K},\ d_2\phi\pi_1^2 < \frac{1}{2}d_1\psi V^*,\ d_3\psi\pi_2^2 < \frac{2}{5}d_2(\gamma_0+\gamma_1 G^*+\gamma_2 S^*),\\
&d_3(\gamma_1 R^*)^2 < \frac{d_7}{5}\frac{\lambda_1 G^*}{K}(\gamma_0+\gamma_1 G^*+\gamma_2 S^*),\ (\alpha A^*)^2 < \frac{d_5}{3}(\alpha_0+\alpha R^*)(\eta_0+\eta_1 R^*),\\
&(d_3\gamma_2 R^* + d_5\eta_1 S^*)^2 < \frac{d_3 d_5}{5}(\eta_0+\eta_1 R^*)(\gamma_0+\gamma_1 G^*+\gamma_2 S^*),\ d_6(\mu U)^2 < \frac{d_7}{2}\frac{\lambda_1}{K}G^*(\delta+\mu G^*),\\
&d_4(\alpha A^*)^2 < \frac{d_3}{15}(\alpha_0+\alpha R^*)(\gamma_0+\gamma_1 G^*+\gamma_2 S^*),\ d_6(\mu U^*)^2 < \frac{d_7}{2}\frac{\lambda_1}{K}G^*(\delta+\mu G^*),\\
&(d_5\mu k G^* + d_6\eta_0\theta) < \frac{d_5 d_6}{2}(\eta_0+\eta_1 R^*)(\delta+\mu G^*),\ d_5(\mu k U^*)^2 < \frac{d_7}{4}\frac{\lambda_1}{K}G^*(\eta_0+\eta_1 R^*)
\end{aligned} \quad (8)$$

On choosing $d_1 = \frac{\phi V^*(\eta_0+\eta_1 R^*)}{A^{*2}}, d_2 = \frac{\psi}{2\pi_1^2}, d_4 = \frac{\alpha_0+\alpha R^*}{A^{*2}}, d_5 = \frac{\gamma_2 R^*}{\eta_1 S^*}d_3, d_6 = \frac{\mu k G^*}{\theta\eta_0}d_5, d_7 = \frac{\gamma_2 k K R^*(\mu U^*)^2}{\lambda_1}$,

the inequalities (8) above reduce to

$$\frac{\alpha^2(\eta_0+\eta_1 R^*)}{\gamma_0+\gamma_1 G^*+\gamma_2 S^*}\max\left\{5\upsilon^2, \frac{15}{\eta_0+\eta_1 R^*}, \frac{3\eta_1 S^* A^{*2}(\gamma_0+\gamma_1 G^*+\gamma_2 S^*)}{\gamma_2 R^*(\alpha_0+\alpha R^*)}\right\} < d_3 < \min\left\{\frac{\delta+\mu G^*}{2\mu}, \frac{\eta_1 G^* S^*(\eta_0+\eta_1 R^*)}{4k}, \frac{\gamma_0+\gamma_1 G^*+\gamma_2 S^*}{5\pi_1^2\pi_2^2}, \frac{\gamma_2 k G^*(\mu U^*)^2(\gamma_0+\gamma_1 G^*+\gamma_2 S^*)}{5\gamma_1^2 R^*}\right\}$$

and

$$\max\left\{\left(\frac{A^*}{V^*}\right)^2, \frac{20\gamma_2\eta_1 S^* R^*}{\gamma_0+\gamma_1 G^*+\gamma_2 S^*}, \frac{2\mu k\theta\eta_0 G^*}{\delta+\mu G^*}\right\} < \eta_0+\eta_1 R^* < \min\left\{\frac{1}{3}\left(\frac{\alpha_0+\alpha R^*}{\alpha\upsilon R^*+\nu\pi G^*}\right)^2, \gamma_2 k G^* R^*\left(\frac{\mu U^*}{2\nu\pi}\right)^2\right\}$$

Hence, dF/dt will be negative definite showing that F is a Lyapunov function, thus proving the theorem.

6. Numerical simulation and discussion

We subject our model to numerical simulation in order to validate the feasibility of our analytical analysis regarding the existence and stability behaviour of E^*. To do this, we choose the following parameter values:

$Q_V=10,\ \phi=2.8,\ \pi=0.6,\ \pi_1=0.03,\ \pi_2=0.002,\ \nu=0.15,\ \upsilon=0.3,\ \alpha=0.75,\ \psi=0.65,\ \gamma_0=0.001,$
$\gamma_1=0.01,\ \gamma_2=0.2,\ Q_A=3.0,\ \alpha_0=0.235,\ Q_S=1.8,\ k=0.4,\ \mu=0.02,\ \eta_0=0.85,\ \eta_1=0.6,\ \theta=0.04,$
$\delta=0.025,\ \lambda_0=0.8,\ \lambda_1=0.65,\ K=100.$

from these values we calculate the equilibrium E^* as

$V^{*,}=6.2462,\ C^*=8.5040,\ R^*=1.9790,\ A^*=2.3170,\ S^*=2.4157,\ U^*=8.6302,\ G^*=10.6000.$

It can further be confirmed that the eigenvalues corresponding to E^*, as obtained from the corresponding Jacobian matrix, are $-17.48936,\ -0.65000,\ -0.59019,\ -1.97275,\ -2.03740,\ -0.23700$ and -0.06890.

This further confirms that E^* is locally asymptotically stable. Furthermore, in the figures 1 – 6, the fertility variation of the mass of fertile topsoil with time is shown for some baseline parameters. The enriching effect of rain on the sustenance of soil fertility of may be noted from figures 1, 2, 3 and 6. It may be observed from these figures that the onset of the rains would suggestively result in the depletion of soil fertility (Fig. 5), probably due to the expected increase in the density of vegetation biomass (Figs. 1). This phenomenon is observed to be short-lived as the trajectory suggests

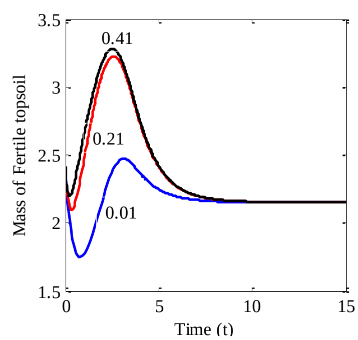

Figure 1. Variation of $S(t)$ with time for varying values of γ_1

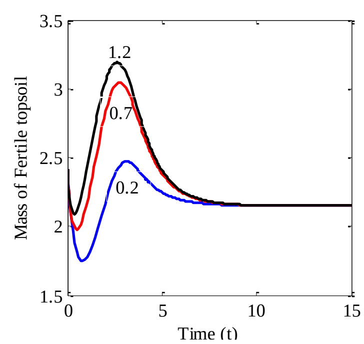

Figure 2. Variation of $S(t)$ with time for varying values of γ_1

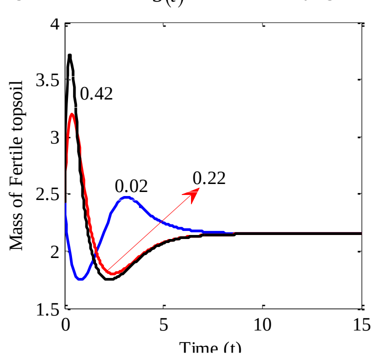

Figure 3. Variation of $S(t)$ with time for varying values of μ

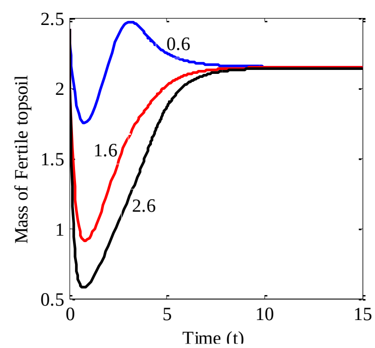

Figure 4. Variation of $S(t)$ with time for varying values of η_1

that the depth of fertile topsoil increases with increase in water uptake by both vegetation biomass and the soil itself. The tendency of the depth of fertile topsoil to deplete may be noted from Figs. 3 and 5.It may be a confirmation that the continues uptake of SOC by vegetation cover can reduce soil fertility (figure 3), the increasing impact of rain drops on the soil surface is inferred by Fig. 4. To have depleting consequence on topsoil (probably due to runoff).

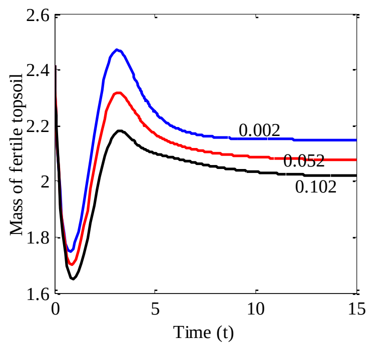

Figure 5. Variation of $S(t)$ with time for varying values of π_2

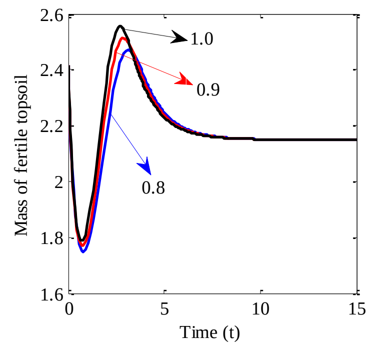

Figure 6. Variation of $S(t)$ with time for varying values of λ_0

It is noted that in all cases the trajectory of the mass of fertile topsoil tends to its steady state level.

7. Conclusion

We proposed in this paper, a mathematical model to study the depletion of topsoil by raindrops and vegetation biomass. We considered the build up to the formation of rain from evaporation and transpiration. Further, we also considered both the stress that rain drops and the pressure that the increasing density vegetation biomass exert on the depth of fertile topsoil.

Computer simulation performed to investigate the effect of some baseline parameters on the depth of fertile topsoil.

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