Transactions of the Nigerian Association of Mathematical Physics
***Volume 11, (January - June, 2020 Issue*), pp41 – 52**
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A MATHEMATICAL MODEL FOR CORRELATING UNAVAILABLE SOLAR RADIATION WITH USEFUL CLIMATOLOGICAL VARIABLES

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Abstract

In this paper we have employed multiple linear regression to model unavailable solar radiation with relative sunshine duration, relative humidity and cloud cover in a four parameter model equation applied to fourteen meteorological stations located in Nigeria, viz. Bauchi, Bida, Enugu, Gusau, Ikom, Jos, Kano, Maiduguri, Minna, Nguru, Potiskum, Yelwa Yola and Zaria. The analysis was carried out for both yearly as well as seasonal variations. High values of correlation coefficients and low standard error of estimates were recorded for most of the stations with our proposed equation than with the five parameter equations in the literature, our proposed equation gave best-fit for 50%, 14% and 71% for yearly, dry season and wet season variation respectively.

Keywords: Mathematical model, unavailable solar radiation, relative sunshine duration, relative humidity, cloud cover

1.0 Introduction

Solar energy is one of the most important forms of energy at available to mankind [1], it is known that all forms of energy in the universe can be traced back to the sun [2-3]. Over the past years, researchers have been occupied with the problem of how best solar radiation can be put to useful forms for the benefit of mankind [4-6], this concept has led to the production of an instrument for measuring the amount of solar radiation received on the surface of the earth known as the pyranometer [7]. The pyranometer is not only too expensive and requires much skills on the part of the user, there is also the need for timely insolation data for its operation [8], these problems have been solved by the use of model empirical equations [9-11] which correlates a dependent variable to one or more independent variables. Some of the commonly used dependent variables are the daily solar radiation H [6, 11] and clearness index [8, 12]H/H_0, where H_0 is the extraterrestrial daily solar radiation, another useful dependent variable introduced in the last decade is the amount of solar radiation not received on the surface of the earth, also referred to as unavailable solar radiation [2, 13-14] H_0-H. Amongst the important climatological independent variables includes; relative humidity R, maximum air temperature [15] T_m, cloud cover [14] C (in oktas), relative sunshine duration S/S_0 (where S and S_0 are respectively the bright sunshine duration and day-length, both measured in hours). Equations have been used to model the data for meteorological stations in Nigeria, model equations for north-eastern Nigeria was investigated by [16], model equations for other stations can be obtained elsewhere [4, 11, 15]. Empirical models for forecasting global solar radiation on horizontal surface using sunshine hours and temperature data was studied by Ilori *et al*. [17]. To the best of our knowledge only few papers reported unavailable solar radiation as a dependent variable in their model equations. Motivated by the works of Aidan *et al*. [2] and Eyube *et al*. [9-10, 14], we are encouraged to model unavailable solar radiation in a four parameter model equation as opposed to the five parameter equations used in previous works [2, 9-10, 14] and to compare results with those in the literature.

2.0 Theoretical Formulation

2.1. Model Equations

Model equations used to correlate solar radiation(unavailable solar radiation, clearness index or daily solar radiation) to important climatological independent variables such as (S/S_0), R and C are generally expressible in the general form [2,]

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$$H' = \sum_{i,j,k=0} \alpha_{ijk} \left(S/S_0\right)^i R^j C^k \qquad (1)$$

where the constant coefficients α_{ijk}, can be determined by multiple linear regression of Eq. (1). Depending on the number of parameters in the model equation, a vast number of equations can be deduced from Eq. (1), Aidan *et al.* [2] considered a five parameter equation in their analysis, the equation used is of the form:

$$H' = \alpha_{000} + \alpha_{100}\left(S/S_0\right) + \alpha_{010}R + \alpha_{001}C + \alpha_{011}RC \qquad (2)$$

this equation has been applied to fourteen (14) meteorological stations in Nigeria [2,13]*viz*: Bauchi, Bida, Enugu, Gusau, Ikom, Jos, Kano, Maiduguri, Minna, Nguru, Potiskum, Yelwa, Yola and Zaria. Eyube *et al.* [13] in their work on correlation of unavailable solar radiation using some climatological parameters, they employed a five parameter equation to study the data for this same fourteen stations using the following model equation:

$$H' = \alpha_{000} + \alpha_{100}\left(S/S_0\right) + \alpha_{010}R + \alpha_{001}C + \alpha_{101}\left(S/S_0\right)C \qquad (3)$$

In the present work, we have proposed a model equation which incorporates only four parameters as opposed to five parameters as was used in previous studies, our proposed four parameter equation assumes the following form:

$$H' = \alpha_{000} + \alpha_{100}\left(S/S_0\right) + \alpha_{103}\left(S/S_0\right)C + \alpha_{001}RC \qquad (4)$$

Eq. (4), to the best of our knowledge has never been used to analyze data for any station, this equation will be used to analyze the data for the fourteen meteorological (Bauchi, Bida, Enugu, Gusau, Ikom, Jos, Kano, Maiduguri, Minna, Nguru, Potiskum, Yelwa, Yola and Zaria) stations all located in Nigeria.To confirm the efficacy of our model equation, we will compare our results with those in the literature where they exist. We will invoke multiple linear regression to determine the constant coefficients α_{ijk} of our proposed equation and also the coefficient of determination (R_d^2). The analysis will be carried out for yearly and seasonal variations, the seasonal variations [8] are classified as dry season and wet seasons, the dry season ranges the months of November-April while the wet season is from the months of May-October. Five goodness-of-fit indices *viz*: the adjusted coefficient of determination (R_a^2), the standard error of estimate (se), residual sum of squares (Δ), the maximum percentage error (MPE) and the absolute average percentage error (AAPE) will be employed in our data analysis. These goodness-of-fit indices defined by Ododo *et al.* [12, 16] as given below:

2.2 Goodness-of-fit Indices

2.2.1. The Standard Error of Estimate

The standard error of estimate in the measurement of H′ is given by:

$$seH' = \left\{ \frac{\sum_i \left(H'_i - H'_{fit}\right)^2}{n-k} \right\}^{\frac{1}{2}} \qquad (5)$$

where n is the number of data points (n = 12 for yearly variation and 6 for seasonal variation), k is the number of parameters determined (k = 5 for Eq. (2), Eq. (3) and Eq. (4)), H'_i is the ith value of observed unavailable solar radiation and H'_{fit} is fitted value of H'_i. A requirement for Eq. (5) is k < n. In our analysis, seH′ > 1 will be regarded as relatively high.

2.2.2. The Adjusted Coefficient of Determination

The coefficient of determination (R_d^2) measures the correlation between the dependent and independent variables, a perfect correlation exist if $R_d^2 = 1$, on the other hand if $R_d^2 = 0$ there is no correlation between the variables. Since R_d^2 necessarily increases as the number of parameters k is increased, a better comparison of the "efficiency" of the equations is the adjusted coefficient of determination (Ododo *et al.*, 1994a), (R_a^2) given by:

$$R_a^2 = \frac{(n-1)R^2 - (k-1)}{n-k} \qquad (6)$$

where k < n

2.2.3. Residual Sum of Squares

This is a measure of the deviation of H'_{fit} from H′ given by:

$$\Delta = \sum_i \left(H'_i - H'_{fit}\right)^2 \qquad (7)$$

The smaller the value of Δ, the better the fit

2.2.4. The Maximum Percentage Error

The absolute percentage error (PE_i) in the ith measurement of unavailable solar radiation is given by:

$$PE_i = \frac{100\left| H_i' - H_{fit}' \right|}{H_i'} \qquad (8)$$

the highest in the set of values of PE_i is defined as the maximum percentage error (MPE), it is given as:

$$MPE = \max\left(PE_i\right) \qquad (9)$$

In this paper, MPE > 5% will be considered relatively high.

2.2.5. The Absolute Average Percentage Error

The arithmetic mean of the set of values given by Eq. (8) is defined as the absolute average percentage error (AAPE), it is given as:

$$AAPE = \frac{\sum_i PE_i}{n} \qquad (10)$$

Our set limit for AAPE is 5%

3.0Spherical Coordinates of the Stations under Consideration

The data in Table 1 shows the spherical coordinates, latitude (lat.), longitude (long.) and altitude [12] of the stations studied in this paper

4.0 Input Parameters

The input parameters [2, 12, 16] for the stations considered in this paper are shown in Tables 2-3, we have computed H′ from the available data.

5.0 Results and Discussion

5.1 Regression Analysis and Parameters

Using MATLAB programming software, a multiple linear regression analysis was carried out on Eq. (4), the entries in Tables 4-8 are the results of regression analysis for all the fourteen stations we have studied, also shown in the tables are results in the literature adopted from Aidan *et al*. [2] and Eyube *et al*. [13], we have used the coefficient of determination in ref. [2] to compute the corresponding adjusted coefficient of determination to enable comparison with our four parameter model equation. Table 9 gives a summary of best-fit-equations and parameters of goodness-of-fit indices for the data for both yearly and seasonal variation.

5.1.1 Yearly Regression Analysis and Parameters

Best-fit equations, parameters of regression coefficients and goodness-of-fit indices are shown in Tables 4-8, summary of best-fit-equation and goodness-of-fit indices are shown in Table 9. Eq. (2) provides best-fit for the data of Gusau, Jos, Maiduguri and Potiskum with relatively low values of seH′, $R_a^2 > 0.94$ Gusau and Maiduguri and Potiskum have relatively large values of MPE, 11.4% for Gusau and 14.1% for Maiduguri, however, these values are isolated cases since they have good values of AAPE of 4.1% and 3.9% respectively. Best-fit for the data of Enugu, Kano and Zaria is provided by Eq. (3), however, Zaria has a relatively large values of seH′(1.3695), Δ(13.13), MPE(11.7%) and AAPE(6.3%), this suggest that Eq. (3), eventhough is the best-fit compared to the other two equations, it may not be suitable to model the data for yearly variation. Enugu and Kano have relatively low values of seH′, Δ, MPE and AAPE with $R_a^2 > 0.97$. On the other hand, Eq. (4) gives best-model equation for 50% of the stations viz. Bauchi, Bida, Ikom, Minna, Nguru, Yelwa and Yola. All the stations have $R_a^2 > 0.92$ with isolated cases of relatively high MPE for Bida, Minna and Yelwa.

5.1.2 Dry Season Regression Analysis and Parameters

The data in Table 9 shows that for dry season variation, Eq. (2) gives the best-fit for Bida, Enugu, Kano, Maiduguri, Potiskum, Yelwa and Yola, all the parameters of goodness-of-fit indices are relatively low with $R_a^2 > 0.95$, it is clear that none of these stations gave best-fit with Eq. (2) for yearly fits, Yelwa has a near perfect fit with Eq. (2) since Δ≈ 0, seH′ ≈ 0.066. Considering Eq. (3), the data for five stations: Gusau, Ikom, Jos, Nguru and Yola have Eq. (3) as the best model equation for the data, all the stations have $R_a^2 < 0.9$ except Jos and Nguru which have excellent fit with Eq. (3). Eq. (4) gives the best model equation for the data of Bauchi and Minna, with relatively high correlation coefficients and relatively low values of seH′, Δ, MPE and AAPE, the fact that Eq. (4) is the best model equation for these two stations for the yearly fits is an affirmation that there is no need to consider seasonal variations for the two stations.

5.1.3 Wet Season Regression Analysis and Parameters

For wet season variation, as revealed in Table 9, model Eq. (2) gives best-fit for two stations: Gusau and Kano where $R_a^2 > 0.99$ with relatively low values of seH′, Δ, MPE and AAPE. Similarly, Ikom and Zaria, Eq. (3) with $R_a^2 > 0.99$ with excellent values of seH′, Δ, MPE and AAPE. For the remaining stations, Bauchi, Bida, Enugu, Jos, Maiduguri, Minna and Nguru, best-fit-equation for the data is provided by Eq. (4) with excellent values of seH′, Δ, MPE and AAPE, however, Bida and Minna have relatively high values of seH′ and MPE.

6. CONCLUSION

We have proposed a new model equation for correlating unavailable solar radiation with relevant climatological parameters, our newly proposed equation employs relative sunshine duration, relative humidity and cloud cover in a four parameter equation comparable to similar equations in the literature, for both yearly and seasonal fits. Our equation gave best-fit for both yearly and seasonal variations for majority of the stations studied. We hope to advance this work to cover more meteorological stations. The results obtained in this paper might be useful in areas of agriculture, aviation and weather forecast.

Table 1 Stations Coordinates

Stations	Lat. (θ° N)	Long. (ϕ° E)	Altitude (m)
Bauchi	10.283	09.817	610
Bida	9.100	6.017	144
Enugu	6.470	7.330	142
Gusau	12.167	6.700	464
Ikom	5.367	8.717	92
Jos	9.867	8.900	1260
Kano	12.050	08.533	472
Maiduguri	11.850	13.083	354
Minna	9.620	9.620	259
Nguru	12.883	10.467	343
Potiskum	11.783	11.033	415
Yelwa	10.9	04.75	244
Yola	9.233	12.467	186
Zaria	11.1	07.68	656

Table 2 Input Parameters C (oktas), H′ (M Jm^{-2} day^{-1})

	Bauchi				**Bida**				**Enugu**				**Gusau**			
	[2, 12, 16]				[2, 12, 16]				[2, 12, 16]				[2, 12, 16]			
	S/S_0	R	C	H′	S/S_0	R	C	H′	S/S_0	R	C	H′	S/S_0	R	C	H′
Jan	0.7365	0.1662	4.11	13.4077	0.5068	0.3338	2.89	15.4885	0.5136	0.5321	2.43	17.2930	0.5975	0.19	4.22	11.5948
Feb	0.7143	0.1343	4.05	13.6902	0.5905	0.3442	3.54	14.4702	0.5235	0.5311	3.31	17.7169	0.6283	0.165	4.54	11.3380
Mar	0.6262	0.2449	4.99	15.4458	0.5227	0.4392	4.96	15.8842	0.471	0.6105	4.63	19.4999	0.5371	0.225	5.26	12.8040
Apr	0.584	0.3032	5.84	16.8053	0.5532	0.544	5.39	17.7315	0.4898	0.7022	5.65	20.0470	0.5263	0.4146	5.86	14.0720
May	0.5982	0.4648	5.94	17.4198	0.5646	0.6459	6.16	18.2347	0.4863	0.76	5.8	19.4907	0.5267	0.5818	5.98	14.6642
Jun	0.5858	0.5866	6.04	18.0464	0.4406	0.7032	6.66	19.8753	0.4264	0.7924	6.06	20.2370	0.4065	0.6925	6.16	17.7752
Jul	0.5084	0.6978	6.41	20.5267	0.3765	0.747	6.8	22.0960	0.3061	0.8081	6.45	22.4238	0.446	0.785	6.486	20.1025
Aug	0.5282	0.7335	6.69	20.1916	0.391	0.7505	6.78	23.7033	0.2936	0.8068	6.4	23.1511	0.5158	0.8207	6.62	21.2745
Sep	0.5764	0.6666	6.19	18.8618	0.4506	0.9419	6.78	20.9040	0.3232	0.8139	6.34	22.3098	0.6227	0.7717	6.25	16.9113
Oct	0.6925	0.4741	5.17	15.3579	0.5841	0.6712	6.03	17.0717	0.445	0.7866	5.88	19.3432	0.664	0.5993	5.38	13.6000
Nov	0.7734	0.239	4.18	13.3025	0.6867	0.475	3.87	12.8495	0.6103	0.6975	4.54	15.6811	0.6911	0.2682	4.36	9.9121
Dec	0.7586	0.1897	3.86	12.6442	0.6023	0.3814	2.45	14.1272	0.5988	0.573	3.33	15.7303	0.6036	0.2056	4.78	10.1211
	Ikom				**Jos**				**Kano**				**Maiduguri**			
Jan	0.3964	0.7522	4.91	17.9133	0.8214	0.1538	3.11	11.8105	0.6926	0.2059	2.13	10.3100	0.783	0.2147	4.07	8.8397
Feb	0.4536	0.6858	5.71	17.4518	0.7837	0.1389	3.55	12.7636	0.7137	0.1655	2.13	10.1905	0.794	0.1672	4.03	8.8460
Mar	0.458	0.6283	6.64	19.2646	0.6495	0.2148	4.94	15.8135	0.6386	0.175	3.27	11.7016	0.7062	0.1519	4.76	10.4899
Apr	0.4841	0.592	6.63	19.4820	0.5451	0.425	5.92	18.4297	0.6325	0.268	4.23	13.3161	0.6537	0.217	5.51	11.4715
May	0.4497	0.6356	6.71	19.0905	0.5277	0.6129	6.43	20.0416	0.6763	0.4232	4.93	14.0940	0.6873	0.351	5.62	11.6635
Jun	0.3915	0.7181	6.88	20.4717	0.513	0.6807	6.46	19.6893	0.6786	0.5515	4.87	14.6363	0.6468	0.4834	5.9	12.9461
Jul	0.2615	0.7704	7.02	23.5640	0.3952	0.78	6.77	22.6960	0.5923	0.677	5.57	17.0736	0.5393	0.6342	6.39	14.5826
Aug	0.2012	0.7981	7.04	25.2523	0.369	0.7909	6.86	23.4756	0.5563	0.7237	5.98	17.3428	0.5025	0.7084	6.45	15.5196
Sep	0.2928	0.8335	6.97	22.4244	0.4894	0.6909	6.53	20.1625	0.6547	0.6512	5.4	14.8668	0.6053	0.6325	5.89	12.6152
Oct	0.3946	0.8367	6.67	20.1226	0.6605	0.464	5.58	16.0560	0.7145	0.4056	3.77	11.7828	0.748	0.3906	5.13	8.0128
Nov	0.4452	0.8177	6.11	17.6564	0.831	0.218	3.89	11.4395	0.7581	0.2225	2.59	10.3897	0.8318	0.2396	4.18	7.4124
Dec	0.4052	0.7929	5.11	17.1083	0.8539	0.1707	3.25	10.8918	0.7339	0.214	2.31	10.0514	0.8131	0.2332	3.98	7.7418

Table 3 Input Parameters C (oktas), H′ (M Jm^{-2} day^{-1})

	Minna				**Nguru**				**Potiskum**			
	[2, 12, 16]				[2, 12, 16]				[2, 12, 16]			
	S/S_0	R	C	H′	S/S_0	R	C	H′	S/S_0	R	C	H′
Jan	0.6908	0.2942	3.44	14.1815	0.7492	0.1618	4.28	11.2243	0.7585	0.1584	4.41	10.9392
Feb	0.7053	0.3225	3.79	13.4888	0.8061	0.1332	4.27	10.4915	0.757	0.1201	4.24	10.9580
Mar	0.6514	0.4876	4.89	15.9196	0.6733	0.1268	4.93	12.0521	0.6737	0.1222	5.02	12.7280
Apr	0.6071	0.6294	6.12	17.5094	0.6413	0.1776	5.4	13.1584	0.6207	0.2166	5.69	14.4496
May	0.6192	0.738	6.47	17.8041	0.7067	0.299	5.26	13.7229	0.6746	0.3574	5.87	15.0680
Jun	0.5323	0.8078	6.67	20.0888	0.7166	0.4221	5.4	14.5898	0.6582	0.494	6.07	15.8485
Jul	0.3609	0.8503	6.86	22.2355	0.635	0.5764	5.87	15.8169	0.5558	0.6529	6.34	17.8562
Aug	0.328	0.8582	6.91	24.8587	0.6122	0.6712	6.35	17.3137	0.5511	0.7236	6.65	18.5435
Sep	0.4569	0.84	6.84	19.4469	0.6967	0.5939	5.79	14.2763	0.6398	0.6479	6.08	15.4791
Oct	0.6569	0.7618	6.04	15.8752	0.7857	0.3394	4.93	11.8527	0.726	0.4113	5.3	12.2990
Nov	0.7649	0.4871	3.95	11.6480	0.8091	0.2007	4.7	10.6350	0.8036	0.1999	4.63	10.8130
Dec	0.7525	0.3471	3.82	12.9761	0.8207	0.1817	4.19	10.7213	0.7611	0.1789	4.28	10.0679
	Yelwa				**Yola**				**Zaria**			
	S/S_0	R	C	H′	S/S_0	R	C	H′	S/S_0	R	C	H′
Jan	0.7745	0.2597	2.77	11.9020	0.7295	0.2003	5.07	12.6301	0.6712	0.2028	6.63	11.5529
Feb	0.7375	0.2486	4	11.6756	0.6968	0.177	5.98	12.1482	0.6553	0.1672	6.72	12.0166
Mar	0.6455	0.3387	4.99	13.3550	0.599	0.2401	6.34	14.1763	0.5527	0.145	6.88	13.8117
Apr	0.6285	0.4516	5.69	15.7912	0.6039	0.3927	6.46	15.2153	0.5708	0.1095	6.98	14.9790
May	0.6416	0.5702	6.34	16.7540	0.6288	0.5432	6.62	16.8151	0.6523	0.1807	6.96	15.8156
Jun	0.6181	0.6626	6.51	17.1942	0.6031	0.6628	6.64	17.4674	0.6017	0.2706	6.99	17.8543
Jul	0.5077	0.726	6.67	19.5236	0.5092	0.7238	6.832	20.0527	0.4986	0.5	6.99	19.0590
Aug	0.5023	0.7705	6.68	20.8558	0.5087	0.7448	6.822	21.2496	0.5046	0.6244	6.99	19.4385
Sep	0.5559	0.7384	6.64	17.1166	0.5609	0.7395	6.77	18.3773	0.5828	0.7045	6.91	17.1079
Oct	0.7083	0.6342	4.41	13.4328	0.6996	0.6334	6.36	14.6385	0.68	0.7261	6.86	13.3124
Nov	0.7928	0.3882	3.58	10.7502	0.7934	0.3575	5.51	11.5625	0.6712	0.2028	6.63	11.5529
Dec	0.7974	0.307	3.57	11.0144	0.751	0.2512	4.88	11.4747	0.6553	0.1672	6.72	12.0166

Table 4Regression parameters and goodness-of-fit indices

	Bauchi			**Bida**			**Enugu**		
	Eq. (2) [2]	Eq. (3) [13]	Eq. (4)	Eq. (2) [13]	Eq. (3) [13]	Eq. (4)	Eq. (2) [2]	Eq. (3) [13]	Eq. (4)
	yearly variation			**yearly variation**			**yearly variation**		
α_{000}	24.0364	15.2298	25.4097	21.9494	19.1365	26.1425	34.4031	34.6261	34.5758
α_{100}	-15.5182	-3.8805	-16.7875	-23.0941	-13.2909	-22.9926	-16.8987	-28.6580	-37.8474
α_{020}	-2.4357	4.3051	…	12.0052	3.2569	…	-23.1812	-11.0006	…
α_{003}	0.0821	1.9687	…	0.8887	1.4079	…	0.5273	0.3839	…
α_{023}	1.1770	…	-0.0977	-1.1519	…	0.4910	1.7114	…	2.9458
α_{103}	…	-2.6053	0.8375	…	-1.4087	0.6975	…	1.6488	-1.1900
seH′	0.2391	0.2333	0.2274	0.9564	0.9451	0.9217	0.3649	0.3431	0.3658
R_a^2	0.9925	0.9928	0.9932	0.9201	0.9220	0.9258	0.9783	0.9808	0.9782
Δ	0.40	0.38	0.41	6.40	6.25	6.80	0.93	0.82	1.07
MPE (%)	2.7	3.4	2.8	7.3	6.7	7.2	2.4	2.3	3.1
AAPE (%)	0.9	1.0	1.0	3.3	3.1	3.2	1.2	1.1	1.2
	dry season variation			**dry season variation**			**dry season variation**		
α_{000}	0.4884	20.8766	16.9718	39.7517	24.0809	22.6529	40.4205	21.2880	31.6330
α_{100}	-1.5303	-23.5028	-19.4908	-11.1185	-24.5847	-13.2704	-21.8692	-10.3316	-29.5638
α_{020}	14.0669	-4.2787	…	-50.9109	12.5460	…	-26.0933	-0.1844	…
α_{003}	3.6878	-0.8880	…	-4.7496	-0.2569	…	-1.4263	2.6126	…
α_{023}	-4.8506	…	3.8480	12.7576	…	-2.1136	4.2092	…	0.3992
α_{103}	…	4.7298	-1.0832	…	0.4170	2.8422	…	-3.9680	0.3963
seH′	0.1888	0.1549	0.1148	0.3514	1.1724	0.6776	0.1190	0.3514	0.3734
R_a^2	0.9857	0.9904	0.9947	0.9562	0.5125	0.8372	0.9958	0.9634	0.9587
Δ	0.04	0.02	0.03	0.12	1.37	0.92	0.01	0.12	0.28
MPE (%)	1.0	0.9	0.9	1.5	4.3	4.2	0.5	1.3	1.7
AAPE (%)	0.4	0.4	0.4	0.9	2.8	2.2	0.2	0.6	1.2
	wet season variation			**wet season variation**			**wet season variation**		
α_{000}	33.7065	24.8707	29.4608	406.7277	-2691.2593	34.0005	150.9741	199.8857	83.5874
α_{100}	-23.7924	-15.5053	-18.7878	-47.0911	3898.7826	10.9676	-20.2197	-295.9624	-176.3613
α_{020}	-1.2337	4.4317	…	-487.7324	-86.1495	…	-164.2833	-81.4789	…
α_{003}	-0.6986	0.7230	…	-54.7613	395.3277	…	-17.2264	-16.1379	…
α_{023}	0.9486	…	-0.7898	72.9938	…	-7.2366	23.3789	…	23.8005
α_{103}	…	-1.2893	0.7103	…	-538.0340	0.6425	…	42.1631	-10.3417
seH′	0.1823	0.1853	0.1314	1.6394	1.5288	1.3274	0.5343	0.4295	0.3396
R_a^2	0.9909	0.9906	0.9953	0.5527	0.6110	0.7067	0.8967	0.9333	0.9583
Δ	0.03	0.03	0.03	2.69	2.34	3.52	0.29	0.18	0.23
MPE (%)	0.8	0.8	0.7	5.4	5.0	5.6	1.6	1.4	1.4
AAPE (%)	0.3	0.3	0.3	2.0	1.9	3.2	0.9	0.7	0.7

Table 5Regression parameters and goodness-of-fit indices

	Gusau			**Ikom**			**Jos**		
	Eq. (2) [2]	Eq. (3) [13]	Eq. (4)	Eq. (2) [13]	Eq. (3) [13]	Eq. (4)	Eq. (2) [2]	Eq. (3) [13]	Eq. (4)
	yearly variation			**yearly variation**			**yearly variation**		
α_{000}	21.1664	-0.2916	16.2458	31.2050	34.3889	34.5580	26.2674	31.1338	30.3376
α_{100}	-7.7936	8.7021	-9.3382	-23.6788	-48.9957	-60.3663	-18.0583	-23.9416	-23.0034
α_{020}	-33.1561	8.5002	…	-15.0941	-5.8192	…	-14.8072	1.8127	…
α_{003}	-1.0923	2.9935	…	0.3550	-0.0085	…	0.4252	-0.0808	…
α_{023}	6.9727	…	-0.4783	1.4491	…	5.2873	2.3234	…	0.0535
α_{103}	…	-3.4715	1.7812	…	3.6416	-0.8803	…	0.0877	0.2615
seH′	0.8238	1.3793	1.1071	0.4203	0.4143	0.3979	0.2762	0.3078	0.2820
R_a^2	0.9523	0.8662	0.9138	0.9733	0.9740	0.9761	0.9962	0.9952	0.9960
Δ	4.75	13.32	9.81	1.24	1.20	1.27	0.53	0.66	0.64
MPE (%)	11.4	10.5	9.2	3.3	3.2	3.2	2.4	3.3	3.1
AAPE (%)	4.1	6.5	5.5	1.4	1.3	1.3	1.1	1.0	1.0
	dry season variation			**dry season variation**			**dry season variation**		
α_{000}	30.3703	-139.0834	23.9219	9.9633	74.7480	33.1881	32.3501	41.3229	32.5734
α_{100}	-18.0277	266.4027	-15.2843	-21.3467	-140.7060	-54.1940	-23.5821	-34.8342	-24.4777
α_{020}	-31.8330	-9.0105	…	13.3271	-4.5764	…	-8.5981	3.1479	…
α_{003}	-1.7853	35.5321	…	3.7680	-8.1593	…	-0.1837	-1.7859	…
α_{023}	6.9415	…	-1.6741	-3.3805	…	5.2793	1.4608	…	-0.2564
α_{103}	…	-62.1787	1.3466	…	21.7112	-1.1830	…	1.6485	0.0066
seH′	1.2991	0.8095	0.9543	0.6649	0.3335	0.3990	0.0847	0.0760	0.0875
R_a^2	0.3332	0.7411	0.6402	0.5479	0.8863	0.8372	0.9992	0.9993	0.9991
Δ	1.69	0.66	1.82	0.44	0.11	0.32	0.01	0.01	0.02
MPE (%)	11.0	5.1	10.6	2.1	1.5	1.7	0.6	0.4	0.7
AAPE (%)	3.5	2.5	3.4	1.5	0.6	1.2	0.2	0.2	0.3
	wet season variation			**wet season variation**			**wet season variation**		
α_{000}	117.3223	-104.4879	7.2233	-22.6917	265.4735	33.5027	34.1775	15.6866	50.9456
α_{100}	-11.3074	153.8658	8.7845	-28.7484	-496.5929	-15.0726	-24.5975	-11.7536	-60.3321
α_{020}	-156.3735	9.6068	…	81.9180	-11.6540	…	-40.5646	-13.2163	…
α_{003}	-17.2610	19.1669	…	8.4615	-31.8070	…	1.0381	4.2350	…
α_{023}	28.0288	…	-3.1186	-12.6888	…	-1.8139	4.3098	…	3.0080
α_{103}	…	-26.0453	3.5574	…	65.5458	-0.4852	…	-2.5402	-2.3645
seH′	0.2312	0.8609	0.9031	0.1517	0.1502	0.1773	0.2248	0.2261	0.1718
R_a^2	0.9940	0.9170	0.9086	0.9958	0.9959	0.9942	0.9926	0.9925	0.9957
Δ	0.05	0.74	1.63	0.02	0.02	0.06	0.05	0.05	0.06
MPE (%)	0.9	2.7	4.7	0.5	0.5	0.7	0.9	0.9	1.0
AAPE (%)	0.4	1.5	2.5	0.2	0.2	0.4	0.4	0.4	0.3

Table 6Regression parameters and goodness-of-fit indices

	Kano			**Maiduguri**			**Minna**		
	Eq. (2) [2]	Eq. (3) [13]	Eq. (4)	Eq. (2) [13]	Eq. (3) [13]	Eq. (4)	Eq. (2) [2]	Eq. (3) [13]	Eq. (4)
	yearly variation			**yearly variation**			**yearly variation**		
α_{000}	15.2105	8.6950	19.2542	21.2899	14.6837	30.2585	25.4302	29.4869	34.8157
α_{100}	-9.8210	-1.7324	-15.4498	-13.3746	-8.4143	-26.5221	-21.5756	-27.2903	-35.8745
α_{020}	-1.4830	2.4226	…	-33.0519	-2.3288	…	-6.8887	-5.4963	…
α_{003}	0.8591	2.6379	…	-0.0080	2.5518	…	1.5308	0.8744	…
α_{023}	0.8669	…	1.0457	5.3935	…	-0.2178	0.1145	…	1.9114
α_{103}	…	-2.2792	0.7740	…	-2.9598	-0.1394	…	0.8039	-0.6202
seH'	0.4330	0.4167	0.4278	0.6685	0.7330	0.7078	0.8782	0.8763	0.8324
R_a^2	0.9731	0.9751	0.9738	0.9401	0.9279	0.9328	0.9509	0.9511	0.9559
Δ	1.31	1.22	1.46	3.13	3.76	4.01	5.40	5.37	5.54
MPE (%)	4.3	3.9	4.6	14.1	15.0	16.1	7.5	7.4	7.1
AAPE (%)	1.9	1.9	2.2	3.9	4.1	4.2	2.8	2.7	3.0
	dry season variation			**dry season variation**			**dry season variation**		
α_{000}	14.7743	1.4861	15.5126	38.6266	16.1249	29.0206	27.0269	31.2183	34.8556
α_{100}	-6.4688	9.1246	-10.6727	-23.9247	-4.7858	-23.7318	-24.1187	-29.6238	-37.2918
α_{020}	-11.0808	2.3739	…	-47.4076	-6.5654	…	-4.8170	-6.4400	…
α_{003}	0.1499	4.9869	…	-2.3193	2.7021	…	1.5730	0.4805	…
α_{023}	4.3861	…	1.0559	9.6735	…	-0.2857	-0.3719	…	2.4967
α_{103}	…	-5.8726	1.4928	…	-4.1705	-0.8999	…	1.4594	-1.0567
seH'	0.1478	0.2594	0.1693	0.1773	0.3065	0.2511	0.6043	0.6034	0.4830
R_a^2	0.9868	0.9593	0.9827	0.9873	0.9620	0.9745	0.9184	0.9187	0.9479
Δ	0.02	0.07	0.06	0.03	0.09	0.13	0.37	0.36	0.47
MPE (%)	1.2	2.0	1.6	1.6	2.8	2.9	3.6	3.6	3.3
AAPE (%)	0.4	0.9	0.7	0.7	1.2	1.4	1.5	1.5	1.8
	wet season variation			**wet season variation**			**wet season variation**		
α_{000}	16.8906	120.5964	27.7371	2.2108	57.5917	24.2532	-625.2750	206.6846	62.2569
α_{100}	-42.8075	-154.7698	-29.2779	-34.8287	-88.1934	-54.3293	-48.1615	-241.9920	-72.2713
α_{020}	57.9475	1.4780	…	48.5065	-6.2584	…	886.8523	-31.4304	…
α_{003}	4.9731	-15.0041	…	6.3680	-4.5036	…	105.9435	-20.8311	…
α_{023}	-10.9639	…	1.9286	-9.8337	…	6.8227	-139.6802	…	5.6639
α_{103}	…	21.4657	-0.0468	…	10.9354	-0.8393	…	29.7339	-4.6342
seH'	0.0973	0.4775	0.6310	0.6219	0.5561	0.4282	1.7560	1.9670	1.4033
R_a^2	0.9978	0.9460	0.9057	0.9439	0.9552	0.9734	0.6961	0.6187	0.8059
Δ	0.01	0.23	0.80	0.39	0.31	0.37	3.08	3.87	3.94
MPE (%)	0.4	2.6	3.5	3.1	2.6	3.3	5.4	5.0	5.8
AAPE (%)	0.2	1.1	2.2	1.5	1.3	1.5	2.5	3.2	3.3

Table 7Regression parameters and goodness-of-fit indices

	Nguru			**Potiskum**			**Yelwa**		
	Eq. (2) [2]	Eq. (3) [13]	Eq. (4)	Eq. (2) [13]	Eq. (3) [13]	Eq. (4)	Eq. (2) [2]	Eq. (3) [13]	Eq. (4)
	yearly variation			**yearly variation**			**yearly variation**		
α_{000}	18.4572	8.0268	20.1332	7.2438	-18.8069	23.4318	26.2400	4.2111	25.8282
α_{100}	-13.2209	0.2164	-14.7429	-5.0909	26.7470	-25.1281	-17.3830	6.8945	-19.3265
α_{020}	5.5434	5.2260	…	-24.1278	-0.7728	…	-7.2096	2.0994	…
α_{003}	0.4299	2.4216	…	1.9142	7.1865	…	-0.4024	3.9346	…
α_{023}	0.0402	…	0.4900	3.9938	…	1.8201	2.0783	…	-0.1787
α_{103}	…	-2.5411	0.9626	…	-6.6666	0.4238	…	-4.6361	0.8601
seH′	0.5908	0.5779	0.5624	0.3657	0.4430	0.5496	0.9173	0.9142	0.8970
R_a^2	0.9285	0.9316	0.9352	0.9837	0.9761	0.9633	0.9274	0.9279	0.9306
Δ	2.44	2.34	2.53	0.94	1.37	2.42	5.89	5.85	6.44
MPE (%)	6.2	6.6	6.4	3.0	4.9	6.6	9.7	8.7	8.9
AAPE (%)	2.7	2.4	2.6	1.8	2.3	3.1	3.5	4.0	4.1
	dry season variation			**dry season variation**			**dry season variation**		
α_{000}	25.4950	-6.4313	21.0188	11.5662	-68.2009	18.5613	34.5885	-61.2882	28.8140
α_{100}	-14.1507	22.9918	-11.8608	-1.1966	94.7281	-23.3611	-13.9857	98.6072	-18.5152
α_{020}	-19.9306	6.2625	…	-72.1641	-16.8226	…	-39.6498	-6.8060	…
α_{003}	-1.0164	5.6071	…	0.5562	16.6166	…	-3.2051	19.0603	…
α_{023}	5.8855	…	-0.6114	13.0569	…	3.1174	10.1737	…	-1.9590
α_{103}	…	-7.6644	1.7975	…	-18.9875	-0.6230	…	-25.2949	2.0174
seH′	0.3754	0.0133	0.2545	0.1574	0.2159	0.4499	0.0657	0.5723	0.7676
R_a^2	0.8699	0.9998	0.9402	0.9906	0.9823	0.9232	0.9988	0.9081	0.8347
Δ	0.14	0.00	0.13	0.02	0.05	0.40	0.00	0.33	1.18
MPE (%)	2.2	0.1	1.7	1.3	1.7	3.9	0.4	2.7	5.8
AAPE (%)	1.2	0.0	1.3	0.4	0.6	1.9	0.2	1.7	3.4
	wet season variation			**wet season variation**			**wet season variation**		
α_{000}	5.6379	4.3776	28.1191	6.4575	16.7456	30.3549	151.2554	3472.7560	46.3397
α_{100}	-13.0134	-4.7069	-32.5372	-17.9980	-27.5086	-46.9395	-31.6012	-4864.4346	-46.9208
α_{020}	1.0297	-3.3951	…	4.5129	-3.0449	…	-183.5730	39.5180	…
α_{003}	3.5640	3.9739	…	3.7282	2.0178	…	-16.8181	-513.5233	…
α_{023}	-0.9001	…	2.4734	-1.2577	…	4.4026	26.8756	…	1.5131
α_{103}	…	-1.8527	-0.1912	…	1.5754	-0.4535	…	712.5534	-1.5655
seH′	0.8799	0.8767	0.6713	0.2132	0.2321	0.2096	1.6491	1.4396	1.1824
R_a^2	0.7758	0.7774	0.8695	0.9908	0.9891	0.9911	0.5841	0.6830	0.7862
Δ	0.77	0.77	0.90	0.05	0.05	0.09	2.72	2.07	2.80
MPE (%)	4.4	5.0	4.5	0.9	1.1	1.2	6.5	6.1	6.1
AAPE (%)	1.9	1.9	2.3	0.5	0.5	0.7	2.9	2.5	2.9

Table 8Regression parameters and goodness-of-fit indices

	Yola			**Zaria**		
	Eq. (2) [2]	Eq. (3) [13]	Eq. (4)	Eq. (2) [13]	Eq. (3) [13]	Eq. (4)
	yearly variation			**yearly variation**		
α_{000}	33.9436	-1.3754	26.8807	-33.2995	-213.0738	26.0452
α_{100}	-17.8756	19.3727	-15.4255	-19.7613	250.8715	-77.1665
α_{020}	-32.1410	6.5605	…	36.2869	2.5280	…
α_{003}	-1.6343	4.4407	…	8.6412	34.4711	…
α_{023}	6.0286	…	-1.2738	-4.8017	…	8.3457
α_{103}	…	-6.6772	1.1430	…	-38.8425	0.5338
seH′	0.6212	0.6588	0.6120	1.4602	1.3695	1.4404
R_a^2	0.9650	0.9607	0.9661	0.7904	0.8156	0.7961
Δ	2.70	3.04	3.00	14.93	13.13	16.60
MPE (%)	4.8	5.9	4.9	13.7	11.7	13.9
AAPE (%)	2.5	2.7	2.8	6.2	6.3	6.9
	dry season variation			**dry season variation**		
α_{000}	33.1079	181.7270	24.1792	-38.4882	-576.3516	23.4552
α_{100}	-13.1678	-231.5741	-14.3770	-9.0982	932.0619	-48.0517
α_{020}	-49.2443	13.1257	…	291.9392	-16.9814	…
α_{003}	-2.0701	-25.5958	…	8.8631	87.7763	…
α_{023}	9.0166	…	-0.6973	-46.0718	…	4.4920
α_{103}	…	34.2572	0.9210	…	-138.4298	0.4206
seH′	0.5785	0.5237	0.5521	0.1945	0.2600	0.7555
R_a^2	0.8538	0.8801	0.8668	0.9883	0.9791	0.8238
Δ	0.33	0.27	0.61	0.04	0.07	1.14
MPE (%)	2.8	2.4	4.6	1.2	1.6	4.9
AAPE (%)	1.8	1.6	1.9	0.5	0.7	2.9
	wet season variation			**wet season variation**		
α_{000}	183.3856	212.4216	47.2295	-167.8404	447.1730	18.5076
α_{100}	-48.0822	-187.3661	4.9357	-15.0819	-1121.4166	-472.6071
α_{020}	-144.5319	-3.2675	…	24.6388	4.2848	…
α_{003}	-20.2263	-23.4070	…	27.7740	-61.3568	…
α_{023}	21.1401	…	-7.6136	-3.3334	…	67.1379
α_{103}	…	18.9322	-0.5257	…	160.0246	0.6512
seH′	0.8206	0.8094	0.6101	0.9366	0.1864	0.4359
R_a^2	0.8791	0.8823	0.9332	0.8305	0.9933	0.9633
Δ	0.67	0.66	0.74	0.88	0.03	0.38
MPE (%)	2.9	2.7	3.0	4.0	0.7	2.9
AAPE (%)	1.3	1.2	1.2	1.9	0.4	1.2

Table 9 Summary of best-fit-equations and parameters

station	**yearly variation**						**dry season variation**						**wet season variation**					
	Eq.	seH′	R_a^2	Δ	MPE (%)	AAPE (%)	Eq.	seH′	R_a^2	Δ	MPE (%)	AAPE (%)	Eq.	seH′	R_a^2	Δ	MPE (%)	AAPE (%)
Bauchi	4	0.2274	0.9932	0.41	2.8	1.0	4	0.1148	0.9947	0.03	0.9	0.4	4	0.1314	0.9953	0.03	0.7	0.3
Bida	4	0.9217	0.9258	6.80	7.2	3.2	2 [13]	0.3514	0.9562	0.12	1.5	0.9	4	1.3274	0.7067	3.52	5.6	3.2
Enugu	3 [13]	0.3431	0.9808	0.82	2.3	1.1	2 [13]	0.1190	0.9958	0.01	0.5	0.2	4	0.3396	0.9583	0.23	1.4	0.7
Gusau	2 [13]	0.8238	0.9523	4.75	11.4	4.1	3 [13]	0.8095	0.7411	0.66	5.1	2.5	2 [13]	0.2312	0.9940	0.05	0.9	0.4
Ikom	4	0.3979	0.9761	1.27	3.2	1.3	3 [13]	0.3335	0.8863	0.11	1.5	0.6	3 [13]	0.1502	0.9959	0.02	0.5	0.2
Jos	2 [2]	0.2762	0.9962	0.53	2.4	1.1	3 [13]	0.0760	0.9993	0.01	0.4	0.2	4	0.1718	0.9957	0.06	1.0	0.3
Kano	3 [13]	0.4167	0.9751	1.22	3.9	1.9	2 [2]	0.1478	0.9868	0.02	1.2	0.4	2 [2]	0.0973	0.9978	0.01	0.4	0.2
Maiduguri	2 [2]	0.6685	0.9401	3.13	14.1	3.9	2 [2]	0.1773	0.9873	0.03	1.6	0.7	4	0.4282	0.9734	0.37	3.3	1.5
Minna	4	0.8324	0.9559	5.54	7.1	3.0	4	0.4830	0.9479	0.47	3.3	1.8	4	1.4033	0.8059	3.94	5.8	3.3
Nguru	4	0.5624	0.9352	2.53	6.4	2.6	3 [13]	0.0133	0.9998	0.00	0.1	0.0	4	0.6713	0.8695	0.90	4.5	2.3
Potiskum	2 [2]	0.3657	0.9837	0.94	3.0	1.8	2 [2]	0.1574	0.9906	0.02	1.3	0.4	4	0.2096	0.9911	0.09	1.2	0.7
Yelwa	4	0.8970	0.9306	6.44	8.9	4.1	2 [13]	0.0657	0.9988	0.00	0.4	0.2	4	1.1824	0.7862	2.80	6.1	2.9
Yola	4	0.6120	0.9661	3.00	4.9	2.8	3 [13]	0.5237	0.8801	0.27	2.4	1.6	4	0.6101	0.9332	0.74	3.0	1.2
Zaria	3 [13]	1.3695	0.8156	13.13	11.7	6.3	2 [13]	0.1945	0.9883	0.04	1.2	0.5	3 [13]	0.1864	0.9933	0.03	0.7	0.4

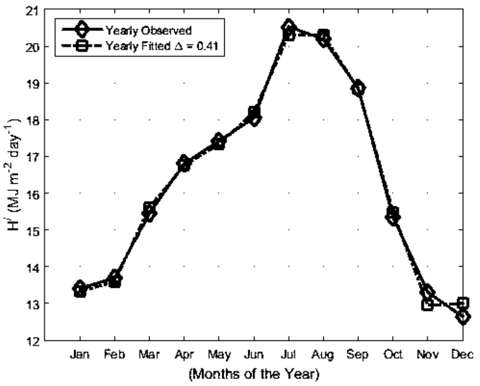

Figure 1. Bauchi yearly

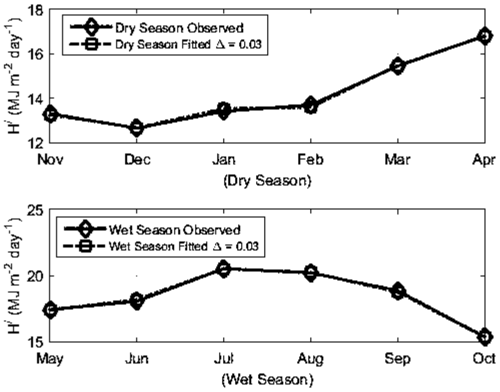

Figure 2. Bauchi seasonal

References

[1] Angstrom, A.J. and Roy, Q.J. (1929). Solar and Terrestrial Radiation. *Meteorol*, 50, 121-126

[2] Aidan, J., Yadima, A. and Ododo, J.C. (2005). Modelling Unavailable Solar Radiation Using Climatological Parameters. *Nigerian Journal of Solar Energy*, 15, 118-126

[3] Prescott, J.A. (1940). Evaporation from Water Surface in relation to Solar Radiation. *Trans. R. Soc. Austr*. 64, 114-118

[4] Fabgenle, R.O. (1990). Estimation of Total Radiation in Nigeria Using Meteorological Data. *Nigerian Journal of Renewable Energy*, 1, 1-10

[5] Page, J.K. (1964). Estimation of Monthly Mean Values of Daily Short Wave Radiation on Vertical and Inclined Surfaces from Sunshine Records for Latitudes 40° N - 40° S. *Prac. UN Conf. on New Sources of Energy*, 4, 378

[6] Swartmann, R.K. and Ogunlade, O. (1967). Solar Radiation Estimates from Common Parameters. *Solar Energy*, 11, 170-172

[7] Ododo, J.C., Agbakwuru, J.A. & Ogbu, F.A. (1995). Correlation of Solar Radiation with Cloud Cover and Relative Sunshine Duration. *Energy Convers. Mgmt*., 37, 1555-1559

[8] Ododo, J.C., Sulaiman, A.T., Aidan, J., Yuguda, M.M. and Ogbu, F.A. (1995). The Importance of Air Temperature in the Parameterization of Solar Radiation in Nigeria. *Journal of Renewable Energy*, 6, 751-763

[9] Eyube, E.S., Alkasim, A. and Najoji, S.D. (2018). Prediction Equations for Parameterization of Clearness Index and Unavailable Solar Radiation in Terms of Some Climatological Parameters for Some Nigerian Stations. . *International Journal of Applied Physics*, 71-77

[10] Eyube, E.S., Sanda, A. and Wadata, U. (2018). Empirical Formulae for Parameterization of Unavailable Solar Radiation in Nigeria. *International Journal of Applied Physics*, 3, 32-48

[11] Ojosu, J.O. (1990). A Correlation of Global Solar Radiation with Cloud Cover and sunshine Hours. *Nigerian Journal of Solar Energy*, 9, 133-142

[12] Ododo, J.C. and Sulaiman, A.T. (1994, May). Angstrom's Regression Coefficient for Nigerian Stations. Paper Presented at the 2nd OAU/STRC Symposium of New, Renewable and Solar Energy Organized in Collaboration with UNESCO/ROSTA, Bamako, Mali

[13] Eyube, E.S., Najoji, S.D. and Alkasim, A. (2018). Correlation of Unavailable Solar Radiation Using Some Nigerian Stations. *International Journal of Renewable Energy Sources*, 3, 60-70

[14] Eyube, E.S., Tanko, P.U. and Alkasim, A. (2018). Use of Relative Sunshine Duration and Cloud Cover for the Prediction of Solar Radiation in Nigeria. *International Journal of Renewable Energy Sources*, 3, 41-59

[15] Ododo, J.C. and Usman, A. (1996). Correlation of Total Solar Radiation with Common Meteorological Parameters for Yola and Calabar. *Energy Convers. Mgmt*., 37, 521-530

[16] Ododo, J.C., Sulaiman, A.T., Aidan, J. and Ogbu, F.A. (1994, May). Modelling of Solar Radiation in North-Eastern Nigeria. Paper Presented at the 2nd OAU/STRC Symposium of New, Renewable and Solar Energy Organized in Collaboration with UNESCO/ROSTA, Bamako, Mali

[17] Ilori, B.Y., Alabi, N.O., Ogun, C.A. and Awofodu, J.O. (2016). Empirical Models for Forecasting Global Solar Radiation on Horizontal Surface Using Sunshine Hours and Temperature Data over Ikeja, Lagos, Nigeria. *Nigerian Journal of Physics*. 26, 18-29.