

**A STUDY OF WAITING AND SERVICE COSTS OF A MULTI-SERVER QUEUING SYSTEM AT NATIONAL HEALTH INSURANCE SCHEME (NHIS) UNIT OF THE GENERAL HOSPITAL, MINNA**

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*Abstract*

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*Queuing is a common situation that occurs in everyday life. Waiting in relation to the time spent by patients to access services in healthcare system is increasingly becoming a major source of concern to most healthcare providers. This is because keeping patients waiting for too long could worsen their health situation and incur more cost (waiting cost) on them. In the other hand, providing more service capacity to operate the queuing system will increase the service cost. In this study, the performance for the queuing system at National Health Insurance Scheme (NHIS) unit of the General Hospital, Minna, was investigated using a multi-server queuing model. The two conflicting cost were balanced and the optimal performance for the queuing system was determined. The relevant data used in the research was collected for a period of four weeks through direct observations and interviews. The results from the research showed that for morning session, the average queue length, waiting time of patients as well as overutilization of Doctors at the unit could be reduced at an optimal server level of 3 Doctors and at a minimum total cost of ₦6219.98 per hour as against the present server level of 2 Doctors with high total of ₦26025.12 per hour which include waiting and service costs and for evening session, the present server level of 1 Doctor should be maintained. The result from this research is an important information to the management of NHIS unit of General Hospital Minna to provide better service to the patients at a minimum cost.*

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**Keywords:** Service Cost, Servers, Customers, Utilization factor, Waiting Cost, Queue.

**1. INTRODUCTION**

Queuing or waiting in line is usually a common situation that occurs in everyday life. Queues (waiting in lines) are usually seen at bus stops, hospitals, filling stations, bank counters and so on [1]. A queuing process consists of customers arriving at a service facility, then waiting in a line (queue), in order to be served by server and after which the customer departs from the service facility. The study of queuing systems has often been concerned with the busy period and the waiting time, because they play a very significant role in the understanding of various queuing systems and their management. There is an increasing demand by highly aware and educated patients due to advances in technology and telecommunications, for demand of quality and efficient service delivery, which have started putting more pressure on the healthcare managers to respond to these concerns [2]. In general, queues form when the demand for service exceeds its supply [3].

According to [4], Queues or waiting lines or queuing theory, was first analyzed by A.K. Erlang a Danish Engineer in 1913 in the context of telephone facilities. He was experimenting with the fluctuating demand for telephone facilities and its effect on automatic dialling equipment at the Copenhagen telephone System. Literature on queuing indicates that waiting in line or queue causes inconvenience to economic costs to individuals and organizations. Healthcare, airline companies, banks, manufacturing firms etc., try to minimize the total waiting cost, and the cost of providing service to their customers.

In the application of queuing models in the hospitals, patients are considered as customers and different units such as National Health Insurance Scheme (NHIS), emergency unit, pharmacy, laboratory or diagnostic imaging can be referred as services facilities. Waiting time depends on the number of customers (human being or objects) in queue, the number of servers serving the customers in queue, and the amount of service time for each individual customer. In healthcare institutions, the effect of queuing in relation to the time spent by patients to access treatment is increasingly becoming a major source of concern to a modern society that is currently exposed to great strides in technological advancement and speed [5]. If the customers are kept for so long, their waiting time could be a cost on them [6].

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The time wasted on the queue would have been used judiciously or utilized elsewhere (opportunity cost of time spent in queuing)[7]. In a waiting line system, managers must decide what level of service to offer. A low level of service may be inexpensive, at least in the short run, but may incur high costs of customer dissatisfaction, such as lost of future business. A high level of service will cost more to provide and will result in lower dissatisfaction costs. When considering improvements in services, the health care manager weighs the cost of providing a given level of service against the potential costs from having patients waiting. The goal of studying queuing system is to minimize the total cost of the system. The two basic costs mentioned are costs associated with patients or customers having to wait for service (Waiting Cost) which include loss of business as some patients might not be willing to wait for service and may decide to go to the competing organizations, cost due to delay in care or the value of the patients time (opportunity cost of the time spent in queuing) and decreased patients satisfaction and quality of care, while Service Cost is the cost of providing service, these includes salaries paid to employees or servers while they wait for service from other servers, Cost of waiting space, facilities, equipment, and supplies. Using the estimation of waiting cost allows decision makers to have the capability of determining the optimal number of servers by minimizing the total cost including the service cost and the waiting cost. The cost of waiting for every individual differs depending on what the individual earns every hour. Some might have their cost of waiting in multiples of other people's value.

A work conducted on how to reduce customer waiting time and improve service delivery. The objective was to reduce the average time a customer spent in the system, focusing on customer waiting time as well as other areas that can be improved[9].

Models for evaluating the impact of bed assignment policies on utilization, waiting time, and the probability of turning away patients was reviewed [10]. The use of queuing theory in pharmacy application with particular attention done by [11] in order to improve customers satisfaction. Customer satisfaction is improved by predicting and reducing waiting times and adjusting staffing. An incremental analysis approach in which the cost of an additional bed is compared with the benefits it generates was proposed by [12]. Beds are added until the increase cost equal the benefits. A pharmacy queuing system with pre-emptive service priority discipline where the arrival of a prescription order suspends the processing of lower priority prescriptions was considered [13]. In this study, Waiting and Service Costs at the Unit using a Multi-server queuing model with a view of determining the optimal service level was studied.

Niger is a state in North Central Nigeria and the largest state in the country. The state capital is Minna, and other major cities are Bida, Kontagora, and Suleja.

Niger State General hospital, Minna is a state owned hospital controlled by the Niger State Government that affords the resident of Minna, to access medical services at a subsides rate.

National Health Insurance Scheme (NHIS) unit, General Hospital, Minna is a branch of the scheme in Minna, Niger state. A corporate body established under Act 35 of 1999 Constitution by the federal government of Nigeria, under the government of President Olusegun Obasanjo GCFR, in order to improve the healthcare of all Nigerian at an affordable cost, through various prepayment systems. National Health Insurance Scheme (NHIS) is an Out Patient Department in General Hospital, Minna that offers medical services for registered members of the unit.

## 2. METHODS

The data used in this research were collected from National Health Insurance Scheme (NHIS) unit of the General Hospital Minna. The methods employed during data collection were direct observation and personal interview administered by the researcher. Data were collected for four weeks. The NHIS unit is open from 8am till 9pm. The following assumptions were made for queuing system at the National Health Insurance Scheme (NHIS) unit of the general hospital Minna which is in accordance with the queue theory. They are:

1. Arrivals follow a Poisson probability distribution at an average rate of  $\lambda$  patients per unit of time.
2. The queue discipline is First-Come, First-Served (FCFS) basis by any of the servers. There is no priority classification for any arrival.
3. Service times are distributed exponentially, with an average of  $\mu$  patients per unit of time.
4. There is no limit to the number of the queue (infinite).
5. The service providers are working at their full capacity.
6. The average arrival rate is greater than average service rate.
7. Servers here represent only doctors but not other medical personnel's.
8. Service rate is independent of line length.
9. Balking and Reneging is not considered in the system.

### 2.1 The M/M/S Model

The model adopted in this work is the (M/M/S) : ( $\infty$ /FCFS) - Multi-server Queuing Model. For this queuing system, it is assumed that the arrivals follow a Poisson probability distribution at an average of  $\lambda$  customers (patients) per unit of time. It is also assumed that they are served on a first-come, first-served basis by any of the servers (in these case Doctors). The service times are distributed exponentially, with an average of  $\mu$  customers (patients) per unit of time and number of servers  $s$ . If there are  $n$  customers in the queuing system at any point in time, then the following two cases may arise:

- (i) If  $n < s$ , (number of patients in the system is less than the number of servers), then there will be no queue. However,  $(s - n)$  number of servers will not be busy. The combined service rate will then be  $\mu_n = n\mu$ ;  $n < s$
- (ii) If  $n \geq s$ , (number of patients in the system is more than or equal to the number of servers) then all servers will be busy and the maximum number of patients in the queue will be  $(n - s)$ . The combined service rate will be  $\mu_n = s\mu$ ;  $n \geq s$ .

From the model, the probability of having  $n$  patients in the system is given by

$$P_n = \begin{cases} \left(\frac{\rho^n}{n!}\right) P_0 & n < s \\ \frac{\rho^n}{(s!s^{n-s})} P_n & n \geq s; \rho = \frac{\lambda}{s\mu} \end{cases} \tag{1}$$

To investigate the performance measure, the following queuing formulas will be used

$$P_0 = \left[ \sum_{n=0}^{s-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!} \frac{\left(\frac{\lambda}{\mu}\right)^s s\mu}{s\mu - \lambda} \right]^{-1} \tag{2}$$

$P_0$  is the probability that there are no patients in the system.

$$Lq = \left( \frac{1}{(s-1)!} \left(\frac{\lambda}{\mu}\right)^s \frac{\mu\lambda}{(s\mu - \lambda)^2} \right) P_0 \tag{3}$$

$Lq$  is the expected number of the patients waiting on the queue.

$$Ls = \left( \frac{1}{(s-1)!} \left(\frac{\lambda}{\mu}\right)^s \frac{\mu\lambda}{(s\mu - \lambda)^2} \right) P_0 + \frac{\lambda}{\mu} \tag{4}$$

$Ls$  is the expected number of patients in the system

$$Wq = \frac{\left( \frac{1}{(s-1)!} \left(\frac{\lambda}{\mu}\right)^s \frac{\mu\lambda}{(s\mu - \lambda)^2} \right) P_0}{\lambda} \tag{5}$$

$Wq$  is the expected waiting time of patients in the queue.

$$Ws = \frac{\left( \frac{1}{(s-1)!} \left(\frac{\lambda}{\mu}\right)^s \frac{\mu\lambda}{(s\mu - \lambda)^2} \right) P_0 + \frac{\lambda}{\mu}}{\lambda} \tag{6}$$

$Ws$  is the average time a patients spends in the system.

$$\rho = \frac{\lambda}{s\mu} \tag{7}$$

$\rho$  is utilization factor, that is the fraction of time Doctors are busy.

Where  $\lambda$  is the arrival rate of patients per unit time,  $\mu$  is the service rate per unit time,  $S$  is the number of Doctors,  $n$  is the number of patients in the system and  $P_0$  is the probability that there are no or zero patients in the system.

### 2.2 Introducing Costs to the Model.

In order to determine and evaluate the optimal number of doctors in the system, two opposing costs must be considered in making these decisions:

- (i) Service costs (the cost of each doctor for service delivery).
- (ii) Waiting time costs of patients (the monetary valued of time spend on queue by the patients).

The analysis of these costs will help the management of NHIS unit of General Hospital to make a trade-off between the increased costs of providing better service and the decreased waiting time costs of patients derived from providing that service.

In order to find these two opposing cost, we calculate the amount an average Niger State medical doctor earns per hour. An average Niger state medical Doctor earns between ₦150,000 and ₦240,000 per month [13], hence taking the average we have ₦195,000 per month, and a Doctor work for at least 8 hours daily for 22 working days, hence he earns ₦195,000/(8×22) = ₦1108 per hour. And the average monthly income of a middle-class Nigerian is between ₦75,000 and ₦100,000 [14], hence taking the average we have ₦87,500 per month and works at least for 8 hours daily for 22 working days, hence he earns ₦87,500/(8×22) = ₦497 per hour.

Therefore,  $C_s = ₦1108$  per hour and  $C_w = ₦497$  per hour.

Expected service cost in the system is given by Equation (8)

$$E(SC) = SC_s \tag{8}$$

Where,  $S$  is the number of Doctors and  $C_s$  is cost of each server.

Expected waiting costs in the system is given by Equation (9)

$$E(WC) = (\lambda W_s) C_w \tag{9}$$

Where  $\lambda$  is the arrival rate,  $W_s$  is the average time a patient spends in the system and  $C_w$  is opportunity cost of waiting by patients.

The expected total cost in the system is given by Equation (10)

$$E(TC) = SC_s + (\lambda W_s) C_w \tag{10}$$

Where,  $E(TC)$  is the expected total cost.

The expected service cost given by Equation (11)

$$E(SC) = SCs \tag{11}$$

Where,  $E(SC)$  is the expected service cost.

Expected service cost is the cost of providing services or service delivery by the doctors.

The expected waiting costs in the system is given by Equation (12)

$$E(WC) = (\lambda W_s) C_w \tag{12}$$

Where,  $E(WC)$  is the expected waiting costs in the system.

Expected waiting cost in the system is the monetary value of the time wasted or spent in the system.

From equation (11) and equation (12),

We have the expected total cost which is given by Equation (13)

$$E(TC) = E(SC) + E(WC) \tag{13}$$

Where,  $E(TC)$  is the expected total cost,  $E(SC)$  is the expected service cost and  $E(WC)$  the expected waiting costs in the system.

### 3. ANALYSIS OF DATA

We compute the performance measures of the multi-server queuing system at the National Health Insurance Scheme (NHIS) unit of the General Hospital Minna for morning session. The parameters obtained from the data collated for one month, the parameters are, arrival rate ( $\lambda$ ) = 21 patients per hour, Service rate ( $\mu$ ) = 11 patients per hour.

From equation (2),

Now for  $S=2$ , that is two Doctors

The probability that there are no patients in the queue is given by Equation (14)

$$P_0 = \left( \sum_{n=0}^{s-1} \frac{1}{n!} \left( \frac{\lambda}{\mu} \right)^n + \frac{1}{s!} \frac{\left( \frac{\lambda}{\mu} \right)^s s\mu}{s\mu - \lambda} \right)^{-1}$$

$$= \left( \frac{1}{0!} \left( \frac{21}{11} \right)^0 + \frac{1}{2!} \frac{\left( \frac{21}{11} \right)^2 \times 2 \times 11}{2 \times 11 - 21} \right)^{-1} + \left( \frac{1}{1!} \left( \frac{21}{11} \right)^1 + \frac{1}{2!} \frac{\left( \frac{21}{11} \right)^2 \times 2 \times 11}{2 \times 11 - 21} \right)^{-1} = 0.0233 \tag{14}$$

From equation (3),

The mean length of patients on the queue is given by Equation (15)

$$Lq = \left( \frac{1}{(S-1)!} \left( \frac{\lambda}{\mu} \right)^s \frac{\mu\lambda}{(S\mu - \lambda)^2} \right) P_0$$

$$Lq = \left( \frac{1}{(2-1)!} \left( \frac{21}{11} \right)^2 \times \frac{11 \times 21}{(2 \times 11 - 21)^2} \right) \left( \frac{1}{0!} \left( \frac{21}{11} \right)^0 + \frac{1}{2!} \frac{\left( \frac{21}{11} \right)^2 \times 2 \times 11}{2 \times 11 - 21} \right)^{-1} + \left( \frac{1}{1!} \left( \frac{21}{11} \right)^1 + \frac{1}{2!} \frac{\left( \frac{21}{11} \right)^2 \times 2 \times 11}{2 \times 11 - 21} \right)^{-1} \tag{15}$$

= 19.5793

From equation (4),

The mean length of patients in the system is given by Equation (16)

$$Ls = Lq + \frac{\lambda}{\mu}$$

$$Ls = \left( \frac{1}{(S-1)!} \left( \frac{\lambda}{\mu} \right)^s \frac{\mu\lambda}{(S\mu - \lambda)^2} \right) P_0 + \frac{\lambda}{\mu}$$

$$Ls = \left( \frac{1}{(2-1)!} \left( \frac{21}{11} \right)^2 \times \frac{11 \times 21}{(2 \times 11 - 21)^2} \right) 0.0233 + \frac{21}{11} = 21.4882 \tag{16}$$

From equation (5),

The waiting time of patients on the queue is given by Equation (17)

$$Wq = \frac{\left( \frac{1}{(s-1)!} \left( \frac{\lambda}{\mu} \right)^s \frac{\mu\lambda}{(s\mu - \lambda)^2} \right) P_0}{\lambda}$$

$$W_q = \frac{\left( \frac{1}{(2-1)!} \left( \frac{21}{11} \right)^2 \times \frac{11 \times 21}{(2 \times 11 - 21)^2} \right) (0.0233)}{21} = 0.9323 \text{hours} \tag{17}$$

The patients spend 55.938minutes on queue

From equation (6),

The waiting time of patients in the system is given by Equation (18)

$$W_s = \frac{\left( \frac{1}{(s-1)!} \left( \frac{\lambda}{\mu} \right)^s \frac{\mu \lambda}{(s\mu - \lambda)^2} \right) P_0 + \frac{\lambda}{\mu}}{\lambda}$$

$$W_s = \frac{\left( \frac{1}{(2-1)!} \left( \frac{21}{11} \right)^2 \times \frac{11 \times 21}{(2 \times 11 - 21)^2} \right) (0.0233) + \frac{21}{11}}{21} = 1.0233 \text{hours} \tag{18}$$

The patients spends 61.398minutes in the system

From equation (7),

The utilization factor is given by Equation (19)

$$\rho = \frac{\lambda}{s\mu}$$

$$\rho = \frac{21}{2 \times 11} \times 100 = 95.45\% \tag{19}$$

From equation (8),

The expected service cost is given by Equation (20)

$$E(SC) = SC_s$$

$$E(SC) = 2 \times \text{₦}1108 = \text{₦}2216 \tag{20}$$

From equation (9),

The expected waiting costs in the system is given by Equation (21)

$$E(WC) = (\lambda W_s) C_w$$

$$E(WC) = (21 \times 1.0233) \text{₦}1108 = \text{₦}23809.12 \tag{21}$$

From equation (10),

The expected total cost is given by Equation (22)

$$E(TC) = E(SC) + E(WC)$$

$$E(TC) = \text{₦}2216 + \text{₦}23809.12 = \text{₦}26025.12 \tag{22}$$

Similarly we solve for when S = 3, 4 and 5.

The summary of the computed performance measure for morning session of the multi-server queuing model at NHIS unit in General Hospital, Minna is given in table 1 and table 2

**Table 1. Performance Measure Of Multi-Server Queuing Model At NHIS Unit In General Hospital, Minna, For Morning Session.**

S(servers)	λ (lamda)	μ (mu)	P <sub>0</sub>	L <sub>s</sub>	L <sub>q</sub>	W <sub>s</sub>	W <sub>q</sub>
2	21	11	0.0233	21.4884	19.5793	1.0233	0.9323
3	21	11	0.1263	2.6137	0.7046	0.1245	0.0336
4	21	11	0.1439	2.0481	0.1391	0.0975	0.0066
5	21	11	0.1474	1.9402	0.0311	0.0924	0.0015

**Table 2. Analysis Of Multi-Server Queuing Model At NHIS Unit In General Hospital, Minna, For Morning Session.**

No. of servers	2 Doctors	3 Doctors	4 Doctors	5 Doctors
Arrival rate (λ)	21	21	21	21
Service rate (μ)	11	11	11	11
System utilization (ρ)	95.45%	63.64%	47.73%	38.18%
L <sub>s</sub>	21.4884	2.6137	2.0481	1.9402
L <sub>q</sub>	19.5793	0.7046	0.1391	0.0311
W <sub>s</sub> in hours	1.0233	0.1245	0.0975	0.0924
W <sub>q</sub> in hours	0.9323	0.0336	0.0066	0.0015
P <sub>0</sub>	0.0233	0.1263	0.1439	0.1474
Expected total cost E(TC)	₦26025.12	₦6219.98	₦6701.4	₦7689.75

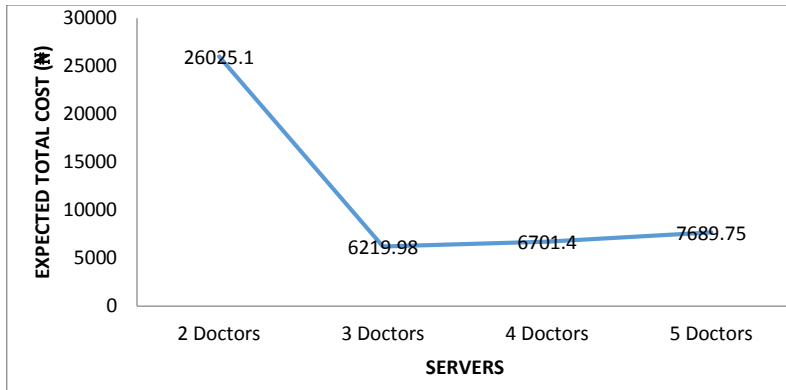


Figure 1: Expected Total Cost Against Level of Service For Morning Session.

Similarly, we compute the performance measures of the multi-server queuing system at the National Health Insurance Scheme (NHIS) unit of the General Hospital Minna for evening session using arrival rate ( $\lambda$ ) = 6 patients per hour, Service rate ( $\mu$ ) = 11 patients per hour and number of server ( $S$ ) = 1.

From equation (2)

The probability that there are no patients in the queue is given by Equation (23)

$$P_0 = \left( \sum_{n=0}^{s-1} \frac{1}{n!} \left( \frac{\lambda}{\mu} \right)^n + \frac{1}{s!} \frac{\left( \frac{\lambda}{\mu} \right)^s s \mu}{s \mu - \lambda} \right)^{-1}$$

$$= \left( \frac{1}{0!} \left( \frac{6}{11} \right)^0 + \frac{1}{1!} \frac{\left( \frac{6}{11} \right)^1 \times 1 \times 11}{1 \times 11 - 6} \right)^{-1} = 0.4545 \tag{23}$$

From equation (3),

The mean length of patients on the queue is given by Equation (24)

$$Lq = \left( \frac{1}{(s-1)!} \left( \frac{\lambda}{\mu} \right)^s \frac{\mu \lambda}{(s \mu - \lambda)^2} \right) P_0$$

$$Lq = \left( \frac{1}{(1-1)!} \left( \frac{6}{11} \right)^1 \times \frac{11 \times 6}{(6 \times 11 - 6)^2} \right) 0.4545 = 0.6545 \tag{24}$$

From equation (4),

The mean length of patients in the system is given by Equation (25)

$$Ls = Lq + \frac{\lambda}{\mu}$$

$$Ls = \left( \frac{1}{(s-1)!} \left( \frac{\lambda}{\mu} \right)^s \frac{\mu \lambda}{(s \mu - \lambda)^2} \right) P_0 + \frac{\lambda}{\mu}$$

$$Ls = \left( \frac{1}{(1-1)!} \left( \frac{6}{11} \right)^1 \times \frac{11 \times 6}{(1 \times 11 - 6)^2} \right) 0.4545 + \frac{6}{11} = 1.2000 \tag{25}$$

From equation (5),

The waiting time of patients on the queue is given by Equation (26)

$$Wq = \frac{\left( \frac{1}{(s-1)!} \left( \frac{\lambda}{\mu} \right)^s \frac{\mu \lambda}{(s \mu - \lambda)^2} \right) P_0}{\lambda}$$

$$Wq = \frac{\left( \frac{1}{(1-1)!} \left( \frac{6}{11} \right)^1 \times \frac{11 \times 6}{(1 \times 11 - 6)^2} \right) 0.4545}{6} = 0.1091 \text{ hours} \tag{26}$$

The patients spend 6.546minutes on queue.

From equation (6),

The waiting time of patients in the system is given by Equation (27)

$$W_s = \frac{\left( \frac{1}{(s-1)!} \left( \frac{\lambda}{\mu} \right)^s \frac{\mu\lambda}{(s\mu - \lambda)^2} \right) P_0 + \frac{\lambda}{\mu}}{\lambda}$$

$$W_s = \frac{\left( \frac{1}{(1-1)!} \left( \frac{6}{11} \right)^1 \times \frac{11 \times 6}{(1 \times 11 - 6)^2} \right) 0.4545 + \frac{6}{11}}{6} = 0.2000 \text{hours} \tag{27}$$

The patients spend 12minutes in the system

From equation (7),

The utilization factor is given by Equation (28)

$$\rho = \frac{\lambda}{s\mu}$$

$$\rho = \frac{6}{1 \times 11} \times 100 = 54.55\% \tag{28}$$

From equation (8),

The expected service cost is given by Equation (29)

$$E(SC) = SC_s$$

$$E(SC) = (1 \times \text{₦}1108) = \text{₦}1108 \tag{29}$$

From equation (9),

The expected waiting costs in the system is given by Equation (30)

$$E(WC) = (\lambda W_s) C_w$$

$$E(WC) = (6 \times 0.2000) \text{₦}1108 = \text{₦}1329.60 \tag{30}$$

From equation (10),

The expected total cost is given by Equation (31)

$$E(TC) = E(SC) + E(WC)$$

$$E(TC) = \text{₦}1108 + \text{₦}1329.60 = \text{₦}2437.6 \tag{31}$$

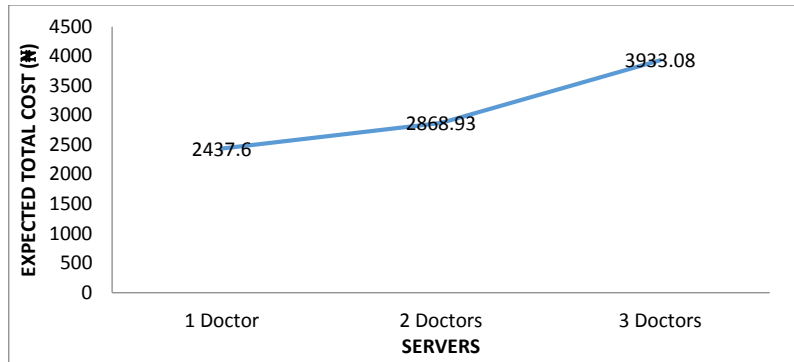
The summary of the computed performance measure for evening session of the multi-server queuing model at NHIS unit in General Hospital, Minna is given in table 3 and table 4

**Table 3. Performance Measure of Multi-Server Queuing Model at NHIS Unit in General Hospital, Minna, For Evening Session.**

S(servers)	(lamda)	μ (mu)	P <sub>0</sub>	L <sub>s</sub>	L <sub>q</sub>	W <sub>s</sub>	W <sub>q</sub>
1	6	11	0.4545	1.2000	0.6545	0.2000	0.1031
2	6	11	0.5714	0.5893	0.0438	0.0982	0.007
3	6	11	0.5789	0.5497	0.0043	0.0916	0.0007

**Table 4. Analysis of a Multi-Server Queuing Model at NHIS Unit at General Hospital, Minna, For Evening Session.**

No. of Servers	1 Doctor	2 Doctors	3 Doctors
Arrival rate( )	6	6	6
Service rate (μ)	11	11	11
System utilization ρ	54.55%	27.27%	18.18%
L <sub>s</sub>	1.2000	0.5893	0.5497
L <sub>q</sub>	0.6545	0.0438	0.0043
W <sub>s</sub> in hours	0.2000	0.0982	0.0916
W <sub>q</sub> in hours	0.1091	0.0073	0.0007
P <sub>0</sub>	0.4545	0.5714	0.5789
Expected total cost E(TC)	₦2437.6	₦2868.93	₦3933.08



**Figure 2; Expected Total Cost Against Level of Service For Evening Session.**

The result of the analysis and the chart shows that for morning session, the average queue length, waiting time of patients as well as overutilization of Doctors at the unit could be reduced at an optimal server level of 3 Doctors and at a minimum total cost of ₦6219.98 per hour as against the present server level of 2 Doctors with high total cost of ₦26025.12 per hour which include waiting and service costs while in the evening session, the present server level of 1 Doctor should be maintained.

## 5. CONCLUSION

The Queuing characteristics at the National Health Insurance Scheme (NHIS) unit of the General Hospital Minna, was analyzed using a Multi-Server Queuing Model and the Waiting and Service Costs was determined with a view to determining the optimal service level. The results of the analysis showed that average queue length, waiting time of patients as well as overutilization of doctors could be reduced when the service capacity level of Doctors at the unit is increased from two to three at a minimum total costs which include waiting and service costs. The operation managers can recognize the trade-off that must take place between the cost of providing good service and the cost of customers waiting time. Service cost increases as a firm attempts to raise its level of service. As service improves, the cost of time spent waiting on the line decreases. This could be done by expanding the service facilities or using models that consider cost optimization.

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