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HOMOTOPY ANALYSIS METHOD FOR STAGNATION POINT FLOW AND CONVECTIVE HEAT TRANSFER OVER AN EXPONENTIAL STRETCHING AND SHIRKING SHEET

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Abstract

This investigation is concerned with the steady boundary layer stagnation-point flow and heat transfer past an exponential stretching sheet. The highly nonlinear coupled partial differential equations are simplified with the help of suitable similarity transformations. The reduced equations are then treated with homotopy analysis method (HAM). The heat transfer problem is modeled using two point convective boundary conditions. The effects of controlling parameters on the dimensionless velocity, temperature, skin friction coefficient and heat transfer rate are analyzed. It is found that the HAM results match well with numerical results obtained by Runge Kutta Fehlberg fourth-fifth order method for different assigned values of parameters. The results indicate that the stretching parameter reduces the hydrodynamic boundary layer thickness whereas Prandtl number reduces the thermal boundary layer thickness.

***Keywords*:** Stagnation point; Convective boundary condition; Heat transfer; stretching sheet; Series solution.

NOMENCLATURE

$\bar{u}, \bar{v}$	velocity component
U_s	straining velocity of the plate
U_w	shrinking/stretching velocity of the plate
N_C	convection parameter
T	Temperature
L	length of the sheet
C_{fx}	local friction factor
Nu_x	local Nusselt number
Pr	Prandtl number
R_{ex}	local Reynolds number

Greek symbols

α	thermal diffusivity of the fluid
σ	electrical conductivity
ρ	density of the base fluid
ν	kinematic viscosity
λ	stretching parameter
ψ	stream function

1. INTRODUCTION

The study of boundary layer flow over a stretching surface has an important bearing on several industrial manufacturing processes such as extrusion of polymer, the cooling of metallic plates, filament extrusion from a dye, and in paper production, Ishak et al. [1]. The flow due to stretching surface is also involved in glass industry for blowing, floating or fibers spinning processes.

Crane [2]was the first to study the flow over a linearly stretching sheet. An analytical similarity solution for the steady two-dimensional problem was obtained. Carragher and Crane [3] have discussed the characteristics of heat transfer in a two dimensional flow over a stretching sheet in the case when the temperature difference between the ambient fluid and the surface is proportional to a power of distance from a fixed point. Dutta*et al*. [4]studied the heat transfer problem for the case of uniform heat flux over a stretching surface. Chiam [5] has analyzed the heat transfer with variable thermal conductivity in a stagnation point over a stretching sheet. Several researchers investigated various physical features and aspects such as three-dimensional flow, magnetic field, suction, and viscoelasticity of the fluid [6,7, 8,9].

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The literature relating to boundary layer flow towards a stretching sheet is constantly growing. Verjravelu[10] provided the analysis of viscous fluid over a nonlinear stretching sheet. Mahapatra and Gupta [11] have studied the heat transfer characteristics in steady two-dimensional stagnation point flow for an incompressible fluid past a stretching sheet with constant surface temperature. The characteristics of heat transfer of viscous fluid past a nonlinear stretching sheet is presented by Cortell[12].A thorough review can be observed in a series of paper [13, 14, 15, 16, 17, 18].Mukhopadhyay [19] investigated unsteady boundary layer flow and heat transfer past a porous stretching sheet in the presence of variable viscosity and thermal diffusivity. Nandeppanavar and Siddalingappa [20]examined nonlinear stretching sheet for the effect of viscous dissipation and thermal radiation on heat transfer. Sharma and Singh [21] studied the effects of variable thermal conductivity on a linear stretching sheet with MHD flow near stagnation point.
There has not been much study on boundary layer flow which is caused by an exponentially stretching sheet though it is significant in many engineering processes. Magyari and Keller[22]were the first to study the heat transfer and boundary layer flow which is due to an exponentially stretching sheet. Numerical treatment of flow and heat transfer over an exponentially stretching surface with wall-mass suction was provided by Elbashbeshy [23]. Some other investigations regarding exponential stretching surface were given by [24, 25,26]. It is to be noticed that these researchers did not use convective boundary conditions in their studies.
A theoretical study on the stagnation point flow past an exponentially stretching sheet via the homotopy analysis method is presented here. As in some other studies, in our study the exponential form of similarity transformation is used and the governing partial differential equations for the flow and heat transfer are transformed into nonlinear ordinary differential equations. The homotopy analysis method is an approximate analytical method that was established by [27, 28],which has been effectively applied on many science and engineering problems [29, 30, 31, 32, 33].

2. MATHEMATICAL FORMULATION

We consider the flow of viscous fluid over an exponentially stretching sheet. We assume that the flow is laminar, steady, incompressible, and two-dimensional boundary layer stagnation-point flow. Under such assumptions the continuity, momentum and energy equations describing the flow can be written as [19]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 v}{\partial y^2}+U_s\frac{dU_s}{dx}, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

where u and v are the x(along the sheet) and y(normal to the sheet) components of the velocities, respectively, $\nu\left(=\frac{\mu}{\rho}\right)$ is the kinematic viscosity of the fluid, ρ is the density, α is thermal diffusivity of the fluid, T is the temperature, σ is the electrical conductivity.
The velocity boundary conditions can be expressed as[19]:

$$u(x,0)=U_w(x),\ \ v(x,0)=0,\ \ u(x,\infty)=U_s(x). \tag{4}$$

The shrinking/stretching velocity U_w and the straining velocity U_s are given by

$$U_w=c\exp\left(\frac{x}{L}\right) \text{ and } U_s=a\exp\left(\frac{x}{L}\right) \tag{5}$$

where c is stretching/shrinking velocity rate with c > 0 for stretching and c < 0 for shrinking, and a > 0 straining velocity rate, L is length of the sheet.
The bottom surface of the plate is assumed to be heated by convection from a hot fluid at temperature T_f and this is deemed to provide aheat transfer coefficient h_f. The boundary conditions at the sheet surface and far into the cold fluid may be written as

$$-k\frac{dT}{dy}(x,0)=h_f\left[T_f-T(x,0)\right] \tag{6}$$

$$T(x,\infty)=T_\infty. \tag{7}$$

To obtain the nondimensionalized form of momentum and energy equations, we define an independent variable η as well as a dependent variable f in terms of the stream function ψ as

$$\eta=\sqrt{\frac{a}{2\nu L}}\,y\exp\left(\frac{x}{2L}\right) \tag{8}$$

$$\psi=\sqrt{2a\nu L}\,f(\eta)\exp\left(\frac{x}{2L}\right). \tag{9}$$

We define a dimensionless temperature θ as

$$\theta=\frac{T-T_\infty}{T_f-T_\infty} \tag{10}$$

Using Eqs. (8) - (10), the momentum and energy equations can be reduced into ordinary differential equations

$$\begin{cases} f''' + f\, f'' + 2\left(1 - f'^2\right) = 0, \\ \theta'' + \Pr f\, \theta' = 0 \end{cases} \tag{11}$$

The boundary conditions of the problem are:

$$\begin{cases} f\left(0\right) = 0,\ f'\left(0\right) = \lambda,\ f'\left(\infty\right) = 1, \\ \theta'(0) = -N_c\left(1 - \theta\left(0\right)\right), \theta\left(\infty\right) = 0 \end{cases} \tag{12}$$

$N_c = \frac{h_f}{k}\sqrt{\frac{2\nu L}{a}}$ is convection parameter, λ is stretching parameter, and $\Pr$ is Prandtl number.

Quantities of the physical interest are the local friction factor, C_{fx} and the local Nusselt number, Nu_x. These quantities can be expressed as

$$C_{fx} = \frac{\mu}{\rho U_s^2}\left(\frac{\partial u}{\partial y}\right)_{y=0} \Rightarrow \sqrt{2\,\mathrm{Re}_x} C_{fx} = f''\left(0\right) \tag{13}$$

$$Nu_x = \frac{-x}{T_f - T_\infty}\left(\frac{\partial T}{\partial y}\right)_{y=0} \Rightarrow \sqrt{\frac{L}{x\,\mathrm{Re}_x}} Nu_x = -\theta'\left(0\right) \tag{14}$$

where $\mathrm{Re}_x = \frac{U_w x}{\nu}$ is the local Reynolds number.

3. HOMOTOPY ANALYSIS SOLUTION

We express $f\left(\eta\right)$ and $\theta\left(\eta\right)$ by a set of base functions

$$\left\{\eta^k \exp\left(-n\eta\right) \middle| k \ge 0, n \ge 0\right\}, \tag{15}$$

in the form

$$\begin{cases} f\left(\eta\right) = a_{0,0}^0 + \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} a_{m,n}^k \eta^k \exp\left(-n\eta\right), \\ \theta\left(\eta\right) = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} b_{m,n}^k \eta^k \exp\left(-n\eta\right) \end{cases} \tag{16}$$

where $a_{m,n}^k, b_{m,n}^k$ are the coefficients. We follow the *rule of solution expression* for determining the initial approximations, auxiliary linear operators, and the auxiliary functions. Therefore, according to the rule of solution expression, we choose the initial guesses $f_0(\eta),\ \theta_0(\eta)$ based on boundary condition (12) and linear operators L_1 and L_2 in the following way:

$$f_0(\eta) = \eta + \left(1 - \lambda\right)\left(e^{-\eta} - 1\right),\quad \theta_0(\eta) = \frac{N_c e^{-\eta}}{1 + N_c} \tag{17}$$

$$L_1\left(f\right) = \frac{d^3 f}{d\eta^3} - \frac{df}{d\eta} \qquad L_2\left(\theta\right) = \frac{d^2\theta}{d\eta^2} - \theta \tag{18}$$

The operators L_1, L_2 have the following properties:

$$L_1\left(C_1 + C_2 e^{\eta} + C_3 e^{-\eta}\right) = 0, \qquad L_2\left(C_4 e^{\eta} + C_5 e^{-\eta}\right) = 0 \tag{19}$$

where $C_i\left(i = 1 - 5\right)$ are arbitrary constants. Let $q \in \left[0, 1\right]$ represent an embedding parameter and $\mathrm{h} \ne 0$ be the auxiliary parameter to adjust the convergence rate of the perturbation series. Then we construct the following zeroth order deformation of the problem, which is:

$$\left(1 - q\right) L_1\left[\hat{f}\left(\eta; q\right) - f_0\left(\eta\right)\right] = q\mathrm{h}_f N_1\left[\hat{f}\left(\eta; q\right)\right], \tag{20}$$

$$\left(1 - q\right) L_2\left[\hat{\theta}\left(\eta; q\right) - \theta_0\left(\eta\right)\right] = q\mathrm{h}_\theta N_2\left[\hat{\theta}\left(\eta; q\right), \hat{f}\left(\eta; q\right)\right], \tag{21}$$

subject to the conditions that:

$$\hat{f}\left(0; q\right) = 0,\quad \hat{f}'\left(0; q\right) = \lambda,\quad \hat{f}'\left(\infty; q\right) = 1 \tag{22}$$

$$\hat{\theta}'\left(0; q\right) = -N_c\left(1 - \hat{\theta}\left(0 : q\right)\right),\quad \hat{\theta}\left(\infty; q\right) = 0 \tag{23}$$

where the nonlinear operators are defined as:

$$N_1\left[\hat{f}\left(\eta; q\right)\right] = \frac{\partial^3 \hat{f}\left(\eta; q\right)}{\partial\eta^3} + \hat{f}\left(\eta; q\right)\frac{\partial^2 \hat{f}\left(\eta; q\right)}{\partial\eta^2} - 2\left(\frac{\partial\hat{f}\left(\eta; q\right)}{\partial\eta}\right)^2 + 2 \tag{24}$$

$$N_2\left[\hat{\theta}\left(\eta; q\right), \hat{f}\left(\eta; q\right)\right] = \frac{\partial^2\hat{\theta}\left(\eta; q\right)}{\partial\eta^2} + \Pr \hat{f}\left(\eta; q\right)\frac{\partial\hat{\theta}\left(\eta; q\right)}{\partial\eta} \tag{25}$$

Setting $q = 0$ and $q = 1$ we have:

$$\hat{f}\left(\eta; 0\right) = f_0\left(\eta\right), \qquad \hat{f}\left(\eta; 1\right) = f\left(\eta\right) \tag{26}$$

$$\hat{\theta}\left(\eta; 0\right) = \theta_0\left(\eta\right), \qquad \hat{\theta}\left(\eta; 1\right) = \theta\left(\eta\right) \tag{27}$$

Defining further:

$$f_m(\eta)=\frac{1}{m!}\frac{\partial^m f(\eta;q)}{\partial q^m}\bigg|_{q=0},\qquad \theta_m(\eta)=\frac{1}{m!}\frac{\partial^m \theta(\eta;q)}{\partial q^m}\bigg|_{q=0} \tag{28}$$

and expanding $\hat{f}(q;\eta)$, $\hat{\theta}(q;\eta)$ by means of Taylor's theorem with respect to q, we obtain:

$$\hat{f}(q;\eta)=f_0(\eta)+\sum_{m=1}^{+\infty} f_m(\eta)q^m \tag{29}$$

$$\hat{\theta}(q;\eta)=\theta_0(\eta)+\sum_{m=1}^{+\infty} \theta_m(\eta)q^m \tag{30}$$

The auxiliary parameters are properly chosen so that series (29)-(30) converge at $q=1$ and thus:

$$f(\eta)=f_0(\eta)+\sum_{m=1}^{+\infty} f_m(\eta) \tag{31}$$

$$\theta(\eta)=\theta_0(\eta)+\sum_{m=1}^{+\infty} \theta_m(\eta) \tag{32}$$

The resulting problems at the m[th] order deformation are:

$$L_1\left[f_m(\eta)-\chi_m f_{m-1}(\eta)\right]=\hbar_f R_m^f(\eta) \tag{33}$$

$$L_2\left[\theta_m(\eta)-\chi_m \theta_{m-1}(\eta)\right]=\hbar_\theta R_m^\theta(\eta) \tag{34}$$

subject to boundary conditions:

$$f_m(0)=0,\ f_m'(0)=0,\ f_m'(\infty)=0,\ \theta_m'(0)=N_c\theta_m(0),\ \theta_m(\infty)=0 \tag{35}$$

$$R_m^f(\eta)=f_{m-1}'''(\eta)+\sum_{k=0}^{m-1} f_k f_{m-1-k}''-2\sum_{k=0}^{m-1} f_k' f_{m-1-k}'+2 \tag{36}$$

$$R_m^\theta(\eta)=\theta_{m-1}''(\eta)+\Pr\sum_{k=0}^{m-1} f_k\theta_{m-1-k}' \tag{37}$$

$$\chi_m=\begin{cases}0, & m\le 1,\\ 1, & m>1.\end{cases} \tag{38}$$

The general solution of Eqs. (33) - (34) are:

$$f_m(\eta)=f_m^*(\eta)+C_1+C_2\exp(\eta)+C_3\exp(-\eta) \tag{39}$$

$$\theta_m(\eta)=\theta_m^*(\eta)+C_4\exp(\eta)+C_5\exp(-\eta) \tag{40}$$

where $f_m^*(\eta)$ and $\theta_m^*(\eta)$ are the particular solution and the constants are to be determined by the boundary condition Eq. (35).

4. CONVERGENCE OF THE HAM SOLUTION

According to Liao [27], the convergence rate of the HAM solution strongly depends on the value of non-zero auxiliary parameter $\hbar$. So it is important to ensure that the solution series Eqs. (33)-(34) are convergent. Note that the solution series contain the auxiliary parameters $\hbar_f$ and $\hbar_\theta$ and, thus, provides us with a simple way to adjust and control the convergence of the solution series. As pointed out by Liao [28], the valid region of $\hbar$ is a horizontal line segment. Convergence of the series solution up to 40[th] order of approximations is presented in Table 1. It is found from Table 1 that the convergence is achieved up to 28[th] order of approximation.

Table 1: Convergence of HAM solution for different order of approximations at $\lambda=0.1, \Pr=1, N_c=0.5$ and $\hbar_f=\hbar_\theta=-1.2$.

Order of approximations	$f''(0)$	$\theta'(0)$
1	1.5330	-0.25470
5	1.5617	-0.26949
10	1.5630	-0.27072
20	1.5630	-0.27627
28	1.5630	-0.27807
30	1.5630	-0.27807
40	1.5630	-0.27807
[RKF-45]	[1.5630]	[-0.27807]

5. RESULTS AND DISCUSSION

The system of (11) with boundary conditions of (12) has been solved analytically via homotopy analysis method (HAM)for various values of different parameters such as the convection parameter N_c , stretching parameter λ and Prandtl number $\Pr$. To illustrate the HAM solution, values of the dimensionless velocity, temperature, friction factor and local Nusselt number have been plotted in Figs. 1-5.The Runge-Kutta Fehlberg fourth-fifth order numerical method has been employed in addition to the HAM to cross-validate the present results. A comparison of the present results of the dimensionless velocity and temperature with the numerical method is presented in Table 2. The results were found to be in good agreement and we therefore have high confidence that our HAM results are accurate. The values of Nusselt number are presented in Table 3fordifferent values of controlling parameters.

In Fig. 1 the effect of stretching parameter λ on the dimensionless velocity is presented. The dimensionless velocity is found to decrease with an increase in stretching parameter λ.Increasing the stretching parameter, due to less difference between the sheet and free stream velocities, the momentum boundary layer gets thinner. According to Fig. 2, we can observe that the increase in convection parameter N_c causes a turn down in the dimensionless temperature and as a result the boundary layer thickness decreased and heat transfer rate will increase. Effects of stretching parameter λ and Prandtl number on the dimensionless temperature have been considered in Fig. 3. The dimensionless temperature is found to be in decreasing manner with both stretching parameter and Prandtl number. It is important to note that the boundary layer thickness decreases with an increase in λ in both cases. An increase in Prandtl number results in the reduction of thermal boundary layer thickness because an increase in Prandtl number would result in a decrease of fluid thermal conductivity.

Fig.4 is prepared to show the influence of stretching parameter λ on skin friction coefficient. We observed that an increase in stretching parameter λ the friction factor monotonically reducing. Finally, in Fig. 5, the variation of Nusselt number, which represent the heat transfer rate at the surface in terms of $(-\theta'(0))$is presented for various values of convection parameter with two values of stretching parameter λ . It is noticed that, the increase in stretching parameter and Prandtl number provide an increase in heat transfer coefficient, but this increase is significant in case of convection parameter N_c .

Table 2 Comparison of HAM and numerical (RKF-45) results for dimensionless velocity and temperature when $\lambda = 0.5, N_c = 0.5, \Pr = 1$

η	$f(\eta)$		$\theta(\eta)$	
	HAM	RKF-45	HAM	RKF-45
0.0	0.0	0.0	0.414270	0.414270
0.5	0.342692	0.342692	0.271431	0.271431
1.0	0.790757	0.790757	0.151232	0.151232
1.5	1.276084	1.276084	0.069778	0.069778
2.0	1.772551	1.772551	0.026204	0.026204
2.5	2.271833	2.271832	0.007919	0.007919
3.0	2.771710	2.771710	0.001911	0.001911
3.5	3.271693	3.271693	0.000366	0.000366
4.0	3.771691	3.771691	0.000054	0.000055
4.5	4.271691	4.271691	0.0000060	0.0000060
5.0	4.771691	4.771691	0.0	0.0

Table 3: Local Nusselt number for different values of N_c , λ, and Pr

Nc	λ	Pr	$-\theta'(0)$
0.1	0	0.72	0.08412
		1.0	0.08582
		2.0	0.08884
0.4	0.2	0.72	0.23367
		1.0	0.24725
		2.0	0.27392
1	1	0.72	0.40371
		1.0	0.44379
		2.0	0.53016

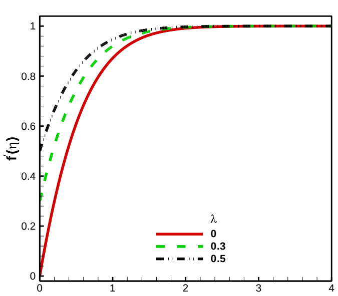

Fig. 1. Effect of stretching parameter λ on dimensionless velocity

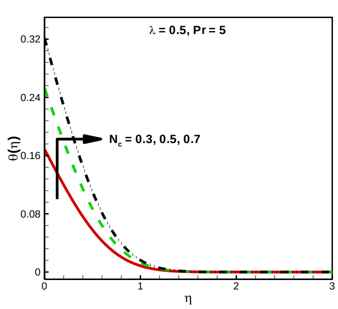

Fig. 2. Effect of convective parameter N_c on dimensionless temperature

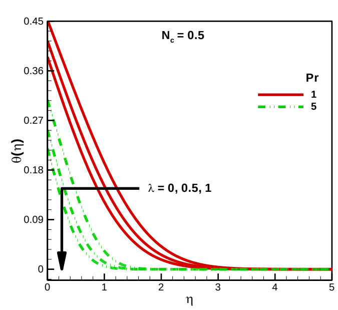

Fig. 3. Effects of stretching parameter and Prandtl numberon dimensionless temperature

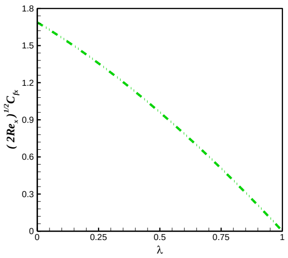

Fig. 4. Variation of skin friction coefficient stretching parameter

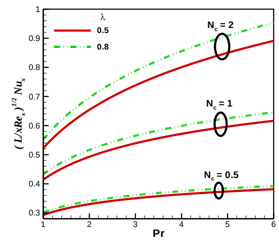

Fig. 5. Variation of heat transfer rate against Prandtl number, stretching and convection parameters

6. CONCLUSION

This paper deal with the analytical study ofstagnation point flow and heat transfer past over anexponential stretching sheet. The validity of our solutions is verified by the numerical results. We analyzed the convergence of the obtained series solutions, carefully. Unlike perturbation methods, the HAM does not depend on any small physical parameters. Thus, it is valid for both weakly and strongly nonlinear problems. Besides, The HAM provides us with a convenient way to control the convergence of approximation series, by means of auxiliary parameter $\hbar$, which is a fundamental qualitative difference in analysis between the HAM and other methods. Graphs are plotted to analyze the variation of the pertinent flow parameters including the stretching parameter λ , Prandtl numberPr, and convection parameter N_C . From the present analysis, we note that the behavior of stretching parameter λ on the dimensionless velocity and temperature is found to be same, i.e. decreases.

It is presumed that, with the help of the present model, the physics of the flow along the vertical channel can be utilized as the basis for many engineering and scientific applications. The findings of the present problem are also of great interest in engineering, industrial, and environmental applications, such as extrusion of polymer, the cooling of metallic plates, filament extrusion from a dye, and in paper production.

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