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A COMPUTATIONAL ANALYSIS OF HEAT TRANSFER IN A THIRD GRADE FLUID FLOW NEAR THE ORTHOGONAL STAGNATION – POINT ON A VERTICAL SURFACE WITH HEAT GENERATION

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Abstract

This paper examined the flow and heat transfer in a third grade fluid flow near the orthogonal stagnation - points on a vertical surface in the presence of heat generation are studied. The governing system of partial differential equations is transformed into a system of ordinary differential equations using the usual similarity transformation method. The resulting equations are then solved numerically by using Galerkin's method of weighted residuals. Effects of buoyancy or mixed convection parameter, Weissenberg parameter, local Reynolds parameter, Prandtl parameter, third – grade parameter, Eckert parameter, and heat generation parameter on the flow velocity and heat transfer characteristics are thoroughly investigated. Also, the effects of the pertinent parameters on the local skin friction coefficient and local Nusselt number are presented numerically in tabular form and compared with the existing results. The results show that the observed parameters have a significant influence on the flow and heat transfer.

Keywords: convective heat transfer; heat generation; stagnation – point flow; third - grade fluid; viscous dissipation.

1. INTRODUCTION

The flow of a stagnation point in a plane was pioneered by Hiemenz[1]. He considered a two – dimensional stagnation flow problem on a stationary plate and used similarity transformation to reduce the Navier – Stokes equation to a system of nonlinear ordinary differential equations. His idea was extended by various researchers to study different aspects of stagnation – point flow problems. Eckert [2] extended the Hiemenz [1] work by considering the energy equation and obtained the exact solution corresponding to the thermal field problem. Stuart [3] studies the viscous flow near a stagnation point when the external flow has uniform vorticity. Tamada [4] investigate two – dimensional stagnation point flow impinging obliquely on a plane wall. The solution of stagnation – point flow with heat transfer analysis by optimal homotopy asymptotic method was studied by Shah, et' al. [5]. Pop, et'al. [6] Studied radiation effects on the flow near stagnation – point of a stretching sheet using Runge – Kutta method couple with a shooting technique. Stagnation – point flow towards a stretching vertical sheet with slip effect was obtained by Khairy, et.al [7]. Magnet hydrodynamic Stagnation – point flow towards a stretching surface was reported by Mahapatra et' al [8]. They obtained a numerical solution of the problem by fourth – order Runge – Kutta integration technique. Hayat, et al [9] studied Peristaltic MHD flow of third – grade fluid with an Endoscope and variable viscosity.

The flows of non – Newtonian fluids have important role to play in several industrial and engineering processes. Unlike Newtonian fluid, the relationship between shear stress and shear rate in the non – Newtonian fluids is nonlinear. In these fluids, the constitutive relationships between shear stress and shear rate are much complicated in comparison to the Navier – Stokes equations. The simplest subclass of non – Newtonian differential type fluids is called second – grade. The second – grade models describe normal stress with no capacity to predict shear thinning / thickening effect which the third – grade fluid model has capacity to attain. Despite the complexities in the constitutive equations of third – grade fluid, several researchers have investigated the flow taking into account various factors. Sahoo [10] numerically investigated the

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Hiemenz flow and heat transfer of a third – grade fluid. Non - Similar series solution for boundary layer flow of a third – order fluid over a stretching sheet was examined by Sajid et' al [11]. Ramachandran [12] studied mixed convention in stagnation flows adjacent to the vertical surface. Stagnation – point flow of a micropolar fluid towards a stretching sheet was extensively discussed by Nazar, et' al [13].
This paper extends the work of Javed et' al [14] by examining the flow and heat transfer in a third – grade fluid near the orthogonal stagnation – point on a vertical surface. To the best of our knowledge no investigation regarding the effects of heat transfer in a third – grade fluid flow near the orthogonal stagnation – point when the heat generation is important. The Galerkin's method of weighted residuals is used to solve the governing mathematical equations and effects of important flow and heat transfer parameters are carefully studied.

2. MATHEMATICAL FORMULATION

We consider a steady two – dimensional convective heat transfer in a third – grade fluid flow near the orthogonal stagnation – point on a vertical surface in the presence of thermal radiation. We select the Cartesian coordinate system in such a manner that $x\ and\ y - axes$ is parallel and perpendicular to the plate. The velocity components of the inviscid fluid in the neighbourhood of the stagnation point are $U_e = ax\ and\ V_e = -ay$ (a is a positive constant). Heat wall temperature T_w higher than the ambient temperature T_∞.
Coleman and Noll [15] defined the incompressible fluid of differential type of grade n as the simple fluid obeying the constitutive equation.

$$T = -pI + \sum_{j=1}^{n} S_j \tag{1}$$

If $n = 3$ the first three tensors S_j are given by

$$S_1 = \mu A_1 \tag{2}$$

$$S_2 = \alpha_1 A_2 + \alpha_2 A_1^2 \tag{3}$$

$$S_3 = \beta_1 A_3 + \beta_2 (A_1 A_2 + A_2 A_1) + \beta_3 (trA_1^2) A_1 \tag{4}$$

Where μ is the coefficient of shear viscosity; $\alpha_i (i = 1,2), \beta_i (i = 1,2,3)$ are material constants. The Rivlin – Ericksen tensors (A_n) are defined by recursion relation:

$$A_n = \frac{dA_{n-1}}{dt} + A_{n-1}(grad\boldsymbol{V}) + (grad\boldsymbol{V})^T A_{n-1}$$
$$n \geq 0 \tag{5}$$

Where d/dt signifies material time differentiation
If the third – grade fluid model involves the thermodynamic constraints [16]

$$\mu \geq 0, \quad \alpha_1 \geq 0, \quad |\alpha_1 + \alpha_2| \leq \sqrt{24\mu\beta_3}, \quad \beta_1 = \beta_2 = 0, \quad \beta_3 \geq 0.$$

Then the Cauchy stress tensor for third – grade takes the form of Eq. (1) is

$$T = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_3 (trA_1^2) A_1.$$

The governing boundary layer equations for the considered flow problem in the presence of viscous dissipation and heat generation are written as:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0; \tag{6}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1}{\rho}\left(u\frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + 3\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + v\frac{\partial^3 u}{\partial y^3}\right) + 2\frac{\alpha_1}{\rho}\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + 6\frac{\beta_3}{\rho}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) = 0 \tag{7}$$

$$-\frac{\partial p}{\partial y} + (2\alpha_1 + \alpha_2)\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^2 \tag{8}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\alpha_1}{\rho c_p}\left(u\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + v\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\right) + 2\frac{\beta_3}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^4 + \frac{Q_o}{\rho c_p}(T - T_\infty) \tag{9}$$

Where $u\ and\ v$ are the velocity components in the $x and y$ directions, respectively, $\rho\ and\ p$ are the fluid density and pressure, $\alpha_1, \alpha_2, and\ \beta_3$ are the material parameters of the fluid, g is the acceleration due to gravity, β is the thermal expansion coefficient, T is the temperature of the fluid within the boundary layer, c_p is the specific heat and Q_o is the heat generation. The last term in the right hand side of Eq. (7) represents the buoyancy force effects. The positive value of the buoyancy force term is taken if the flow is directly vertical upward(''assisting '' flow) and the negative value of the buoyancy force term is taken if the flow is directed vertically downward(''opposite'' flow). This paper considered only, the positive value of the buoyancy or mixed convection force term.

From Eq. (8) the modified pressure are obtained as:

$$\dot{p} = p - (2\alpha_1 + \alpha_2)\left(\frac{\partial u}{\partial y}\right)^2$$

From Eqs. (7) and (8) the following expressions are obtained

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial \dot{p}}{\partial x} + v\frac{\partial^2 u}{\partial y^2} +$$
$$\frac{\alpha_1}{\rho}\left(u\frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + v\frac{\partial^3 u}{\partial y^3}\right)$$
$$+ 6\frac{\beta_3}{\rho}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty); \qquad (10)$$
$$\frac{\partial \dot{p}}{\partial y} = 0$$

By expression for the free-stream velocity u_e , we get for Eqs. (9) and (10) in the form

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + \alpha\frac{\partial^2 T}{\partial y^2} + \frac{v}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\alpha_1}{\rho c_p}\left(u\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + v\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\right)$$
$$+ 2\frac{\beta_3}{\rho c_p}(\frac{\partial u}{\partial y})^4 + \frac{Q_o}{\rho c_p}(T - T_\infty) \qquad (11)$$
$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{\partial u_e}{\partial x} + v\frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1}{\rho}\left(u\frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + v\frac{\partial^3 u}{\partial y^3}\right)$$
$$+ 6\frac{\beta_3}{\rho}(\frac{\partial u}{\partial y})^2 + \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty); \qquad (12)$$

The boundary conditions in the presence of velocity and thermal slip are

$$y = 0; \quad u = \frac{\gamma_1}{\mu}\tau_{xy,} v = 0, \quad T = T_w + \gamma_2\frac{\partial T}{\partial y}$$
$$y \to \infty: \quad u = u_e = ax, \quad v = v_e = -ay, \quad T = T_\infty$$

Where $\gamma_1 and \gamma_2$ are the velocity and thermal slip parameters.

$T_w = T_\infty + \Delta T x$ ($\Delta T is the temperature\ difference$)is equal to the wall temperature which varies linearly along the x axis.

Introducing the following dimensionless quantities, the mathematical analysis of the problem is simplified by using similarity transforms:

$$\eta = \sqrt{\frac{a}{v}}y, \quad u = axf'(\eta), \quad v = -\sqrt{av}f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}.$$

The governing equations (11) and (12), which are nonlinear partial differential equations transform into a system of ordinary differential equations.

$$f''' - f'^2 + ff'' + 1 + We\left(2f'f'' - ff'''' - f''^2\right) + 6\varepsilon Re_x f'''f''^2 + \lambda\theta = 0$$
$$\theta'' + Pr[f\theta' - f'\theta + SPr\theta]$$
$$+ PrEc\left[f''^2 + We\left(ff''^2 - ff''f'''\right) + 2\varepsilon Re_x f''^4\right] = 0 \qquad (13)$$

Where the prime denotes differentiation with respect to η, $\Pr = \frac{v}{\alpha}$ is the Prandtl number, $We = a\alpha_1/\mu$ is the Weissenberg number, $\varepsilon = \beta_3 a^2/\mu$ is the third – grade parameter, $Re_x = ax^2/v$ is the local Reynolds number, $\lambda = Gr_x/Re_x^2$ is the buoyancy or mixed convection parameter, $Gr_x = g\beta(T_w - T_\infty)x^3/v^2$ is the local Grashof number, $Ec = a^2x^2/[c_p(T_w - T_\infty)]$ is the Eckert number and $S = Q_o/a\rho C_p$ is the heat generation.

The corresponding boundary conditions take the form

$$f(0) = 0, \quad f'(0) = \gamma_v f''(0)[1 + 3Wef'(0) + 2\varepsilon Re_x(f''(0))^2],$$
$$f'(\infty) = 1, \quad f''(\infty) = 0, \quad \theta(0) = 1 + \gamma_t\theta'(0), \quad \theta(\infty) = 0, \qquad (14)$$

Where $\gamma_v = \gamma_1\sqrt{a/v}\ and \gamma_t = \gamma_2\sqrt{a/v}$ are the dimensionless velocity and thermal slip parameters.

The important physical quantities of interest in this problem are local skin friction C_{fx} and the local Nusselt number Nu_x which are defined as:

$$C_{fx} = \frac{\tau_{xy}}{\rho {u_e}^2}, \qquad Nu_x = \frac{xq_w}{k(T_w - T_\infty)} \qquad (15)$$

(k is the thermal conductivity of the fluid); where the shear stress τ_{xy} and the wall heat flux q_w are given by the formulas

$$\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + \alpha_1\left(u\frac{\partial^2 u}{\partial x \partial y} + u\frac{\partial^2 v}{\partial x^2} + v\frac{\partial^2 u}{\partial y^2} + v\frac{\partial^2 v}{\partial x \partial y} + 2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + 2\frac{\partial v}{\partial x}\frac{\partial v}{\partial y}\right) + \beta_3\left\{\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\left[4(\frac{\partial u}{\partial x})^2 + 2(\frac{\partial u}{\partial y})^2 + 4\frac{\partial v}{\partial x}\frac{\partial u}{\partial y} + 2(\frac{\partial v}{\partial x})^2 + 4(\frac{\partial v}{\partial x})^2\right]\right\}|_{y=0},$$

$$q_w = -k\frac{\partial T}{\partial y}|_{y=0}$$

In the dimensionless form, Eq. (15) is reduced to

$$C_{fx} = Re_x^{-1/2}[f'' + We(3f'f'' - ff''') + 2\varepsilon Re_x f''^3]|_{\eta=0},$$

$$Nu_x/ Re_x^{1/2} = -\theta'(0)$$

$$f(0) = 0, \quad f'(0) = \gamma_v f''(0)[1 + 3Wef'(0) + 2\varepsilon Re_x(f''(0))^2],$$

$$f'(\infty) = 1, \ f''(\infty) = 0, \ \theta(0) = 1 + \gamma_t\theta'(0), \ \theta(\infty) = 0 \qquad (16)$$

Where $\gamma_v = \gamma_1\sqrt{a/v}\, and \gamma_t \gamma_2\sqrt{a/v}$ are the dimensionless velocity and thermal slip parameters.

3. METHOD OF THE SOLUTION

In order to solve the coupled nonlinear system of ordinary differential equations (13) subject to the boundary conditions (14), the Galerkin's method of weighted residual is used [17 – 19]. The idea of Galerkin's method of weighted residuals is to seek an approximate solution, in form of a polynomial, to the differential equation of the form

$$L[u(x)] = f in the domain \Omega \qquad (17)$$

$$B_\mu[u] = \gamma_\mu on \partial\Omega \qquad (18)$$

Where $L[u]$ denotes a general differential operator (linear or nonlinear) involving spatial derivatives of dependent variable u, f is a known function of position, $B_\mu\,[u]$ represents the appropriate number of boundary conditions and Ω is the domain with boundary $\partial\Omega$.

The function u (i.e. solution) must satisfy the operator equation (18), and also required to satisfy the boundary condition (19)

$$u = u_0 + \sum_{i=1}^{n} c_i u_i \qquad (19)$$

Where c_i are constants to be determined which satisfies the given boundary conditions (19), is assumed to be the solution of (18). The trial function is chosen in a way to satisfies all the boundary conditions including those at infinity. In this case, functions such as $e^{-nx}, for n > 0$ are included in the trial function. This will make the trial function satisfy the boundary condition naturally.

Substitution of equation (20) into equation (18) gives the residual function$R(x)$. The idea is to minimize the residual function to be as small as possible.

In the Galerkin's method the weight function $u_j where j = 1,2, \dots n$, are used to multiply the residual $R(x)$, that is

$$\int_D u_j R(x)dx = 0, \quad j = 1,2, \dots n. \qquad (20)$$

We now use the Gauss – Laguerre formula to integrate each of the equations (21) to sets of simultaneous equations which are used to solve for the parameter $c_{i.}$

Table1. Comparison of results of skin friction number $f''(0)$ and Nusselt number $-\theta'(0)$ for $\Pr = \lambda = 0.2, Ec = Re_x = \varepsilon = \gamma_t = \gamma_v = S = 0$, and different values of We

$f''(0)$

We	Resultfrom Li et al[20]	Resultfrom Hayat et al [21]	Resultfrom Javed et al [15]	Present result
0	1.35426	1.3543	1.35426	1.35426
0.5	0.98230	0.9821	0.98230	0.982297
1.0	0.71694	0.8174	0.81738	0.817376
1.5	0.71694	0.7171	0.71694	0.716944
2.0	0.64713	0.6474	0.64713	0.647134
		$-\theta'(0)$		
0	0.44198	0.4420	0.44198	0.441983
0.5	0.40990	0.4097	0.40990	0.4099
1.0	0.39180	0.3920	0.37922	0.391892
1.5	0.37922	0.3793	0.37922	0.379223
2.0	0.36944	0.3698	0.36944	0.369438

In the present study the following parameter values are adopted for computations: $\lambda = 1, \Pr = 0.5,\ Ec = 0.05,\ We = 0.3, Re = 0.1, \varepsilon = 1,\ \gamma_1 = 0.2,\ \gamma_2 = 0.1\ and S = 1$.All tables and graphs therefore correspond to these values unless specifically indicated on the appropriate graph and table.

Table2 are prepared to illustrate the effects of Prandtl, buoyancy or mixed convection and heat generation parameters.

Table2. Computed values of skin friction coefficient $f''(0)$ for various values of $Pr, \lambda, and Rd$.

Pr	λ	S	$f''(0)$
0.5	1. 0	1. 0	0.995434
1.0	1. 0	1. 0	0.995573
0.5	3. 0	1. 0	1.27199
0.5	1. 0	5. 0	1.15071

Table 3. Computations showing the local skin friction $f''(0)$ and the local Nusselt number $-\theta(0)$.

Pr	λ	We	$f''(0)$	$-\theta'(0)$
0.5	1.0	0.5	1.15071	-0.820408
1.0	1.0	0.5	0.739863	1.5252
0.5	3.0	0.5	1.44509	-0.243772
0.5	1.0	0.8	0.982596	-0.41675

Table 2 presents the variation of the skin friction coefficient in relation to Prandtl, mixed convection and heat generation parameters. On observing the table, as both the value of mixed convection and heat generation parameters increases, the value of skin friction coefficient increase. However, the skin friction coefficient increases as the value of the Prandtl parameter increases although marginally. Table 3 has been prepared to illustrate the effects of Prandtl, mixed convection and the Weissenberg numbers on the skin friction coefficient and the local Nusselt numbers. The first two entries shows that for a fixed buoyancy or mixed convection $\lambda = 1$, the skin friction coefficient number decreases with increase in Prandlt number. However, the local Nusselt number increases as λ are increased from 0.5 to 1.0. The last two entries in Table 3 shows that when the value of buoyancy or mixed convection increases from 1.0 to 3.0 the skin friction increases sharply while the local Nusselt number also increases. The increment in Weissenberg number from 0.5 to 0.8 leads to a decrease in skin friction coefficient and increase in the local Nusselt number compared with the first value on Table 3.

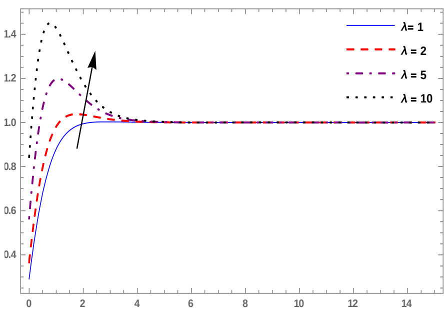

Figure 1 Velocity profiles for different values of λ

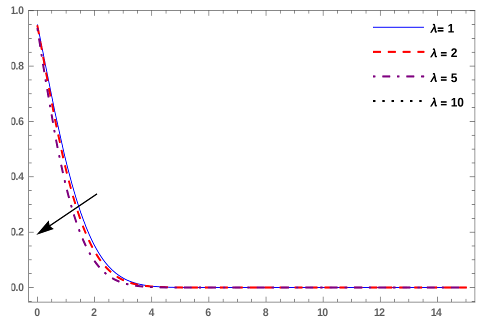

Figure 2 Temperature profiles for different values of λ

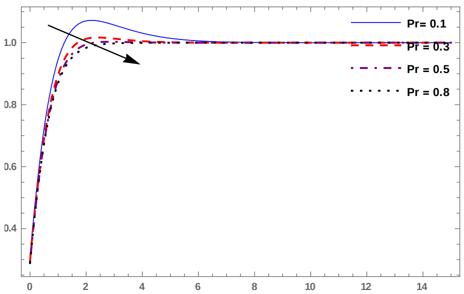

Figure 3 Velocity profiles for different values of Pr

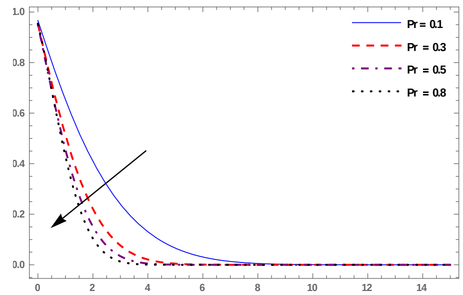

Figure 4 Temperature profiles for different values of Pr

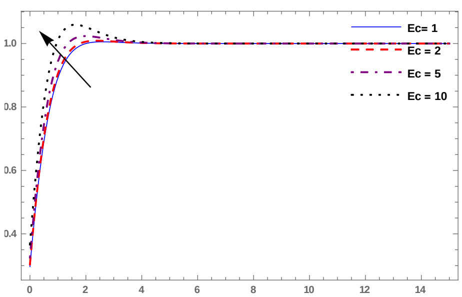

Figure 5 Velocity profiles for different values of Ec

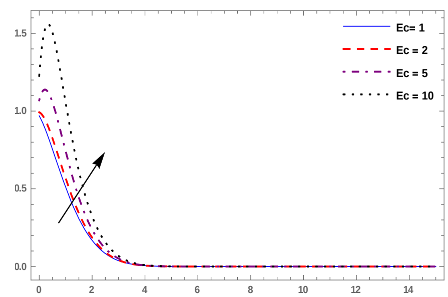

Figure 6 Temperature profiles for different values of Ec

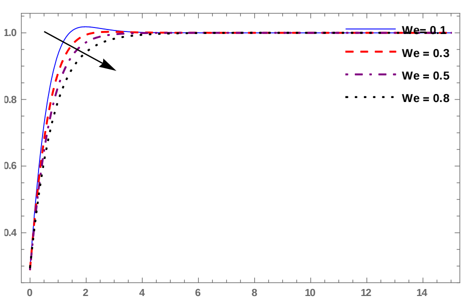

Figure 7 Velocity profiles for different values of We

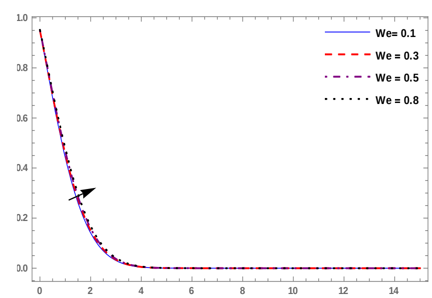

Figure 8 Temperature profiles for different values of We

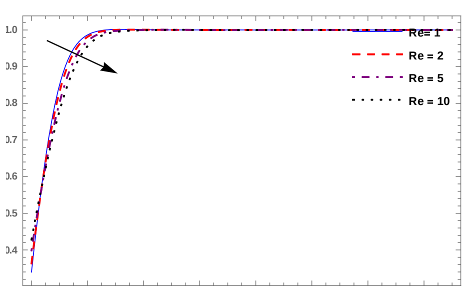

Figure 9 Velocity profiles for different values of Re

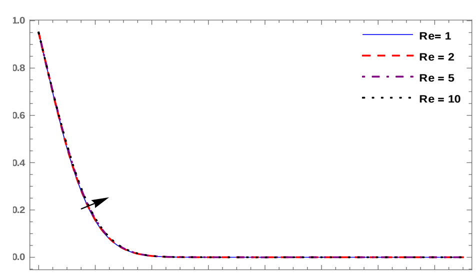

Figure 10 Temperature profiles for different values of Re

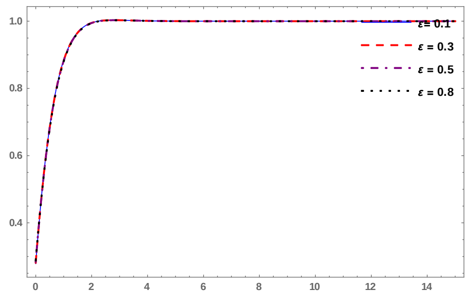

Figure 11 Velocity profiles for different values of ε

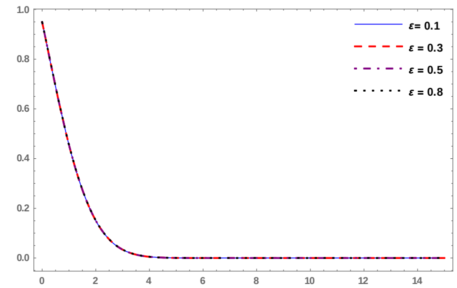

Figure 12 Temperature profiles for different values of ε

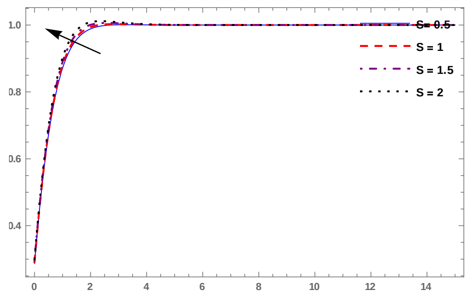

Figure 13 Velocity profiles for different values of S

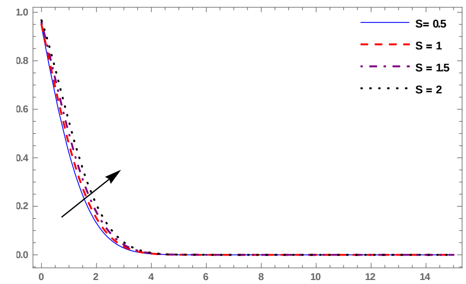

Figure 14 Temperature profiles for different values of S

Figures 1 and 2 demonstrate the effect of buoyancy or mixed convection parameter λ on the velocity and temperature profiles. As an output of figures, it is seen that the velocity of the fluid across the boundary layer increase by increasing the buoyancy or mixed convection parameter λ and the temperature of the fluid across the thermal boundary layer decreases by decreasing the buoyancy or mixed convection parameter. Prandtl parameter Pr effect on the velocity and temperature profiles are shown in figures 3 and 4 The effects of Prandtl Pr are decreasing in both the velocity and temperature profiles. Figures 5 and 6 depicts for the effects of Eckert number on velocity and temperature profiles. It is clear from the figures that velocity and temperature distribution increases in both profiles with an increase in Eckert number Ec. Figures 7 and 8 are the graphical representation of the velocity and temperature profiles for different value of Weissenberg number We**.** It can be observed from these figures that the velocity profile decreases with an increase in Weissenberg number We and thermal boundary layer thicken with an increase in Weissenberg parameter We.
The effect of the local Reynolds number Re is to reduce the velocity profiles and thickens thermal boundary layer with an increase in the local Reynolds numberRe. Figures 13 and 14 presents the various velocity and temperature profiles for various values of heat generation S. From the graphs, it is obvious that the velocity and temperature of the fluid accelerates with an increase in the heat generation.

4. CONCLUSION

A numerical study of heat transfer in a third – grade fluid flow near the orthogonal stagnation – point on a vertical surface with heat generation has been performed. A similarity transformation is been used to transform the governing partial differential equations into ordinary differential equations, a convenient form for numerical computation. These equations are solved numerically using the Galerkin's method of weighted residual. The main findings of the study are summarized as follows:

a. The velocity boundary layer increases with an increase in the buoyancy or mixed convection parameter.
b. The thermal boundary layer decreases with an increase in buoyancy or mixed convection parameter.
c. The velocity boundary layer decreases with an increase in the Prandtl number
d. Thermal boundary layer thickness decrease due to increase in Prandtl number Pr.
e. The velocity boundary layer thickens with an increase in the heat generation sources.
f. The thermal boundary layer increases with an increase in the values of the heat generation parameter.

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