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PARAMETRIC SENSITIVITY ANALYSIS OF A MATHEMATICAL MODEL OF THE EFFECT OF CO_2 ON CLIMATE CHANGE

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Abstract

Mathematical modeling is one of the most powerful methods for the study and understanding of the Earth's climate system and its components. Modern climate models used in a variety of applications are derived from a set of multi-dimensional nonlinear differential equations in partial derivatives, which describe dynamical, physical and chemical processes and cycles in the climate system. Climate models contain a wide number of model parameters that can describe external forcing that can strongly affect the behavior of the climate. It is imperative to estimate the influence of variations in parameters on climate change. The methods of 1-norm, 2-norm, and infinity-norm were used to quantify different forms of the sensitivity of model parameters. The study shows that the most sensitivity parameters in the model are the concentration of a suitable absorbent and the rate of inflow of absorbent in the absorption chamber.

Keywords: Sensitivity Analysis, Mathematical model, climate change

1. Introduction

One of the biggest issues facing humanity today is the observed ongoing global climate change. Consequently, the prediction of future climate as well as changes in climate due to changes in natural processes and human-caused factors (e.g. greenhouse gas emissions) are issues that have deservedly received significant attention. The essential, powerful and effective methodology for solving this class of problems is computational simulation of the Earth's climate system (ECS) and its components with the use of mathematical climate models that can range from relatively simple to fairly complex. Over the past several decades, the use of climate models as an aid in the understanding past, present and future climates has been substantial. State-of-the-art mathematical climate models used in variety of applications represent systems of multi-dimensional, non-linear differential equations in partial derivatives that are the mathematical statements of basic physical laws, primarily the conservation laws for momentum, mass and energy. Such models also include a variety of empirical and semi-empirical relationships and equations that are based on observations and experience rather than theories.

Mathematical climate models are mostly deterministic with a large-phase space dimension, containing a vast number of various parameters [1-7]. Equations that describe the evolution of the ECS and its processes are quite complicated. Therefore, in the majority of situations, we, unfortunately, cannot solve them analytically with an arbitrary set of initial conditions, even for very simple cases. We can only find an approximate solution using numerical methods such as, for example, Galerkin projection or finite-difference technique. Consequently, climate models have finite space and time resolutions. Due to the limited resolutions of climate models, many physical processes those are very important for climate dynamics cannot be adequately resolved by the model space-time grid and, therefore, should be parameterized,

In dealing with real-life or physical problems, mathematical modelling is always of great advantage because of its power to predict system behaviour and a clear insight of the important inputs and outputs. Mathematical models are of various forms such as deterministic, stochastic, fuzzy, and uncertain forms [8-9] .However, there is a need to model problems arising from the Global warming due to carbon dioxide potentially affects human and the environment, the mitigation of carbon dioxide from the atmosphere is absolutely necessary. There is no dispute whether the presence of CO_2 in the atmosphere influences temperature rise. What is therefore an open question is the degree of impact of carbon dioxide has on the temperature.

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Researchers have carried out several investigations on ways to reduce the concentration of carbon dioxide into the atmosphere significantly by introducing liquid droplet [10]. But [11] made a theoretical analysis to study the dynamics of carbon dioxide uptake by a liquid droplet and have observed the abatement in carbon dioxide concentration absorbed by liquid droplets. On their part, [12] have established an experimental set-up to calculate the liquid phase mass transfer coefficient of carbon dioxide absorption by single water droplet. The result indicated that the mass transfer coefficient decreases as droplet formation time increases. Also, researches have studied the removal of carbon dioxide by using suitable absorbents such as sodium hydroxide, amines and aqueous ammonia solution [13-16]. It has also been discovered that the reduction in carbon dioxide concentration by aqueous ammonia solution is more predominant. As a result of this, [16] have made an experiment to compare the performance of two absorbents, aqueous ammonia solution and mono-ethanolamine (MEA), for scrubbing carbon dioxide in terms of high CO_2 removal efficiency and absorbing capacity.

Mathematical modelling represents a very powerful and effective instrument to study complex processes occurring in technical, economic, social and natural systems. State of the art mathematical models used in various branches of natural science are defined as a set of (partial) differential equations that contain a large number of parameters some of which can be inaccurate. Parameter errors and their time and space variability generate parametric uncertainty in mathematical models. It is therefore important to estimate the influence of parameter variations on the model output results. Sensitivity analysis, which is an essential element of model building and quality assurance, addresses this very important issue

2. Materials and Methods

Consider an atmosphere affected by global warming gases containing CO2. To successfully model the phenomenon Mathematically, the following assumptions are followed [17].

1. The rate of emission of carbon dioxide is constant.
2. There exists a threshold concentration of carbon dioxide below which harmful effects are insignificant.
3. The rate of introduction of liquid species in the atmosphere is in direct proportion to the difference of cumulative and threshold concentrations of carbon dioxide. Here, cumulative concentration stands for global average temperature of carbon dioxide in the atmosphere.
4. The rate of inflow of absorbent in absorption chamber is in direct proportion to the cumulative concentration of carbon dioxide.
5. The decrease in cumulative concentration of carbon dioxide is directly proportional to the cumulative concentration of CO2 itself and the concentration of externally introduced liquid species. The decrease in concentration of CO2 is also proportional to the product of cumulative concentration of carbon dioxide and the concentration of suitable absorbent.
6. During the interaction of carbon dioxide and externally introduced liquid species, particulate matter is formed which is removed from the atmosphere by gravity, lowering the concentration of carbon dioxide.
7. The natural depletion rates of carbon dioxide and externally introduced liquid species are assumed to be directly proportional to their respective concentrations.

From the above assumptions, the following dynamical systems of nonlinear ordinary differential equations are formulated

$$\frac{dK}{dt} = M - \varphi_0 K - \lambda_1 KA - \eta_1 KK_c \qquad (1)$$

$$\frac{dA}{dt} = -\lambda_1 (K - K_0) - \lambda_0 A - \lambda_1 KA \qquad (2)$$

$$\frac{dK_p}{dt} = \omega\lambda_1 KA - \omega_0 K_p \qquad (3)$$

$$\frac{dK_c}{dt} = \eta K - \eta_0 K_c - \eta_1 KK_c \qquad (4)$$

With the following conditions satisfied $K(0) = K_0 \geq 0, A(0) \geq 0, K_p \geq 0, K_m(0) \geq 0, \quad K > K_0$

K is the cumulative concentration of carbon dioxide, A is the concentration of externally introduced liquid species, K_p is the density of particulate matter formed due to interaction of carbon dioxide with liquid species and K_c be the concentration of a suitable absorbent. Also, M Is the rate of discharge of carbon dioxide from different sources with its natural depletion rate K_0. The depletion of carbon dioxide due to interaction with liquid species is assumed to be in direct proportion to the cumulative concentration of carbon dioxide and the concentration of liquid species.

Furthermore, the constant η_0 is the natural depletion rate coefficient of liquid species. Let ω be the rate by which particulate matter is formed in the atmosphere as a result of interaction of carbon dioxide with liquid species, and ω_0 is its natural depletion rate coefficient. Let η be the rate of inflow of absorbent in absorption chamber with its natural depletion rate $\eta_0 K_m$. The depletion of carbon dioxide due to interaction with absorbent is assumed to be in direct proportion to the cumulative concentration of carbon dioxide and that of absorbent and by the same amount absorbent is also used. The constant η_1 is the interaction rate coefficient of carbon dioxide with absorbent. All the above constants are assumed to be positive.

2. Numerical Simulation

In this section, we perform a numerical simulation of model (1) – (4) with respect to E^*, for the different values of parameters for the validation of analytical results and to study the dynamical behavior of the model system. For that the system (1) – (4) is integrated numerically with the help of MAPLE 7 by considering the following set of parameter values, we shall adopt the model parameter values as proposed by

$$M = 1,\ \varphi_O = 0.1, \lambda_1 = 0.5, \lambda = 0.4, \lambda_0 = 0.2, \omega = 0.8, \omega_0 = 0.7, K_0 = 0.60, \eta = 1, \eta_0 = 0.02, \eta_1 = 0.6$$

3. Method Analysis

The approximate model formulation was constructed on the basis of these simplifying assumptions. The method of sensitivity analysis which we have used in this study has been adapted from recent research reports [18]. This involves coding the given system of continuous non-linear first order ordinary differential equation in a Matlab programming language, varying a single parameter one-at-a-time while other model parameters are fixed, formulating an appropriate ODE solver which will simulate the program designed, using the program designed to calculate the 1-norm, 2-norm, 3-norm and infinity norm of the solution trajectories and finally, interpret the result quantitatively. That is, the parameter which when varied a little and produces the biggest cumulative effect on the solution trajectories is called a most sensitive parameter.

4. Results and Discussion

Efforts shall be made to present and discuss our results which we have achieved in this study.

Table 1: Percentage variations of A

Variation		1 *percent*	2 *percent*	3 *percent*	4 *percent*	5 *percent*	10 *percent*	20 *percent*	30 *percent*
$1-norm$	16.721	21.8460	21.783	21.7214	21.6594	21.597	21.293	20.7021	20.1345
$2-norm$	3.9307	5.0983	5.0840	5.0697	5.0555	5.0414	4.9716	4.8363	4.7066
$3-norm$	2.4597	3.1685	3.1598	3.1511	3.1424	3.1338	3.0913	3.0090	2.9301
$\infty-norm$	1.1575	1.4250	1.4215	1.4181	1.4147	1.4113	1.3958	1.3664	1.3377

Table 2: Percentage variations of K_c

Variation		1 *percent*	2 *percent*	3 *percent*	4 *percent*	5 *percent*	10 *percent*	20 *percent*	30 *percent*
$1-norm$	16.721	53.9557	53.110	52.2854	51.4790	50.690	47.007	40.7412	35.6403
$2-norm$	3.9307	12.7639	12.554	12.3509	12.1520	11.958	11.055	9.5380	8.3190
$3-norm$	2.4597	7.9692	7.8351	7.7045	7.5773	7.4534	6.8787	5.9193	5.1554
$\infty-norm$	1.1575	3.3376	3.2786	3.2219	3.1665	3.1123	2.8688	2.4767	2.1778

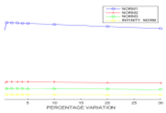

Figure1: A graph of Mathematical Norms of the Solution trajectories against percentage variation of parameter A

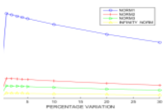

Figure 2: A graph of Mathematical Norms of the Solution trajectories against percentage variation of parameter K_c

Table 3: Percentage variations of λ_1

Variation		$1\, percent$	$2\, percent$	$3\, percent$	$4\, percent$	$5\, percent$	$10\, percent$	$20\, percent$	$30\, percent$
$1-norm$	16.7213	53.9557	53.110	52.2854	51.4790	50.690	47.007	40.7412	35.6403
$2-norm$	3.9307	12.7639	12.5548	12.3509	12.1520	11.9580	11.0556	9.5380	8.3190
$3-norm$	2.4597	7.9692	7.8351	7.7045	7.5773	7.4534	6.8787	5.9193	5.1554
$\infty-norm$	1.1575	3.3376	3.2786	3.2219	3.1665	3.1123	2.8688	2.4767	2.1778

Table 4: Percentage variations of η_1

Variation		$1\, percent$	$2\, percent$	$3\, percent$	$4\, percent$	$5\, percent$	$10\, percent$	$20\, percent$	$30\, percent$
$1-norm$	16.7213	21.8460	21.7836	21.7214	21.6594	21.5978	21.2931	20.7021	20.1345
$2-norm$	3.9307	5.0983	5.0840	5.0697	5.0555	5.0414	4.9716	4.8363	4.7066
$3-norm$	2.4597	3.1685	3.1598	3.1511	3.1424	3.1338	3.0913	3.0090	2.9301
$\infty-norm$	1.1575	1.4250	1.4215	1.4181	1.4147	1.4113	1.3958	1.3664	1.3377

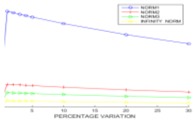

Figure 3: A graph of Mathematical Norms of the Solution trajectories against percentage variation of parameter λ_1

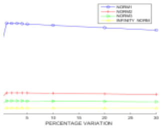

Figure 4: A graph of Mathematical Norms of the Solution trajectories against percentage variation of parameter η_1

From Figures 1–4, it is convenient to classify the identified parameters with respect to their degree of sensitivity and relative significance.

5. Discussion of results

From our results which have been presented in the previous section, we observe that the parameters K_c and λ_1 which represents be the concentration of a suitable absorbent and the rate of inflow of absorbent in absorption chamber respectively can be classified as the most sensitive parameters using the 1-norm and 2-norm estimated sensitivity values while the other parameters can be classified as relatively least sensitive parameters.

6. Conclusion

The sensitivity of the parameters in the model shows that the higher the degree of the concentration of a suitable absorbent and the rate of inflow of absorbent in absorption chamber, the higher the quantity of carbon dioxide in the atmosphere which implies that the more of these you have , the more the temperature rise in the atmosphere which is in total agreement with result in recent literatures that when the carbon dioxide concentration goes up, temperature goes up and when the carbon dioxide concentration goes down, temperature goes down. This study has shown the more sensitive parameter in the model that can bring about climate change is K_c .

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