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ON THE PROPERTIES OF NEW WEIBULL-INVERSE LOMAX DISTRIBUTION

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Abstract

In this paper, we proposed another extension of the Inverse Lomax distribution and study some of its statistical properties. This properties include the quantile function, median through the quantile function, moments, moments generating function, entropies, and ordered statistics. The distribution is a fat-tail, with decreasing hazard rate function and right skewed density. The higher moments does not exist and can be approximated with the Euler's constant. We estimated the parameters of the distribution by the method of maximum likelihood.

Keywords: Inverse Lomax, Moments, Euler's constant, Entropies, ordered statistics.

1. Introduction

A mixture of exponential distributions and gamma mixing weights is the Pareto distribution (Lomax). Although of the gamma distribution relationship, Pareto's distribution is a highly customized distribution. The distribution of Pareto is therefore ideal for modeling extreme losses, e.g. rare but potentially devastating losses. The distribution is said to be a heavy tail distribution when a distribution significantly increases the probability of higher values. And the distribution of Inverse Lomax is one of them. The distribution of Pareto has a negative bias and a large tail to the left. It is an excellent model for extreme events, like, for example, the long tail or more chances. Initially it was used as a framework to describe the income and wealth of a nation. The large-scale delivery study provides information on the potential for financial failure and financial ruin in financial systems [1].

Inverse Lomax distribution belongs to the Generalized Beta family of distributions [2]. If a random variable say R has Lomax distribution, then $X = \frac{1}{R}$ has an Inverse Lomax distribution (ILD). It has been utilized to get the Lorenz ordering relationship among ordered statistics and also have applied this model on geophysical data, specifically on the sizes of land fires in California state of United States,[3]. Apart from this, it has also many applications in economics, actuarial sciences, and stochastic modeling see[2]. Moreover, [4] have discussed the estimation and prediction challenges for the inverse Lomax distribution via Bayesian approach. Also, [5] have used this distribution for reliability estimation based on Type-II censored observations. Inverse-Lomax distribution reside in inverted family of distributions and established to be very useful to analyze the situation where the non-monotonicity of the failure rate has been accomplished [6]. Some details about the Inverse Lomax distribution including its applications are available in [7], [8], [9] and [10].

2. The Inverse-Lomax distribution and the New Weibull G- Family

In this section we discussed about the baseline and the generator used in this study. The probability density function (pdf) and cumulative distribution function(cdf) of ILD are given by the following equations as define by [5] as:

$$g(x;\gamma,\lambda) = \frac{\gamma\lambda}{x^2}\left(1+\frac{\gamma}{x}\right)^{-(1+\lambda)} \quad (1)$$

$$G(x;\gamma,\lambda) = \left(1+\frac{\gamma}{x}\right)^{-\lambda} \quad (2)$$

where $x > 0$, $\gamma > 0$ is a scale parameter and $\lambda > 0$ is a shape parameter respectively.

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Let $g(x;\eta)$ and $G(x;\eta)$ denote the probability density (pdf) and cumulative functions (cdf) of a baseline model with parameter vector η. Based on this cdf, Tahir et al. [11] replaced the argument $W[G(x)] = -log[G(x;\eta)]$ and defined the cdf and the pdf of the New Weibull-G family by:

$$F(x;\alpha,\beta,\eta) = 1 - \int_0^{-log[G(x;\eta)]} \alpha\beta v^{\beta-1} \exp - \alpha v^{\beta} dv = \exp\{-\alpha\{-log[G(x;\eta)]\}^{\beta}\} \tag{3}$$

where $G(x;\eta)$ is any baseline cdf which depends on a parameter vector η.

$$f(x;\alpha,\beta,\eta) = \alpha\beta \frac{g(x;\eta)}{G(x;\eta)} \{-log[G(x;\eta)]\}^{\beta-1} \exp\{-\alpha\{-log[G(x;\eta)]\}^{\beta}\} \tag{4}$$

A random variable X with density function is denoted by X: NWeib-G(α, β, η). The extra parameters induced by the New Weibull generator are sought just to increase the flexibity of the distribution.

2.1 The New Weibull Inverse Lomax (NWIL) Distribution

Defination 1: let X be a random variable that follows a New Weibull-Inverse lomax distribution i.e $X :\ NWIL(\alpha,\beta,\gamma,\lambda)$ then, its probability density function (pdf) is given by:

$$f(x;\theta) = \begin{cases} \alpha\beta\gamma\lambda x^{-2}\left(1+\frac{\gamma}{x}\right)^{-1}\left\{-log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]\right\}^{\beta-1} \exp\left\{-\alpha\left\{-log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]\right\}^{\beta}\right\}, & x \geq 0 \\ \end{cases} \tag{5}$$

where $\theta = (\alpha,\beta,\gamma,\lambda)$. Also, consider a continuous random variable X with pdf $f(x;\alpha,\beta,\gamma,\lambda)$. Then, for all $x \in \mathrm{R}$:

i). $f(x;\alpha,\beta,\gamma,\lambda) \geq 0$.

ii). $\int_{-\infty}^{\infty} f(x;\alpha,\beta,\gamma,\lambda)dx = 1$.

Prove:

we proceed as follows to show that the NWIL pdf is proper:

$$\int_0^{\infty} f(x)dx$$

$$= \int_0^{\infty} \alpha\beta\gamma\lambda x^{-2}\left(1+\frac{\gamma}{x}\right)^{-1}\left\{-log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]\right\}^{\beta-1} \exp\left\{-\alpha\left\{-log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]\right\}^{\beta}\right\}dx \tag{6}$$

let $y = -log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]$ $\quad dx = \frac{\left(1+\frac{\gamma}{x}\right)}{\gamma\lambda x^{-2}}dy$

by substituting back in equation (6) and simplifying we have

$$\int_0^{\infty} f(x)dx = \alpha\beta\int_0^{\infty} y^{\beta-1} exp\left\{-\alpha y^{\beta}\right\}dy \tag{7}$$

let $u = \alpha y^{\beta}$ and $dy = \frac{du}{\alpha\beta y^{\beta-1}}$ by replacing back in equation (7) it reduces to

$$\int_0^{\infty} f(x)dx = \int_0^{\infty} exp\left\{-u\right\}du = \Gamma(1) = 1 \tag{8}$$

The cdf of NWIL is given below

$$F(x;\beta,\alpha,\lambda,\gamma) = \exp\left\{-\alpha\left\{-log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]\right\}^{\beta}\right\} \tag{9}$$

where $\beta > 0$, and λ are the shape parameters while α and γ are the scale parameters. Henceforth, we denote a random variable X having pdf (5) by X: N WIL(α, β, γ, λ). The hazard function $h(x)$, cummulative hazard function $H(x)$, survival function $s(x)$ and reversed hazard rate $r(x)$ of X are given by

$$h(x;\alpha,\beta,\gamma,\lambda) = \frac{\alpha\beta\gamma\lambda x^{-2}\left(1+\frac{\gamma}{x}\right)^{-1}\left\{-log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]\right\}^{\beta-1} \exp\left\{-\alpha\left\{-log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]\right\}^{\beta}\right\}}{1-\exp\left\{-\alpha\left\{-log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]\right\}^{\beta}\right\}} \tag{10}$$

$$H(x;\alpha,\beta,\gamma,\lambda) = -\alpha\left\{-log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]\right\}^{\beta} \quad (11)$$

$$s(x;\alpha,\beta,\gamma,\lambda) = 1-\exp\left\{-\alpha\left\{-log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]\right\}^{\beta}\right\} \quad (12)$$

$$r(x;\alpha,\beta,\gamma,\lambda) = \alpha\beta\gamma\lambda x^{-2}\left(1+\frac{\gamma}{x}\right)^{-1}\left\{-log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]\right\}^{\beta-1} \quad (13)$$

2.2 Shapes of the Weibull-Inverse Lomax pdf and hazard functions

The graphs below shows the shapes of the NWIL density at various selected parameter values. The shapes of the density and hazard rate functions of the NWIL distribution can easily be traced analytically. The critical points of the NWIL density are the roots of the equation below:

$$\gamma(1+\lambda)-2x\left(1+\frac{\gamma}{x}\right)+\gamma\lambda+\frac{(1-\beta)\gamma\lambda}{log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]}+\alpha\beta\gamma\lambda\left\{-log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]\right\}^{\beta-1}=0 \quad (14)$$

and the critical points of $h(x)$ are obtained from the equation below:

$$\gamma(1+\lambda)-2x\left(1+\frac{\gamma}{x}\right)+\gamma\lambda+\frac{(1-\beta)\gamma\lambda}{log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]}+\alpha\beta\gamma\lambda\left\{-log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]\right\}^{\beta-1}$$

$$+\frac{\alpha\beta\gamma\lambda\left\{-log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]\right\}^{\beta-1}\exp\left\{-\alpha\left\{-log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]\right\}^{\beta}\right\}}{1-\exp\left\{-\alpha\left\{-log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]\right\}^{\beta}\right\}}=0 \quad (15)$$

Another indication of heavy tail weight is that the distribution has a decreasing risk rate function. On the other hand, a distribution with an increasing risk level function has a light tailored distribution. If the risk frequency function decreases (within time, if the random variable is a time variable), then the population dies at a decreasing rate, resulting in a heavier tail for distribution. By using optimization software in R, we can easily examine equations (13) and to (14) to determine the local minimums and maximums inflexion points.

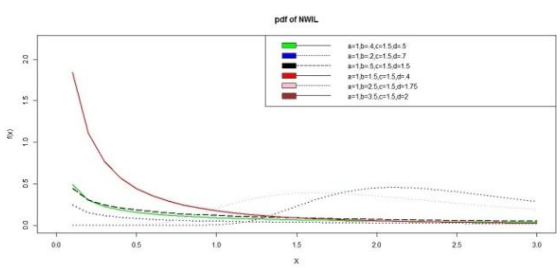

Figure 1: Density of NWIL at $\alpha = 1$ and $\gamma = 1.5$

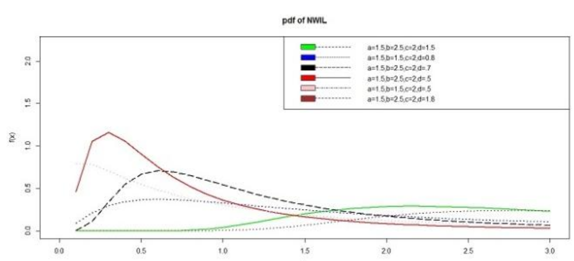

Figure 2: Density of NWIL at $\alpha = 1.5$ and $\gamma = 2$

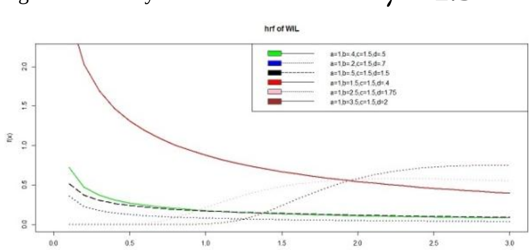

Figure 3: Hazard rate of NWIL at $\alpha = 1.5$ and $\gamma = 2$

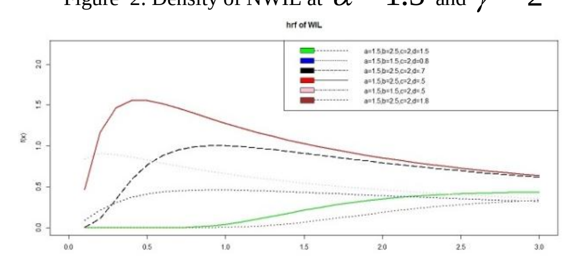

Figure 4: Hazard rate of NWIL at $\alpha = 1.5$ and $\gamma = 2$

2.3 Some Expansions

Let E be equal to equation (9), by expanding E using power series

$$E=\sum_{i=0}^{\infty}\frac{(-1)^i\alpha^i}{i!}\left\{-log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]\right\}^{i\beta}$$

then we can re-write the above equation as

$$F(x;\beta,\alpha,\lambda,\gamma)=\sum_{i=0}^{\infty}\frac{(-1)^i\alpha^i}{i!}\left\{-log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]\right\}^{i\beta} \quad (15)$$

for $i\geq 1$, the following holds

$$\left\{-log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]\right\}^{i\beta}=\sum_{r,l=0}^{\infty}\sum_{j=0}^{r}\frac{(-1)^{j+r+l}i\beta}{(i\beta-j)}$$

$$\times\binom{r-i\beta}{r}\binom{r}{j}\binom{i\beta+r}{l}q_{j,r}\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]^{l} \quad (16)$$

for $r=1,2,3,\ldots$ and $q_{j,0}=1$ for $j\geq 0$

$$q_{j,r}=r^{-1}\sum_{n=1}^{r}\frac{(-1)^n[n(j+1)-r]}{(n+1)}q_{j,r-n}. \quad (17)$$

by inserting the above expansion in equation (15) gives

$$F(x;\beta,\alpha,\lambda,\gamma)=\sum_{l=0}^{\infty}c_l\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]^{l}=\sum_{l=0}^{\infty}c_lT_l(x;\gamma,\lambda), \quad (18)$$

where $T_l(x;\eta)=\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]^{l}$ i.e for $l\geq 1$, is the Exponentiated-G density function by Cordeiro et al. [14] with the power parameter l, $T_0(x;\eta)=1$ and

$$c_l=\sum_{i,r=0}^{\infty}\sum_{j=0}^{r}\frac{(-1)^{i+j+r+l}i\beta}{i!(i\beta-j)}\binom{r-i\beta}{r}\binom{r}{j}\binom{i\beta+r}{l}q_{j,r} \quad (19)$$

We can also write the pdf of New Weibull Inverse lomax distribution as a mixture of Exponetiated-G densities

$$f(x;\alpha,\beta,\gamma,\lambda)=\sum_{l=0}^{\infty}c_{l+1}t_{l+1}(x;\gamma,\lambda), \quad (20)$$

where $t_{l+1}(x;\gamma,\lambda)=(l+1)\frac{\gamma\lambda}{x^2}\left(1+\frac{\gamma}{x}\right)^{-(1+\lambda)}\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]^{l}$ is the Exponentiated-G density function with $l+1$ as the power parameter Tahir et al. [11].

3.0 Statistical properties

3.1 Quantile function and median

In probability and statistics, the quantile function associated with a random variable's probability distribution specifies the random variable's value so that the variable's probability of being below or equal to that value is equal to the given probability. Quantile function is used in drawing a sample from a particular distribution function. The quantile function of the NWIL distribution is the inverse of equation (9) and is given by

$$Q(u)=\frac{\gamma}{\left[\exp\left\{-\left[-\frac{log(u)}{\alpha}\right]^{\frac{1}{\beta}}\right\}\right]^{-\frac{1}{\lambda}}-1} \quad (21)$$

where u : uniform(0,1) and random numbers can easily be generated from the WIL distribution using

$$x=\frac{\gamma}{\left[\exp\left\{-\left[-\frac{log(u)}{\alpha}\right]^{\frac{1}{\beta}}\right\}\right]^{-\frac{1}{\lambda}}-1} \quad (22)$$

The median of the WIL distribution can be derived by setting $u=0.5$ in equation (3.5.1) to be

$$Median = \frac{\gamma}{\left[\exp\left\{-\left[-\frac{log(0.5)}{\alpha}\right]^{\frac{1}{\beta}}\right\}\right]^{-\frac{1}{\lambda}} - 1} \tag{23}$$

3.2 Moments

The existence of positive moments occurs only until a certain value of a positive integer p means that the distribution has a heavy right tail. Must of the basic features and characteristics of a distribution can be studied through moments(for example kurtosis,tendency, skewness and dispersion).

Theorem2 If X: NWIL(θ) then the p^{th} moments of X is given as

$$\mu_p^{'} = E(X^p) = \sum_{l=0}^{\infty}(l+1)c_{l+1}\lambda\gamma^p \frac{\Gamma(1-p)\Gamma(\lambda(l+1)+p)}{\Gamma(\lambda(l+1)+1)} \qquad p = 1,2,3..... \tag{24}$$

where c_{l+1} is given in equation (19).

Proof:

Let start the prove with the well known definition of the p^{th} moment of the random variable X with probability density function $f(x;\theta)$ given by

$$\mu_p^{'} = \int_0^{\infty} x^p f(x)dx$$

By substituting in the above, we get

$$\mu_p^{'} = \sum_{l=0}^{\infty}(l+1)c_{l+1}\phi_{p,l}$$

where

$$\phi_{p,l} = \int_0^{\infty} x^{p-2}\left(1+\frac{\gamma}{x}\right)^{-\lambda(l+1)-1} dx$$

By letting $y = \frac{\gamma}{x}$ and some simplifications, we have equation 5.4.

$\Gamma(-p) = \frac{(-1)^p}{p!}\psi(p) - \frac{(-1)^p}{p!}\varphi$, φ denotes Euler's constant, $\Gamma(0) = -\varphi$ and $\psi(p) = \sum_{k=1}^{p}\frac{1}{k}$ as in Fisher and K [12].

The mean of X follows by setting p=1 in equation (24) as

$$\mu_1^{'} = E(X) = \sum_{l=0}^{\infty}(l+1)c_{l+1}\lambda\gamma \frac{\Gamma(0)\Gamma(\lambda(l+1)+1)}{\Gamma(\lambda(l+1)+1)} \tag{25}$$

and the p^{th} central moment of X is given by

$$\mu_1^{'} = E(X-\mu_1^{'})^p \tag{26}$$

It should be noted that the skewness and kurtosis measures can easily be derived from the central moments i.e equation (26) by a well known relationships.

3.3 Moment generating function

Theorem3 If X: NWIL(α, β, γ, λ) then the moment generating fuction (mgf) of X is given as

$$M_x(t) = \sum_{p=0}^{\infty}\frac{t^p}{p!}\mu_p^{'} \tag{27}$$

This implies that $\mu_p^{'}$ is $M_x^p(0)$, that is, the p^{th} derivative of M, at 0. Note that its also possible to have $\mu_p^{'}$ exist for all r and yet $M_x(t)$ not exist.

Proof

By defination, the mgf of a random variable X with density $f(x)$ is given

$$M_x(t) = \int_{-\infty}^{\infty} e^{tx} f(x)dx$$

By substituting $e^{tx} = \sum_{p=0}^{\infty}\frac{(tx)^p}{p!}$ in the above defination, we have

$$M_x(t)=\sum_{p=0}^{\infty}\frac{t^p}{p}x^p f(x)dx=\sum_{p=0}^{\infty}\frac{t^p}{p!}\int_{-\infty}^{\infty}x^p f(x)dx=\sum_{p=0}^{\infty}\frac{t^p}{p!}\mu_p'.$$

3.4 Order statistics

Order statistics are used in many areas of statistical theories and practices, for instance, detection of outlier in statistical quality control processes. In this section, we derive the closed form expressions for the pdf of the i^{th} order statistic of the Weibull Inverse Lomax distribution. Suppose is a random sample from a distribution with pdf $f(x)$ and Let denotes the corresponding order statistics obtained from this sample. Then

$$f_i:_n(x)=\frac{f(x)}{B(i,n-i+1)}F(x)^{i-1}\left[1-F(x)\right]^{n-i} \tag{28}$$

where $f(x)$ and $F(x)$ are the pdf and cdf of the New Weibull Inverse Lomax distribution. Using the binomial expansion on $\left[1-F(x)\right]^{n-i}$, we have $\left[1-F(x)\right]^{n-1}=\sum_{k=0}^{(\infty)}(-1)^k F(x)^k$ by substituting back in the above equation

$$f_{i:n}(x)=\frac{f(x)}{B(i,n-i+1)}\sum_{r=0}^{\infty}(-1)^r\binom{n-i}{r}F(x)^r F(x)^{i-1}$$

$$=\frac{f(x)}{B(i,n-i+1)}\sum_{r=0}^{\infty}(-1)^r\binom{n-i}{r}F(x)^{i+r-1} \tag{29}$$

Equation (3.5.8) can also be writing as

$$f_{i:n}(x)=\frac{n!f(x)}{(i-1)(n-i)!}\sum_{r=0}^{\infty}(-1)^r\binom{n-i}{r}F(x)^{i+r-1} \tag{30}$$

By equations (3.2.1) and (3.4.1) and the fact that

$$F(x;\theta)^{r+i-1}=\sum_{s=0}^{r+i-1}(-1)^s\frac{(r+i-1)!}{(r+I-s-1)!s!}exp\left\{-\alpha s\left\{-log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]\right\}^{\beta}\right\} \tag{31}$$

the order statistics becomes

$$f_{i:n}(x)=Qx^{-2}\left\{-log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]\right\}^{\beta(1+i)-1}\left(1+\frac{\gamma}{x}\right)^{-1}exp\left\{-\alpha s\left\{-log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]\right\}^{\beta}\right\} \tag{32}$$

where $Q=\frac{n!\alpha^{i+1}\beta\gamma\lambda}{(i-1)(n-i)!i!}\frac{(r+i-1)!}{(r+I-s-1)!s!}\sum_{i=0}^{\infty}\sum_{r=0}^{n-1}\sum_{s=0}^{r+i-1}(-1)^{i+r+s}\binom{n-i}{r}$

3.5 Rényi Entropy

We defined Rényi Entropy by

$$I(\zeta)=\frac{1}{(1-\zeta)}log\left[\int_0^{\infty}f^{\zeta}(x)dx,\right]\zeta>0 and \zeta\neq 1$$

also

$$f^{\zeta}(x)=(\alpha\beta\gamma\lambda)^{\zeta}x^{-2\zeta}\left(1+\frac{\gamma}{x}\right)^{-\zeta}\left\{-log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]\right\}^{\zeta(\beta-1)}\exp\left\{-\zeta\alpha\left\{-log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]\right\}^{\beta}\right\}$$

by simplifications and E, we have

$$f^{\zeta}(x)=\sum_{l=0}^{\infty}Z_l(\lambda\gamma)^{\zeta}x^{-2\zeta}\left(1+\frac{\gamma}{x}\right)^{-\zeta(2\lambda+1)-\lambda l},$$

where

$$Z_l=\sum_{i,j,r=0}^{\infty}\frac{(-1)^{1+j+r+l}(\alpha\zeta)^i[\zeta(\beta-1)+i\beta]}{i![\zeta(\beta-1)+i\beta-j]}$$
$$\times q_{j,r-m}\binom{r}{j}\binom{r-\zeta(\beta-1)-i\beta}{r}\binom{r+\zeta(\beta-1)+i\beta}{l}$$

hence, the Rényi entropy becomes

$$I(\zeta)=\frac{1}{(1-\zeta)}log\left[\int_0^{\infty}f^{\zeta}(x)dx,\right]$$

finally, after integrating and simplifying we have,

$$I(\zeta)=\frac{1}{(1-\zeta)}log\left[\sum_{l=0}^{\infty}Z_l\gamma^{1-\zeta}\lambda^{\zeta}B\left(2\zeta-1,l+1+\frac{\zeta}{\lambda}\right)\right] \quad (33)$$

which can also be re-written in terms of gamma function as

$$I(\zeta)=\frac{1}{(1-\zeta)}log\left[\sum_{l=0}^{\infty}Z_l\gamma^{1-\zeta}\lambda^{\zeta}\frac{\Gamma(2\zeta-1)\Gamma(l+1+\frac{\zeta}{\lambda})}{\Gamma(2\zeta+l+\frac{\zeta}{\lambda})}\right] \quad (34)$$

3.6 Maximum Likelihood Estimation

Furthermore, the MLEs have desirable properties and can be used to establish confidence intervals.The estimate of normality for these estimators is readily treated either numerically or analytically in the theory of large sample distribution.In this section, we determine the maximum likelihood estimates (MLEs) of the parameters of the New Weibull Inverse Lomax distribution from complete samples only. Let $x_1,x_2,x_3 \ldots x_n$ be the observed values from the NWIL distribution with parameter space $\theta=(\alpha,\beta,\gamma,\lambda)^T$ be the $p\times 1$ parameter vector. The log-likelihood function for θ is given by

$$l(\theta)=nlog(\alpha\beta\gamma\lambda)-2\sum_{i=1}^{n}logx_i-\sum_{i=1}^{n}log\left(1+\frac{\gamma}{x}\right)+(\beta-1)\sum_{i=1}^{n}log\left[-log\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]$$

$$-\alpha\sum_{i=1}^{n}\left[-log\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]^{\beta} \quad (35)$$

Differentiating $l(\theta)$ with respect to each parameter $\alpha,\beta,\gamma and \lambda$ and setting the result equals to zero, we obtain maximum likelihood estimates (MLEs). The partial derivatives of $l(\theta)$ with respect to each parameter or the score function is given by:

$$W_n(\theta)=\left(\frac{\partial l}{\partial\alpha},\frac{\partial l}{\partial\beta},\frac{\partial l}{\partial\gamma},\frac{\partial l}{\partial\lambda}\right)$$

where

$$\frac{\partial l}{\partial\alpha}=\frac{n}{\alpha}-\sum_{i=1}^{n}\left[-log\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]^{\beta} \quad (36)$$

$$\frac{\partial l}{\partial\beta}=\frac{n}{\beta}-\sum_{i=1}^{n}log\left[-log\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]$$

$$-\alpha\sum_{i=1}^{n}\left[\left\{-log\left(1+\frac{\gamma}{x}\right)^{\lambda}\right\}^{\beta}log\left\{-log\left(1+\frac{\gamma}{x}\right)^{\lambda}\right\}\right] \quad (37)$$

$$\frac{\partial l}{\partial\gamma}=\frac{n}{\gamma}-\sum_{i=1}^{n}\left[\frac{x^{-2}}{\left(1+\frac{\gamma}{x}\right)}\right]-(\beta-1)\sum_{i=1}^{n}\left[\frac{1}{x\left(1+\frac{\gamma}{x}\right)log\left(1+\frac{\gamma}{x}\right)}\right]$$

$$+\alpha\beta\sum_{i=1}^{n}\left[\frac{\left\{\lambda log\left(1+\frac{\gamma}{x}\right)\right\}^{\beta-1}}{x\left(1+\frac{\gamma}{x}\right)}\right] \quad (38)$$

$$\frac{\partial l}{\partial\lambda}=\frac{n}{\lambda}+\frac{n(\beta-1)}{\lambda}-\alpha\beta\sum_{i=1}^{n}\left[\left\{\lambda log\left(1+\frac{\gamma}{x}\right)\right\}^{\beta-1}log\left(1+\frac{\gamma}{x}\right)\right] \quad (39)$$

4. Conclusions

We successfully proposed an extension of inverse Lomax distribution and also derived some of its properties including moments, quantiles, entropy and ordered statistics. We observed by the plots and the moments (i.e the higher moments does not exist) that the NWIL distribution is a fat tail distribution. We also estimated the parameters of the model by maximum likelihood method.

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