Transactions of the Nigerian Association of Mathematical Physics
Volume 9, **(***March and May, 2019***), pp171 – 176**

LINEAR CODES OF INCIDENCE STRUCTURES OF THE NON-ASSOCIATIVE STRUCTURAL PROPERTY OF THE AUNU INTEGER SEQUENCE.

Chun P.B[1], *Ibrahim A.A*[2], *Garba A.I*[3], *Magami M.S*[4] *and Gau, P.D*[5]

[1] Department of Mathematics, Plateau state University Bokkos, Jos Nigeria.
[2,3,4]Department of mathematics, Usmanu Danfodiyo University Sokoto, Sokoto Nigeria.
[5] Department of Mathematics and Statistics, Plateau state Polytechnic Barkin/Ladi, Jos, Nigeria.

Abstract

The non-associative and non-commutative structural property of the Audu and Aminu (AUNU) integer sequence has been enumerated by the authors, where the graph theoretic applications of the AUNU integers was generated in which sub-graphs were deduced from the main graph and their adjacency matrices and incidence matrices constructed. In this paper, we establish a relationship between the incidence matrices of some of such Sub-graphs and the incidence matrix, $\underline{M_D}$ ***of incidence structures. Some important classes of linear codes of incidence structures of*** $t-designs$ ***are constructed and analysed from such incidence matrices.***

***Keywords*:** Aunu numbers, Non - associativity, Non-commutative, Graph, Adjacency matrix, Incidence Structure, Incidence matrix, Integer sequence.

I. INTRODUCTION

An overview of AUNU integer sequence, AUNU permutations patterns, the (123)/(132) - avoiding patterns and their applications was reported by the authors in [2] as cited in [1]. This algebraic structures has found wide applications in almost all facets of applied mathematics such as association/succession schemes, thin cyclic design, latin squares, lattices, automate theory, Graph theory, Coding theory e.t.c [3]. Williem H. Haemers reported that the adjacency matrix of a graph can be interpreted as the incidence matrix of a design or a generator matrix of a binary code [4]. This assertion in respect to generator matrix of a code has been demonstrated by the Authors in [3], [4]. In this paper, we first establish a relationship between the incidence matrices of some Sub-graphs of AUNU integer sequence as non-associative structure and the incidence matrix, $\underline{M_D}$ of incidence structures. Some important classes of linear codes of incidence structures of $t-designs$ [6], are then constructed and analysed from such incidence matrices.

II. SOME BASIC CONCEPTS

Some important basic concepts are reviewed in this section to make the paper self explanatory and understanding.

Incidence Structures

An incidence structure $\mathbb{D}$, is a triple $\mathbb{D}=(\mathcal{P}, \beta, \mathcal{T})$, where $\mathcal{P}$ is a set of elements called points and β is a set of elements called blocks (lines), and $\mathcal{T} \subseteq \mathcal{P} \times \beta$ is a binary relation , called incidence relation. $\mathbb{D} = (\mathcal{P}, \beta, \mathcal{T})$ is termed a finite incidence structure if both $\mathcal{P}$and β are finite sets.

Remark

A finite incidence structure with number of points in $\mathcal{P}$ equal to number of blocks in β is called a square.

Correspondence Author: Chun P.B., Email: chunpamsongmail.com, Tel: +2348035929091

Example 1

Let $\mathcal{P}$ = { 1, 2, 3, 4, 5, 6, 7 } and β = { {1,2,3}, {1,4,5}, {1,6,7}, {2,4,7}, {2,5,6}, {3,4,6}, {3,5,7} } and define $\mathcal{T}$= $\mathcal{P}\times\beta$. Then ($\mathcal{P}$, β, $\mathcal{T}$) is an incidence structure obtained from the Fano plane which is clearly a square.

Incidence Matrices

Let $\mathbb{D}$ = ($\mathcal{P}$, β, $\mathcal{T}$) be an incidence structure with $v \geq 1$ points and $b \geq 1$ blocks. Indexed the points of $\mathcal{P}$ with $p_1, p_2, \ldots, p_v$ and denote the blocks of β by $B_1, B_2, \ldots, B_b$. Then the incidence matrix $M_D = m_{ij}$ of $\mathbb{D}$ is a $b \times v$ matrix where $m_{ij} = 1$ if p_j is on Bi and $m_{ij} = 0$ otherwise. It is Clear that the incidence matrix M_D depends on the labelling of the points and blocks of $\mathbb{D}$, but is unique up to row and column permutations.

Example 2

Let's, consider the incidence structure $\mathbb{D}$ = ($\mathcal{P}$, β, $\mathcal{T}$) of example 2 above. Let the labelling of the points and blocks of $\mathbb{D}$ be in the order of their appearance in $\mathcal{P}$and β.

Then the incidence matrix of $\mathbb{D}$ is given by

$$M_D = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Let p be a Prime. The p -rank of an incidence structure $\mathbb{D}$ = ($\mathcal{P}$, β, $\mathcal{T}$) is the rank of an incidence matrix over $GF_{(P)}$.

Linear Codes of Incidence Structures

Let $\mathbb{D}$ = ($\mathcal{P}$, β, $\mathcal{T}$) be an incidence structure with $v \geq 1$ points and $b \geq 1$blocks, and let M_D be its incidence matrix with respect to a labelling of the points and blocks of $\mathbb{D}$. The linear code of $\mathbb{D}$ over a field F is the subspace $C_F(D)$ of F^v spanned by the row vectors of the incidence matrix M_D. By definition, $C_F(D)$ is a linear code over F with length v and dimension at most b.

Note The code $C_F(D)$ depends on the labelling of the points and blocks of $\mathbb{D}$ = ($\mathcal{P}$, β, $\mathcal{T}$). Whatever the labelling, these codes are all equivalent.

Example 3

Consider the incidence Structure in example 2 and the labelling of points in example 1. Then the code $C_F(D)$ over $GF_{(2)}$ is the binary Hamming code with parameters [7,4,3] and has generator matrix

$$M_D = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Remark

The inner product of two code words $u \; and \; v$ in C denoted by $u.v$ and defined as $u.v = uv^T = \sum_{i=1}^{n} a_i b_i$ where v^T denotes the transpose of the code word v.

Algorithm I

$Input$: A non-empty subset S of F_2^n .

$Output$: A basis for $C = \langle S \rangle$ the linear code generated by S .

$Description$: We form a matrix A whose rows are the words of S . Find a REF (Row echelon form) matrix for A. Then the non-zero rows of the REF of A is a basis for C .

Example 1: Let $q = 3$. Find a basis for $C = \langle S \rangle$ where

$S = \{(12101), (20110), (01122), (11010)\}$

$Solution$:

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 & 1 \\ 2 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 2 & 2 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 & 1 & 0 & 1 \\ 0 & 2 & 2 & 1 & 1 \\ 0 & 1 & 1 & 2 & 2 \\ 0 & 2 & 2 & 1 & 2 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 & 1 & 0 & 1 \\ 0 & 1 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The last matrix is in REF is obtained by applying the following row operations on rows of A;

$$1st\ Operation: R_2 = R_1 + R_2$$
$$R_4 = 2R_1 + R_4$$

$$2nd\ Operration: R_2 = R_3$$
$$R_3 = R_3 + R_4$$
$$R_4 = R_2 + R_3$$

Therefore $\{$ (12101), (01122), (0001) $\}$ by algorithm 1 forms a basis for C .

III. METHODOLOGY

(a) We consider in what follows, Fig.1: a subgraph of the network of fig.2.1, Table 1 and Table 2, the Adjacency and Incidence matrices of Sub-graph of the netwok of Fig. 2.1 respectively as in [1]

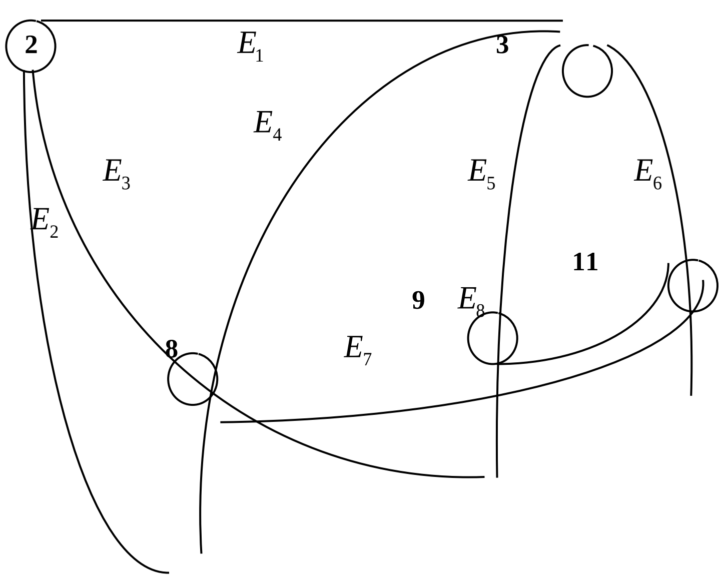

Fig.1: Sub graph of the network of fig.2.1[1]

TABLE 1: Adjacency matrix of Sub-graph in Fig.1

	2	3	8	9	11
2	0	1	1	1	0
3	1	0	1	1	1
8	1	1	0	1	1
9	1	1	0	0	1
11	0	1	1	1	0

TABLE 2: Incidence matrix of Sub-graph in Fig.1

	e1	e2	e3	e4	e5	e6	e7	e8
2	1	1	1	0	0	0	0	0
3	1	0	0	1	1	1	0	0
8	0	1	0	1	0	0	1	0
9	0	0	1	0	1	0	0	1
11	0	0	0	0	0	1	1	1

The Tables 1 and 2 above are expressed as matrices A and B below;

$$A=\begin{bmatrix}0&1&1&1&0\\1&0&1&1&1\\1&1&0&1&1\\1&1&0&0&1\\0&1&1&1&0\end{bmatrix} \text{ and } B=\begin{bmatrix}1&1&1&0&0&0&0&0\\1&0&0&1&1&1&0&0\\0&1&0&1&0&0&1&0\\0&0&1&0&1&0&0&1\\0&0&0&0&0&1&1&1\end{bmatrix}$$

We next obtain the Possible Reduced echolon form of matrices A and B as,

$$\mathrm{A}=\begin{bmatrix}0&0&0&0&0\\1&1&0&0&1\\1&0&1&0&1\\0&0&0&1&0\\0&0&0&0&0\end{bmatrix} \quad and \quad \mathrm{B}=\begin{bmatrix}1&0&0&1&1&1&0&0\\0&1&0&1&0&0&1&0\\0&0&1&0&1&1&1&0\\0&0&0&0&0&1&1&1\\0&0&0&0&0&1&1&1\end{bmatrix}$$

respectively, and on considering only the non-zero rows in accordance with algorithm I in case of A and avoiding repeated rows in case of B, we have;

$$\mathrm{A}^*=\begin{bmatrix}1&1&0&0&1\\1&0&1&0&1\\0&0&0&1&0\end{bmatrix} \quad and \quad \mathrm{B}^*=\begin{bmatrix}1&0&0&1&1&1&0&0\\0&1&0&1&0&0&1&0\\0&0&1&0&1&1&1&0\\0&0&0&0&0&1&1&1\end{bmatrix}$$

Next, the rows of the matrices $\mathrm{A}^* \; and \; \mathrm{B}^*$ above are bases for codes generated by the rows of A and B as subspaces of $F_2^5 \; and \; F_2^8$ respectively. Let $C^A and \; C^B$ be such two codes, then it is easily seen that C^A is a [5 3 1]-linear code of size $M = 8$ which is *an* [*n, k, d* − *r*]*-linear code over* **F***q for any* 1 ≤ *r* ≤ *d* – 1 version of the [5 3 2] linear code [5] and C^B, a [8 4 2] code of size $M = 16$ and is an extended version of the [7 4 2]-linear code.[9]

IV. FINDINGS/REULTS

Clearly, C^B is traced to be a linear code of the sub graph of fig.1 as an incidence structure with parameters,

$v = n = 8 \; and \, b = k = 4$

where $v =$ number of points in an incidence structure,

$n =$ length of the linear code of an incidence structure

while $b \, and \, k$ are their blocks and dimension respectively and as such, the subgraphs are not $t-(v\,k\,\lambda)$ designs, but $t-(v \;\; \lambda)$ designs.

V. CONCLUSION

We thus conclude that there exist some sort of isomorphism between the incidence matrices of some of such Sub-graphs and the incidence matrix, $\underline{M_D}$ of incidence structures. Moreover, some codes C^B of incidence structures of $t-designs$ are constructed and analysed from such incidence matrices. Further analysis of such incidence matrices of the other sub-graphs is adviced to allow generalization.

REFERENCES

[1] Ibrahim, A.A. and Abubakar, S.I. (2016) AUNU Integer Sequence as Non-associative Structure and their Graph theoretic properties. *Advances in Pure Mathematics*, **6**, 409-419. http://dx.doi.org/10.4236/apm.2016.66028

[2] Ibrahim A.A., and Abubakar S.I. (2016) Non-Associative Property of 123-Avoiding Class of Aunu Permutation Patterns. Advances in Pure Mathematics, **6**, 51-57. http://dx.doi.org/10.4236/apm.2016.62006

[3] Chun, P. B, Ibrahim, A.A, Garba, A.I (2016). Algebraic Theoretic Properties of the Non-associative Class of (132)-Avoiding Patterns of AUNU Permutations: Applications in the Generation and Analysis of a General Cyclic Code. *Computer Science and Information Technology, 4* , 45 - 47. doi: 10.13189/csit.2016.040201.

[4] Van Steen, M. (2010) An Introduction to Graph Theory and Complex Networks. *Amsterdam*

[5] Chun, P.B, Ibrahim, A.A, and Garba, A.I, (2016) Algebraic theoretic properties of the avoiding class of AUNU permutation patterns: Application in the generation and analysis of linear codes. *International Organization for Scientific Research (IOSR), Journal of Mathematics12* (1) pp 1-3.

[6] Ding, C.(1962) Codes from Difference sets World Scientific Publishing Co. Pte. Ltd 5 Toh Tuck Link Singapore.

[7] Ibrahim (A.A. (2006)). *Some Graph theoretical properties of (132) – avoiding patterns of certain class* Nigerian Journal of renewable energy 14, 21 – 24.

[8] Shmuel, .F (2015) Matrices: Algebra, Analysis and Applications. World Scientific publishing Co. Pte. Ltd, Singapore pp. 399.

[9] Ibrahim A.A, Chun P.B, Garba A.I, Abubakar S.I and Mustafa .A (2017) A [7 4 2] linear code due to the cayley table for $n = 7$ of the generated points of Ω as permutations of the (132) and (123) - avoiding patterns of the Non-associative AUNU Schemes *(Accepted 2017, yet to be published) Quest Journal of Research in Applied Mathematics.Vol.3,Issue 8.*