Transactions of the Nigerian Association of Mathematical Physics
***Volume 9, (March and May, 2019*), pp85 – 90**

THE THERMAL ENERGY, PRESSURE AND ENTROPY OF PAIR-PRODUCTION IN THE LATE EVOLUTION OF MASSIVE STARS

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Abstract

In this work, we made calculations of specific thermal energy, pressure, and entropy of electron-positron pairs in massive stars. Electron-Positron ($e^{\pm}$) equation of state (EoS) based on table interpolation of Helmholtz free energy was used. Our consideration refers explicitly to stars of masses $13, 15, 20, 25$ and $75M_{\odot}$ under the assumptions of arbitrary nuclear degeneracy as well as thermal equilibrium. The pair production occur at high temperature and relatively low density in the centers of the massive stars. The calculated pressure was found to rapidly increase with increase of density before attaining a stage where the star is unstable. The increase in the central temperature of the star leads to its collapse as pair production reached its maximum temperature of about $3 \times 10^9 K$. Whilst at this temperature, the star is sensitive to its entropy, more massive stars showed instability at lower temperature near their central regions. The high temperature in the center of the massive stars is maintained by the internal thermal energy that holds the stars life. This thermal energy was therefore negligible at early production of the $e^{\pm}$ pair production, but very significant as the temperature and density approaches the maximum pair production limit. This result will contribute in understanding the effect of pair production on the stability and collapse of massive stars.

***Keywords*:** Electron-Positron pairs, Equation of State; Massive Stars

1. Introduction

One of the great challenges in the late evolution of massive stars is posed by the question: how the electron-positron pairs are produced and what is their behavior after production? Inside the stars is an environment of matter and radiation interactions at high temperatures that produce pressure to oppose gravitational contraction. All chemical compositions of stars are in form of Plasmas at high temperature. At this extreme conditions; atoms are ionized, electrons become degenerate and ultra-relativistic while the radiation pressure might be significant. Energy is produced via fusion process to maintain stars' high temperature and emits energy as heat or quantum light. Whilst, the temperature at which a star is unstable is sensitive to its entropy near the center[1], more massive stars with higher entropies become unstable at lower temperatures. With these complexities however, we can understand many properties of the stellar interior by means of a thermodynamic system of properties. Various thermodynamic properties at different densities, compositions and temperatures are vital in modelling stellar events. However, these thermodynamic properties may be macroscopically described by temperature T, pressure P and its chemical potential μ, which will further leads to thermodynamic equilibrium characterized by uniform temperature, pressure and chemical potential that can then be related by an equation of state. At high temperatures and comparatively low densities inside massive stars; the interactions of radiation with matter produces two important processes; the production of electron-positron pairs and photo disintegration of atomic nuclei. In this work, the equation of state relating pressure, energy and entropy to temperature and density allows for electrons and positrons be relativistic and arbitrarily degenerate. The most accurate, speedily executable and thermodynamically consistent electron-positron ($e^{\pm}$) equation of state (EOS) based on table interpolation of Helmholtz free energy was used to calculate the specific thermal energy, pressure and entropy of the selected massive stars due to the electron-positron pairs. These calculations were carried out for the stellar constituents' of hydrogen, helium, carbon, oxygen, neon and silicon were electron-positron process take charge.

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Understanding the nature and evolution of stars serves as important pre-requisite for interpreting the universe. Massive stars are usually defined as star which has reached or will reach a mass greater than 10 $M_{\odot}$and their cores after collapse become neutron stars (NS) or black holes (BH). They go beyond carbon burning phase and produces many elements heavier than oxygen. Though low in number, massive stars have great role in the evolution of interstellar medium and galaxies. The first generation stars formed are conceived to be all massive or even very massive[2]. One important property of massive star is their effective temperatures which exceeds about 25,000 K on the main sequence, and they can be cool to about 3,500 K at their early evolutionary stage (in the red supergiant phase). They create necessary elements to existing life and are fundamental in lightening up stellar birth. The formation of massive stars is stricken by observational confusion as they occur in distant and highly obscured regions. For example, the time it takes to form a massive star is disputative as such they are theoretically difficult to analyze due to the many processes acting concurrently. Generally, a star consisting of average particle mass $\bar{m}$is formed when its average density exceeds a critical value given by;

$$\rho = \frac{3}{4\pi M^2}\left(\frac{3kT}{2G\bar{m}}\right)^3 \quad (1)$$

And it attains a maximum temperature given by;

$$kT \approx \left[\frac{G^2\bar{m}^{8/3}m_e}{h^2}\right] M^{4/3} \quad (2)$$

where M is mass of the star, G; gravitational constant, m_e; is electron mass, k; Boltzmann constant and h; Planck's constant. From this, we can see that the only bodies with a mass greater than $0.08M_{\odot}$ can achieve the temperature necessary to ignite hydrogen fusion and become a genuine star. The maximum mass $(50-100)M_{\odot}$ of a normal star is due to the radiation pressure increase in massive stars. Hence, the mass of a star determines its evolution and fate. Massive stars evolve rapidly and end their life with ruinous collapse and their out layers are ejected as supernovae and the remaining matter forms a neutron star and black hole[3].

In massive stars, the internal kinetic energy and gravitational energy is conserved. This implies that the hydrodynamic equilibrium is unstable inside the massive stars. Hence, the kinetic energy increases as the star core contracts. And the temperature of this contracting star of mass M and average particle mass $\bar{m}$; will increase as the radius R of the core decreases or as the stars contracts;

$$kT \approx \frac{GM\bar{m}}{3R} \propto M^{2/3}\rho^{1/3} \quad (3)$$

But a maximum temperature T(equation 2) is attained at a lower densityρ (equation 1) when the mass M is much higher.However, theoretical models of highly evolved stars suggest that the iron core has a temperature of about $8e9\ K$ and a density of about $4e13\ gcm^{-3}$ just before collapse.The mass of the main sequence star determines its core pressure and temperature. Stars of higher mass have higher temperature while stars of lower mass have cooler cores. A star's mass determines its entire life because it determines its core temperature. High mass stars with >8M have short lives eventually becoming hot enough to make iron, and end in supernovae explosion. Low mass stars with mass <2M have long lives, never become hot enough to fuse carbon nuclei, and end as white dwarfs. Intermediate mass stars can make elements heavier than carbon but end as white dwarfs. In this work, we used the equation of state relating pressure, energy and entropy to temperature, density and composition -which allows for electrons and positrons be relativistic and arbitrarily degenerate- to evaluate the quantitative values of the thermonuclear energy, pressure and entropy due to the $e^{\pm}$ pairs in some selected massive stars . The most accurate, speedily executable and thermodynamically consistent electron-positron ($e^{\pm}$) equation of state (EOS) based on table interpolation of Helmholtz free energy was applied. These computations were carried out for the stellar constituents' of hydrogen, helium, carbon, oxygen, neon and silicon.

This paper is organized as follows. In §2 we present the electron-positron pair productions in massive stars and the electron-positron EoS and the massive stellar composition used, is described in §3. §4 is results and discussions while conclusion is given in §5.

2. Electron-Positron Pairs in Massive Stars

Electro-magnetic radiation field is the major source for creating electron-positron pairs at high temperatures inside stars which makes most of the elements completely ionized, this perhaps compelled the electrons, positrons, ions and photons be at any given radius in the star which would then collide and exchange energy very frequently so that thermal equilibrium can be achieved. However, a large scale temperature gradient may be attained across the star which we assume here to be negligible for thermodynamic purposes. The pair annihilation has been confirmed as most important neutrino process in massive stars[4]. This annihilation occurs at high temperatures of about $T > 10^9\ K$ where energetic photons undergo pair creation and in every 10^9 cases the pair annihilates into neutrinos. The number densities of the electrons and positrons are given by;

$$n_{e^{\pm}} = \rho N_{e^{\pm}} = \frac{64\pi^4\sqrt{2}}{\hbar^3}(m_e c)^3 \propto^{3/2} F_n(\mu,\alpha) \quad (4)$$

Where

$$F_n(\mu,\alpha) = \int_0^\infty \frac{y^n(1+0.5\alpha y)^{1/2}}{1+e^{(y-\mu)}} dy \quad (5)$$

is Fermi-Dirac integral, m_e is electron rest mass, $\hbar = 2\pi h$, $\alpha = \frac{kT}{m_e c^2}$ is relativity parameter and the normalized chemical potential is μ.So, the individual number density for free electron is given by [3-5]

$$n_{e^-} = \frac{64\pi^4\sqrt{2}}{\hbar^3}(m_e c)^3 \propto^{3/2} \left[F_{1/2}(\mu,\alpha) + F_{3/2}(\mu,\alpha)\right] \quad (6)$$

While for the positron, the chemical potential must have the rest mass terms which was subtracted in the case of electrons, and therefore it is given by

$$n_{e^+} = \frac{64\pi^4\sqrt{2}}{\hbar^3}(m_e c)^3 \propto^{3/2} \left[F_{1/2}(-\mu - 2/\alpha, \alpha) + \alpha F_{3/2}(-\mu - 2/\alpha, \alpha)\right] \quad (7)$$

The chemical potential μ (which is the only unknown in this equation) can be found by applying the boundary condition for complete ionization of the matter present[6];

$$n_0 = n_{e^-} - n_{e^+} = N_a \frac{\rho Z}{A} = Z n_{ion} \quad (8)$$

Where N_a is Avogadro's number and ρ, Z and A are the mass density, atomic number and atomic weight of the matter excluding electron-positron pairs. However, many methods can be used for this one-dimensional root finding. While absolute accuracy and thermodynamic consistency are primarily the major concern, Timmes EOS evaluated the Fermi-Dirac integrals and their derivatives with respect to the chemical potential and relativity parameter[7], whereas, the chemical potential was calculated using Newton-Raphson scheme to at least 15 significant figures[5].After finding the value for the chemical potential by using newton-Raphson iteration method, the electron and positron pressures, thermal energies and entropy can be given in the following equations:

$$P_{e^-} = \frac{128\pi^4\sqrt{2}}{3\hbar^3} m^4{}_e c^5 \alpha^{5/2}\left[F_{3/2}(\mu,\alpha) + \frac{1}{2}\alpha F_{5/2}(\mu,\alpha)\right] \quad (9)$$

$$P_{e^+} = \frac{128\pi^4\sqrt{2}}{3\hbar^3} m^4{}_e c^5 \alpha^{5/2}\left[F_{3/2}(-\mu - 2/\propto, \alpha) + \frac{1}{2}\alpha F_{5/2}(-\mu - 2/\propto, \alpha)\right] \quad (10)$$

$$E_{e^-} = \frac{64\pi^4\sqrt{2}}{\rho\hbar^3} m^4{}_e c^5 \alpha^{5/2}\left[F_{3/2}(\mu,\alpha) + \alpha F_{5/2}(\mu,\alpha)\right] \quad (11)$$

$$E_{e^+} = \frac{64\pi^4\sqrt{2}}{\rho\hbar^3} m^4{}_e c^5 \alpha^{5/2}\left[F_{3/2}(-\mu - 2/\propto, \alpha) + \alpha F_{5/2}(-\mu - 2/\propto, \alpha)\right] + \frac{2m_e c^2}{\rho} n_{e^+} \quad (12)$$

$$S_{e^-} = \frac{P_{e^-}/\rho + E_{e^-}}{T} + \frac{\mu k N_{e^-}}{\rho} \quad (13)$$

$$S_{e^+} = \frac{P_{e^+}/\rho + E_{e^+}}{T} + \frac{(\mu + 2/\alpha) k N_{e^+}}{\rho} \quad (14)$$

Equations (9) & (10) represents the electron and positron pressure respectively. While equations (11) and (12) as well as equations (13) and (14) represents the electron and positron energy and entropy respectively.

3. Electron-Positron Equation of State (EoS)and Initial Composition

Stellar equation of state determines many aspect of stellar physics, like the electron degeneracy and electron-positron pair formation[3]. Proper equation of state for different nuclear densities is crucial in the studies of the explosion mechanisms of core-collapse supernovae[8-11]. Given the stellar composition, temperature and density, the EOS produces the energy, pressure, entropy and many more thermodynamic quantities. Practically, an equation of state is constructed by a three dimensional table of the related thermodynamic quantities as functions of the inputs[12].In the case of massive stars, the EOS relating the energy and pressure to temperature, density and composition has been developed and used. At high temperatures; the electrons and electron-positron pairs are described as perfect thermal gas with arbitrary relativity and degeneracy. Many subroutines have been given for the massive stellar EOS at high temperatures[7, 13]. The equation of state subroutines used in this work is described in details by [5, 7].In this work six composition were considered as major contributor for the creation/annihilation processes of the electron-positron pairs in the massive stars as in table I. The eos is developed such that for an isotope i with Z_i and A_i as its protons and nucleons number respectively, the total isotope$i$ has a mass and number densities to be ρ $(g\ cm^{-3})$ and $n_i(in\ cm^{-3})$ respectively and a temperature $T(K)$. For this, the dimensionless mass fraction for individual isotope i is

$$X_i = \frac{A_i n_i}{\rho N_A} \quad (13)$$

And the dimensionless number density is $Y_i = \frac{X_i}{A_i} = \frac{n_i}{\rho N_A}$ where N_A is the Avogadro's number[7, 13].

4. Results and Discussions

Initially, the compositions (by mass) of the $13, 15. 20, 25$ and $75M_\odot$ stars were taken to be 71% hydrogen, 27% helium and 2% heavy elements as reported by [14]. However, the temperature and densities for their nuclear burning phases were reported by [13].

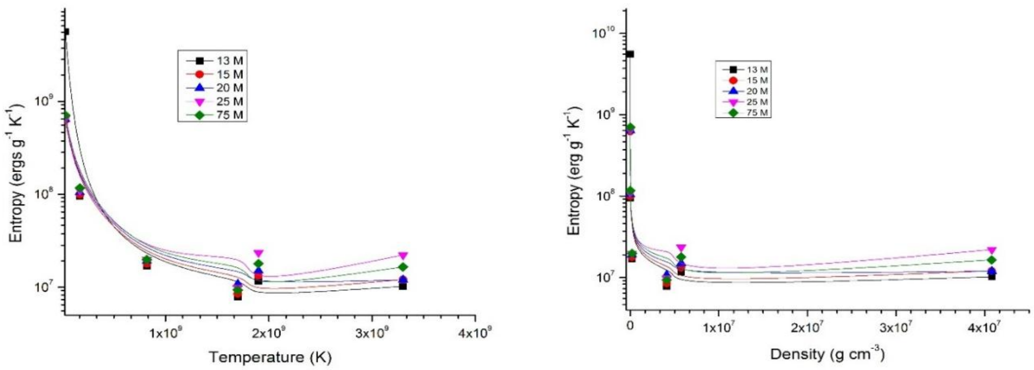

Figure 1: Electron-Positron specific thermal energy with stars Density & Temperature for ^{1}H,^{4}He,^{12}C,^{16}O,^{20}Ne&^{23}Si

The temperature segments in Figures 1 and 2 clearly shows the production of the electron-positron pairs in the massive stars. Starting from carbon, neon to oxygen and down to silicon nuclear burning stages; the pair production take place at temperature limit $1 \times 10^9 K$ to $4.5 \times 10^9 K$ which is basically in line with the prediction made in §2. This is evident to the fact that $e^{\pm}$ pairs contribute only at advanced nuclear burning phases and have no effect whatsoever at densities lower than$1 \times 10^7\ g\ cm^{-3}$.

Table I: Massive star's pressure, energy and entropy at different nuclear burning stages

Massive stars$(M_{\odot})$	**Burning stage**	**Pressure $(ergs\ cm^{-3}) \times 10^{15}$**	**Specific thermal energy $(ergs\ g^{-1}) \times 10^{15}$**	**Entropy $(ergs\ g^{-1}K^{-1}) \times 10^{6}$**
13	Hydrogen	1.47E+01	03.32	508.12
	Helium	4.17E+02	03.74	94.82
	Carbon	7.53E+05	04.16	16.95
	Oxygen	4.00E+07	10.60	11.62
	Neon	3.38E+07	06.26	07.79
	Silicon	3.54E+08	21.70	10.23
15	Hydrogen	1.70E+01	04.43	622.69
	Helium	3.54E+03	03.96	99.21
	Carbon	5.81E+05	04.22	17.81
	Oxygen	3.54E+07	12.49	13.15
	Neon	2.19E+07	06.16	08.48
	Silicon	3.68E+08	26.62	12.02
20	Hydrogen	1.32E+01	04.43	641.26
	Helium	2.61E+03	04.19	105.52
	Carbon	4.34E+05	04.52	19.16
	Oxygen	3.27E+07	15.11	15.01
	Neon	9.76E+06	07.34	10.84
	Silicon	3.68E+08	26.62	12.02
25	Hydrogen	1.11E+01	04.43	653.63
	Helium	2.15E+03	04.42	110.13
	Carbon	3.18E+05	04.34	19.76
	Oxygen	3.24E+07	29.29	23.48
	Neon	1.19E+07	06.58	09.88
	Silicon	5.17E+08	57.92	22.15
70	Hydrogen	5.56E+00	04.43	704.34
	Helium	1.46E+03	04.65	117.45
	Carbon	3.55E+05	04.57	19.81
	Oxygen	3.27E+07	19.86	17.90
	Neon	1.61E+07	06.57	09.34
	Silicon	4.66E+08	40.62	16.47

Figure 2 represent all the evaluated pressure within the density and temperature ranges: $(0-5)\times 10^{7} g\ cm^{-3}$ and $(0-5)\times 10^{9} K$ respectively.It is important to note that, in all the massive stars; the $e^{\pm}$ pressure explicitly manifest itself at around $2.5\times 10^{9} K$ and rapidly grow to the silicon burning where the density and temperature are relatively high. Table I shows the evaluated pressure, specific energy and entropy found in this work. The low entropy indicates where the temperature and density are relatively high (Fig. 3) for the pair production range near the advanced nuclear reactionsto the stage where collapse of the massive star dominate.

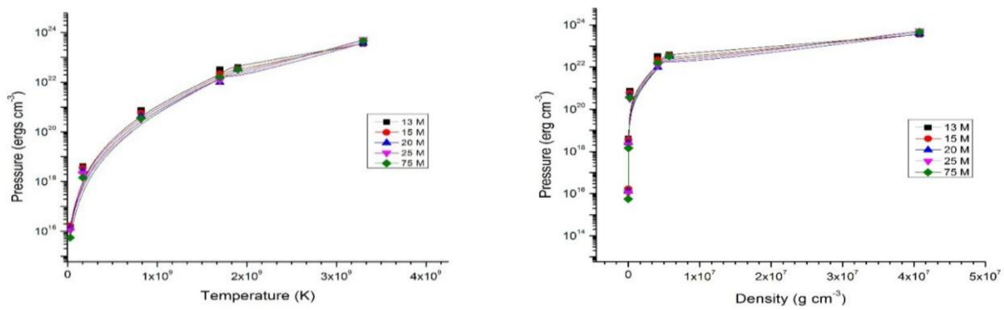

Figure 2: Electron-Positron pressure with stars density & Temperature for ^{1}H,^{4}He,^{12}C,^{16}O,^{20}Ne&^{23}Si

The collapse occur at the Si burning where the internal energy is high and its time is comparably longer than the time for collapse in the central core of the massive stars. The entropy increases in the process of developing the core of the massive stars as can be seen in both the column/bar and line-scatter of Figure 3, whereas, the energy (Fig. 1) and pressure (Fig. 2) remain low at this ignition stage of the nuclear burning. The increase in the entropy however, is macroscopically responsible for the uncertainties in some nuclei to reach the pair production limit.

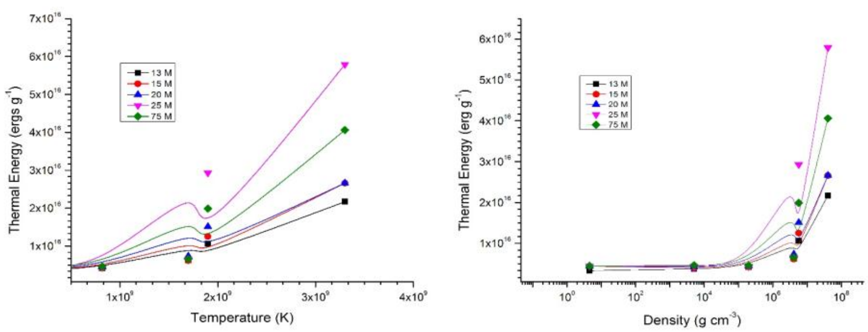

Figure 3: Electron-Positron specific entropy with stars Density & Temperature for ^{1}H,^{4}He,^{12}C,^{16}O,^{20}Ne&^{23}Si

5. Conclusion

Electron-positron pairs greatly affects the evolution and collapse of massive stars. In this paper we have considered the effects of the pressure, specific thermal energy and entropy of the electron-positron pairs in some selected massive stars. We found that, while the pairs are produced at high temperature-specifically within the range $1\times 10^{9} K$ to $4.5\times 10^{9} K$- and relatively low density; there is a critical temperature where the pairs affects the massive stars most.The decrease of energy at this temperature, is responsible for the gravitational contraction of the stars. However, the uncertainties affecting many of the nuclei at the early formation of a massive star is caused by their low energy to reach the pair production limit. This results will not only benefits the research in the evolution of massive stars, but also fill the gap on the understanding of the pair production in the center of the massive stars.

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