# STATISTICAL ANALYSIS OF BREAD INVENTORY, USING THE USEFUL LIFE TIME BASED ORDERING POLICY.

***Izevbizua O. and Emunefe O.J.***

**Department of Mathematics, University of Benin/Department of General Studies PTI Effurun, Delta State.**

***Abstract***

***Bread is a fixed lifetime product with a useful lifetime of 4 days. In this work the useful lifetime ordering policy was applied to the bread inventory and statistical analysis carried out using the statistical software R, to determine the best statistical distribution that fit the demand distribution and the best ordering policy for the inventory manager.***

***Keywords*****:** Bread, lifetime, distributions, ordering, policy.

## 1. Introduction

Bread is one of the most common food item in Nigeria. It is classified as a fixed lifetime product because, it outdate after 4days if not used to meet demand by the inventory mangers and most be disposed off. Consuming outdated bread is dangerous to the consumers. Outdating as it affect the bread inventory is a major concern to the inventory manager. In this work, we use statistical analysis to derive the ordering policy for bread based on the number of useful lifetime remaining on the items on hand.

Data was collected from a shop owner in Benin City (unpublished Ph.D thesis by [1]).The period of observation was between January 8, 2014 – February 9, 2014. The quantity of items received and the demand for each period was recorded as shown in Table1. For simplicity $d_{p_{i,j}} = d_{i,j}$ demand in period $i$ for order $j$. Wherever we have items from two orders together, items from the previous order must be used to meet demand before items from the current order. This is indicated in the Table1 by the ratio sign. The observation showing quantity ordered and periodic demand for items on hand was recorded using useful lifetime ordering policy (that is, order when one useful period remain on the items on hand) .

In Table1 the first order received was 50. The demand in period1 was 15, demand in period2 was10, demand in period3 was 20 and demand in period4 was 16. The number of items brought into period4 from the first order was 5 and these five items must be used first to meet demand in period4 before items from the new order. The process continues.

**Table1: Ordered quantity, demand and outdates for bread Inventory.**

| Day | Order(s) | On hand inventory | Demand | outdates |
|---|---|---|---|---|
| 1 | 1 | 50 | 15 | - |
| 2 | 1 | 35 | 10 | - |
| 3 | 1,2 | 255,30 | 20 | - |
| 4 | 1,2 | 5,30 | 16(5:11) | - |
| 5 | 2 | 19 | 8 | - |
| 6 | 2,3 | 112,35 | 9 | - |
| 7 | 2,3 | 2,35 | 14(2:12) | - |
| 8 | 3 | 23 | 10 | - |
| 9 | 3,4 | 138,60 | 5 | - |
| 10 | 3,4 | 8,60 | 28(8:20) | - |
| 11 | 4 | 40 | 27 | - |
| 12 | 4,5 | 133,32 | 10 | - |
| 13 | 4,5 | 3,32 | 12(3:9) | - |
| 14 | 5 | 23 | 8 | - |
| 15 | 5,6 | 1510,40 | 5 | - |
| 16 | 5,6 | 10,40 | 23(10:13) | - |

Correspondence Author: Emunefe O.J., Email: maorobo@yahoo.com, Tel: +2348062334547 (IO)

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|---|---|---|---|---|
| 17 | 6 | 27 | 10 | - |
| 18 | 6,7 | 178,35 | 9 | - |
| 19 | 6,7 | 8,35 | 6(6:0) | 2 |
| 20 | 7 | 35 | 15 | - |
| 21 | 7,8 | 204,40 | 16 | - |
| 22 | 7,8 | 4,40 | 22(4:18) | - |
| 23 | 8 | 22 | 12 | - |
| 24 | 8,9 | 100,50 | 15(10:5) | - |
| 25 | 8,9 | 0,45 | 19(0:19) | - |
| 26 | 9 | 26 | 10 | - |
| 27 | 9,10 | 163,42 | 13 | - |
| 28 | 9,10 | 3,42 | 20(3:17) | - |
| 29 | 10 | 25 | 14 | - |
| 30 | 10,11 | 110,45 | 12(11:1) | - |
| 31 | 10,11 | 0,44 | 21 | - |
| 32 | 11 | 23 | 18 | - |
| 33 | 11,12 | 570, | 15 | - |

**Empirical Data Analysis.**

We collected demand data for bread as shown in Table1, Using the statistical software R, we computed the relevant statistics for the data and the result is shown in Table2.

**Table 2: Statistics for the demand data for bread**

| S/N | Statistic | Computed value |
|---|---|---|
| 1 | Mode | 10 |
| 2 | Median | 13.5 |
| 3 | Mean | 14.089 |
| 4 | SD | 6.0170 |
| 5 | Variance | 36.20 |
| 6 | Kutorsis | -0.5256 |
| 7 | Skewness | 0.5726 |

Also, the demand rate per period per order is shown inTable3

**Table3: Demand rate per order for bread Inventory.**

| Order | $d_1$ | $d_2$ | $d_3$ | $d_4$ | $t$ | $\lambda$ |
|---|---|---|---|---|---|---|
| 1 | 15 | 10 | 20 | 5 | 50 | 12.5 |
| 2 | 11 | 8 | 9 | 2 | 30 | 7.5 |
| 3 | 12 | 10 | 5 | 8 | 35 | 8.75 |
| 4 | 20 | 27 | 10 | 3 | 60 | 15 |
| 5 | 9 | 8 | 5 | 10 | 32 | 8 |
| 6 | 13 | 10 | 9 | 6 | 38 | 9.5 |
| 7 | 0 | 15 | 16 | 4 | 35 | 8.75 |
| 8 | 18 | 12 | 10 | 0 | 40 | 10 |
| 9 | 19 | 10 | 13 | 3 | 45 | 11.25 |
| 10 | 17 | 14 | 11 | 0 | 42 | 10.5 |
| 11 | 1 | 21 | 18 | 15 | 55 | 13.75 |
| Average | 12.27 | 13.18 | 11.45 | 4.64 | | |

Next, we determine the demand distribution for the data in Table3.

We pass the data through a series of known distributions to determine the best fit. Using the Mathematical software R, we computed the AIC (Akaiki Information Criterion) and the k-statistic (Kolmogorov-Smirnov statistic) for the data (these are performance evaluation measures used to determine best fit) and thereafter plot the histogram and Q-Q plot for the data.

Three of the known statistical distributions were able to fit the data. They are the Gamma distribution, the Weibull distribution and the exponential distribution.

**Exponential distribution is as follows**

$$pdf = f(x) = \lambda e^{-\lambda x},\ x > 0 \ and \ \lambda > 0$$

$$CDF = F(x) = 1 - e^{-\lambda x},\ x > 0 \ and \ \lambda > 0$$

$$where \ \lambda = rate \ \ parameter$$

**Weibull distribution is as follows**

$$pdf\ =f(x)=\frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1}\exp\{-(\frac{x}{\alpha})^{\beta}\},\ \beta>0,\ x>0\ \ and\ \ \alpha>0$$

$$CDF=F(x)=1-\exp\{-(\frac{x}{\alpha})^{\beta}\},\ \beta>0,\ \ x>0\ and\ \alpha>0$$

$\alpha = scale\ \ parameter,\ \beta = shape\ \ parameter,$

**Gamma distribution is as follows**

$$pdf\ =f(x)=\frac{(\frac{x}{c})^{\alpha}\exp\{-(\frac{x}{c})\}}{x\Gamma(\alpha)},\ x>0,\ \alpha>0,\ \ c>0$$

$$CDF=F(x)=\zeta(\alpha,(\frac{x}{c}))$$

$\alpha = shape\ \ parameter,\ c = scale\ \ parameter,\ \Gamma(.)\ \ gamma\ \ function\ \ and\ \ \zeta(.,.)\ incomplete\ \ gamma\ \ function.$

The AIC and k-statistics for the three distributions were computed and are as follows; Gamma distribution (AIC=217.5775, k-satistic=0.1154), Weibull distribution (AIC=218.9821, k-statistic=0.1195) and exponential distribution (AIC=249.8831, k-statistic=0.3450). Also the histogram plot and Q-Q plot for the data in Table3 are shown in Figure1.

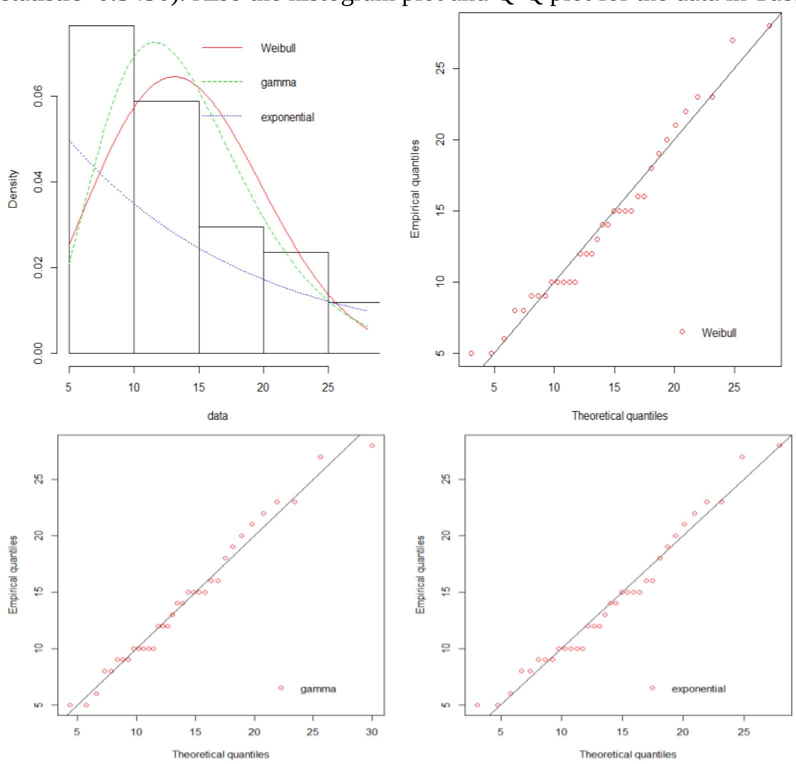


Figure1. Histogram and Q-Q plots for data in Table3.

All of these distributions belongs to the family of life distributions and are suitable for our analysis. The distribution with the lowest AIC value is best for the data.

Next using the total cost function

$$C(x,y)=\min_{y\geq 0}\{ky+h\int_{0}^{x+y}(x+y-t)f^{*}(t)dt+v\int_{x+y}^{\infty}(t-(x+y))f^{*}(t)dt+\theta\int_{0}^{y}(y-t)f^{*}(t)dt\}$$

*where*

$f^{*}(t)=\ distribution\ \ of\ \ total\ \ demand\ \ in\ \ periods\ 1,2,\ .\ .\ .\ m..\ that\ \ is$

$f^{*}(t)\ is\ \ distribution\ of\ \ t=\sum_{i=1}^{m}d_{i}$

obtained by [2], we have.

$$\bar{\lambda} = \sum_{i=1}^{4} average\ demand\ in\ period\ i\ for\ 11\ orders$$

$$12.27 + 13.18 + 11.45 + 4.64$$

$$41.54$$

$$with\ k = 120, v = 40, \theta = 30, h = 2, \bar{\lambda} = 41.54,\ we\ have$$

$$y = 42$$

Therefore, we order 42 when the useful life remaining is one period.

**Conclusion**

Statistical analysis, using the statistical software R can be used to determine the best statistical distribution for the demand data of a fixed lifetime product. Also, results from the statistical analysis can be used to derive the ordering policy, which determine how much to order and when to order. This will help minimize the quantity of products outdating in a fixed lifetime inventory system.

References.

[1] Unpublished Ph.D thesis (2017) by Izevbizua Orobosa. Department of Mathematics University of Benin, Benin City.

[2] **Izevbizua O.** and Emunefe, O.J (2017). Comparing the ordering policies of useful lifetime based and quantity based, fixed lifetime inventory system. The Transactions of the Nigeria Association of Mathematical Physics. Vol. 5, PP 51-60.